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THE APPLICATION OF MOTOR INPUT VOLTAGE FEEDBACK IN CONTROL SYSTEM COMPENSATION

ALBERT K. GLOVER, JR.
and
ALBERT W. HOUSTON

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THE [illegible]TION OF MOTOR INPUT [illegible] FEEDBACK TO CONTROL SYSTEM COM[illegible]TION

* * * * *

Albert F. Glover, Jr.

and

Albert [illegible]. Houston

[illegible]

by

Albert [illegible]. Flowers, Jr.

Lieutenant, United States Navy

and

Elliott [illegible]. Houghton

Lieutenant, United States Navy

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1 9 6 2

ABSTRACT

Many varied methods may be used to compensate a control system. This paper is a study of the effects of using the motor input voltage as a feedback quantity. It uses a unity feedback positioning servomechanism as the uncompensated system and investigates the effect on root location with the motor input voltage fed back through various compensation devices, such as a lag or lead network.

Only type zero and type one systems are considered. These vary from second order to fourth order. Root loci approach is used with some of the systems checked on the analog computer.

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1. Introduction.

The purpose of this project was to investigate and report on various means and methods of using motor input voltage as a method of compensating a control system. Various methods of analysis were considered, but the root locus approach was decided upon, with various systems checked by analog computer. The obvious, hoped-for result would then be a series of curves from which a practicing servo engineer might obtain the best type of compensator for his particular case and also a good idea of the range of values to use.

To present the problem, a standard block diagram was drawn with each block a component in itself which could be varied to suit each individual case. This is as follows:

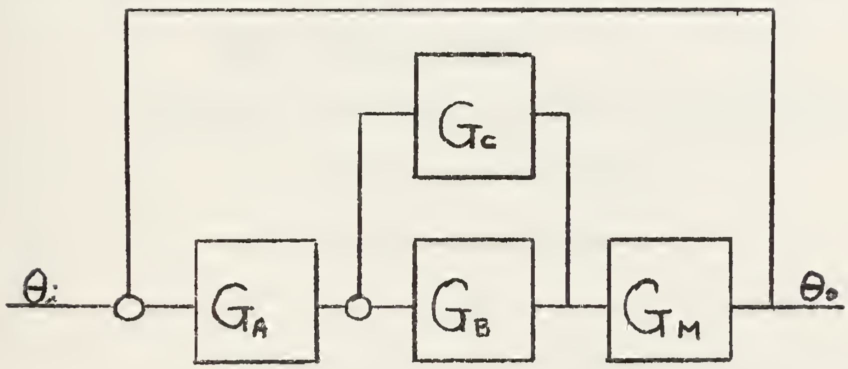


Figure 1-1

where G_m = motor function

G_a and G_b = system components

G_c = motor voltage feedback compensating device

In other words, the input to the G_m box is the motor voltage supply and this voltage is fed back through some type of compensator as shown. The actual functions of G_c and G_a may be varied according

to the order of the system and to the number of time constants desired to feed around. These might be anything from amplifiers to amplidynes.

The problem was approached as if the servo was a position output type; however it could represent other types. The "motor" function was also simplified to the point where it represents both the motor and a standard mechanical system (if the motor inductance is assumed to be negligible). Thus, $G_m = \frac{1}{s(s+1)}$. Although this actually represents a special or individual case, the value of the gain and time constant give a somewhat "normalized" function. The values of the other function gains and time constants were also chosen with this in mind, but varied in an effort to minimize any pole and zero cancellation. For a further discussion of a normalizing method of approach, see section 10.

A numbering system was then chosen to represent the various systems investigated. This consisted of a four digit code as follows:

<u>1st</u> <u>2nd</u> <u>3rd</u> <u>4th</u>

where <u>1st</u> designates the type of system as:

0 = type zero system

1 = type one system

3 = special case of a quadratic in the G_m function

<u>2nd</u> designates the type of function in the G_b box as:

0 = 10

1 = $\frac{10}{s+2}$

2 = $\frac{s+1.5}{s+2}$

3 = $\frac{s+4}{s+2}$

4 = $\frac{(s+4)(s+3)}{s+2}$

5 = $\frac{s+2}{(s+3)(s+4)}$

6 = $\frac{10}{s+2}$ but $G_a = 10(s+3.5)$ and $G_m = \frac{1}{s(s+1)}$

$7 = \frac{10}{s+2}$ but $G_a = 10$ and $G_m = \frac{s+3.5}{s(s+1)}$

$8 = \frac{10}{(s+3)(s+4)}$

$9 = \frac{10}{(s+1.2)(s+3)(s+4)}$

<u>3rd</u> designates the type of compensator G_c as:

$1 = k_c \frac{s+a}{s+1}$

$2 = k_c \frac{s+a}{s+4}$

$3 = k_c (s+a)$

$4 = k_c (s^2+a)$

$5 = k_c \frac{(s^2+5s+a)}{s+4}$ (referred to as "50" compensator)

$6 = k_c \frac{(s+a)}{(s+3)(s+4)}$ (referred to as "60" compensator)

<u>4th</u> designates the value of the variable, a in the compensator.

For example, the system 1242 means:

1st digit = 1 or type one or $G_m = \frac{1}{s(s+1)}$

2nd digit = 2 or $G_b = \frac{s+1.5}{s+2}$

3rd digit = 4 or $G_c = k_c(s^2+a)$

4th digit = 2 or $a = 2$

or in block diagram form, system 1242 is:

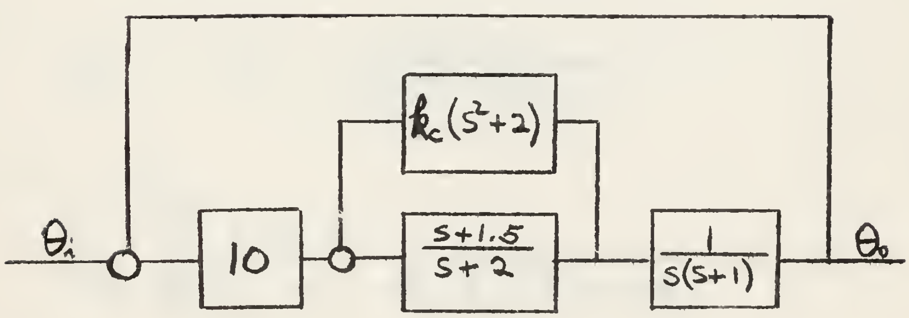


figure 1-2

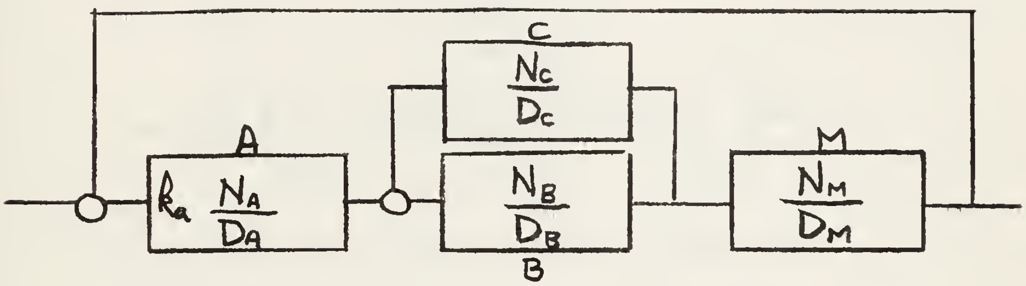
In addition, a block diagram is included for each system discussed in sections 2 through 4.

Two methods of analysis by root locus approach were considered. These were as follows:

a) Method I

F_{o_u} $\triangleq$ open loop transfer function for uncompensated system

F_{o_c} $\triangleq$ compensated open loop transfer function



$$F_{o_u} = k_a G_A G_B G_M = k_a \frac{N_A N_B N_M}{D_A D_B D_M}$$

$$F_{o_c} = \frac{k_a G_A G_B G_M}{1 + G_B G_C} = \frac{k_a \frac{N_A}{D_A} \frac{N_B}{D_B} \frac{N_M}{D_M}}{1 + \frac{N_B N_C}{D_B D_C}}$$

$$F_{o_c} = \frac{k_a N_A N_B N_M}{D_A D_B D_M} \times \frac{D_B D_C}{D_B D_C + N_B N_C}$$

or
$$F_{o_c} = F_{o_u} \times \frac{D_B D_C}{D_B D_C + N_B N_C}$$

or for roots:

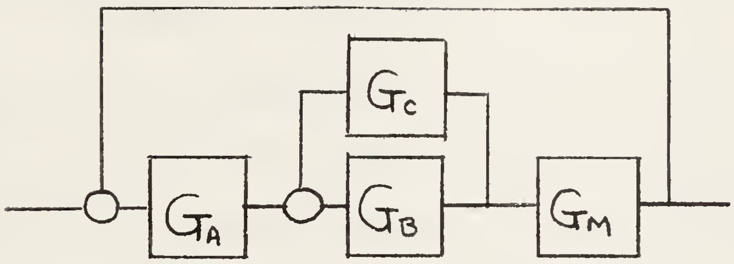
$$F_{o_u} \frac{\prod^{B,C} (s+p)}{\prod^{B,C} (s+p) + k_B k_C \prod^{B,C} (s+z)} = -1$$

and k_a may be used as the variable.

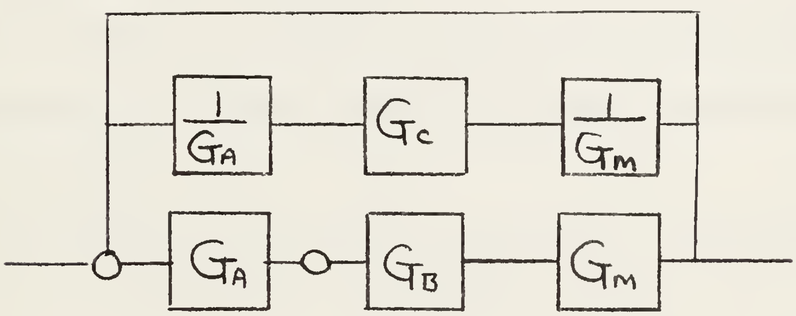
Some of these loci were drawn and are shown in appendix C. However,

this method has the disadvantage of having the compensator function lost in the algebra. It would be in the interest of clarity if the locus equation were to have G_c in series so that its poles and zeros could then be superimposed on the systems. Therefore, method II, which follows, was decided upon.

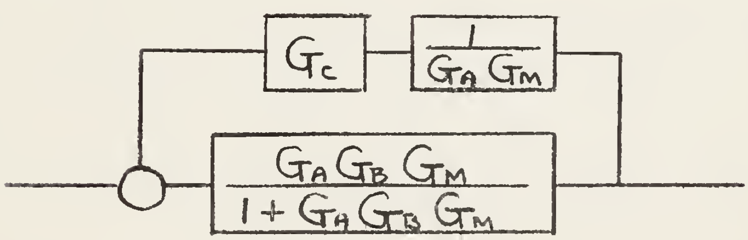
(b) Method II



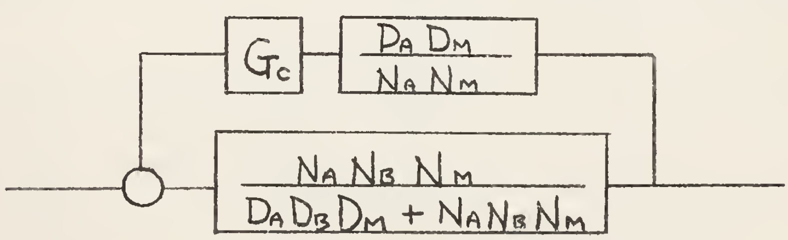
This may be manipulated by block algebra to:



and also to:



or:



or for roots:

$$\frac{\cancel{N_A} N_B \cancel{N_M}}{D_A D_B D_M + N_A N_B N_M} \times \frac{D_A D_M}{\cancel{N_A} \cancel{N_M}} \times G_c = -1$$

or:

$$\frac{D_A D_M N_B}{D_A D_B D_M + N_A N_B N_M} G_c = -1$$

The variable may be chosen as the coefficient of the G_c function (k_c). This is the system which has been used as the basis for the analyses. These locus equations were set up on the digital computer with k_c as the variable. Their plots are given in sections 2 through 8 along with a discussion of each.

Also included on the loci are plots of constants k_c and K_v. The derivation of K_v is given in appendix B and gives an indication of velocity lag error.

Analog computer checks were made on some of the systems with both lead, lag or derivative type compensators. Tapes showing the servo output to a step input are included in section 9 along with a discussion of their significance.

Ten basic systems were investigated for compensation purposes. Seven of these systems were initially unstable, while the other three, although stable, were not so to any great degree. In other words, they had a small value for ζ. The type and order of the ten systems, according to the four digit code previously explained, were as follows:

1. Type 0, Third order : 0100
2. Type 1, Second order: 1000
3. Type 1, Third order : 1100, 1200, 1300, 1400

4. Case 1, fourth order : 1[illegible]00, [illegible]

5. Case 1, fifth order : 1900, [illegible]

Compensation of each of these systems was accomplished with the previously listed compensators. These are also listed in table 1-1 along with a physical description of what might be incorporated with these compensators.

After the root loci of the systems investigated were plotted, an initial inspection and comparison revealed the fact that analysis can best be conducted by separating the systems into groups. The most obvious groupings would have been by the order [illegible] the motor function. However, it was apparent from inspection that the curves very closely resembled each other when grouped by the excess of the number of poles over the number of zeros of the h_p function. This was therefore used as the criteria for group analysis, and the remaining sections are concerned with the compensation of these groups. They are shown in table 1-2.

TABLE 1-1

G_m	Description	Digit Type
$k_c \frac{s+a}{s+b}$	Lead network, $b > a$	10 or 20
$k_c \frac{s+a}{s+b}$	Lag network, $a > b$	10 or 20
$k_c (s+0)$	First derivative feedback	30 $(a=0)$
$k_c (s+a)$	First derivative plus proportional feedback	30 $(a \neq 0)$
$k_c (s^2+0)$	Second derivative feedback	40 $(a=0)$
$k_c (s^2+a)$	Second derivative plus proportional feedback	40 $(a \neq 0)$
$k_c \frac{(s+a)}{(s+3)(s+4)}$	Lead or lag network in series with low band pass filter	60
$k_c \frac{(s^2+5s+a)}{(s+4)}$	Lead or lag network in series with derivative plus proportional feedback	50

[illegible] 1-2

[illegible]	[illegible]	[illegible]	$\Sigma P - \Sigma Z$ of d_b	[illegible]
I	0	-	-	0100
II	1	3	-	2600
III	1	2	-1	1400
IV	1	2	0	1000 1200 1300
V	1	2	+1	1100 1500
VI	1	2	+2	1800
VII	1	2	+3	1900

2. Group I.

A. General.

This group differs from all the remaining groups in that it is a type zero system, which could represent a speed regulator. It consists of system 0100, a block diagram of which is shown in figure 2-1. This system also differs somewhat from the remaining systems in that the gain of the G_h box was increased to 100 to insure instability of the uncompensated system. Thus the true value of the type of compensator would be more easily recognized.

B. Completely satisfactory compensators.

(1) Lead network.

The effect of this compensator is shown in figures 2-2 and 2-3 for $\underline{a} \leq 1$ and $\underline{a} \leq 4$ respectively. Although it is capable of stabilizing the system, the range of ζ's obtainable is very limited. In general, the following characteristics apply to the system: (a) Increasing the value of the compensator pole tends to extend the range of obtainable ζ's. This is, in effect, increasing the ratio of pole to zero and is limited in practical respects to about 10 to 1. (b) There is a minimum value of computer gain, k_{cr}, necessary in each case to stabilize the system. These values are given in table 2-1. Increasing k_c beyond this value tends to increase the stability. (c) For a given value of k_c, the smaller the value of $\underline{a}$, the larger is the obtainable ζ. This will also tend to make ω_n smaller.

(2) First derivative feedback.

This locus is shown in figure 2-4 (for $\underline{a} = 0$). This compensator gives a complete range of obtainable ζ's from zero to one and thus gives the designer considerable flexibility. For the given system there is a minimum compensator gain of $k_{cr} =$ 0.2. Increasing

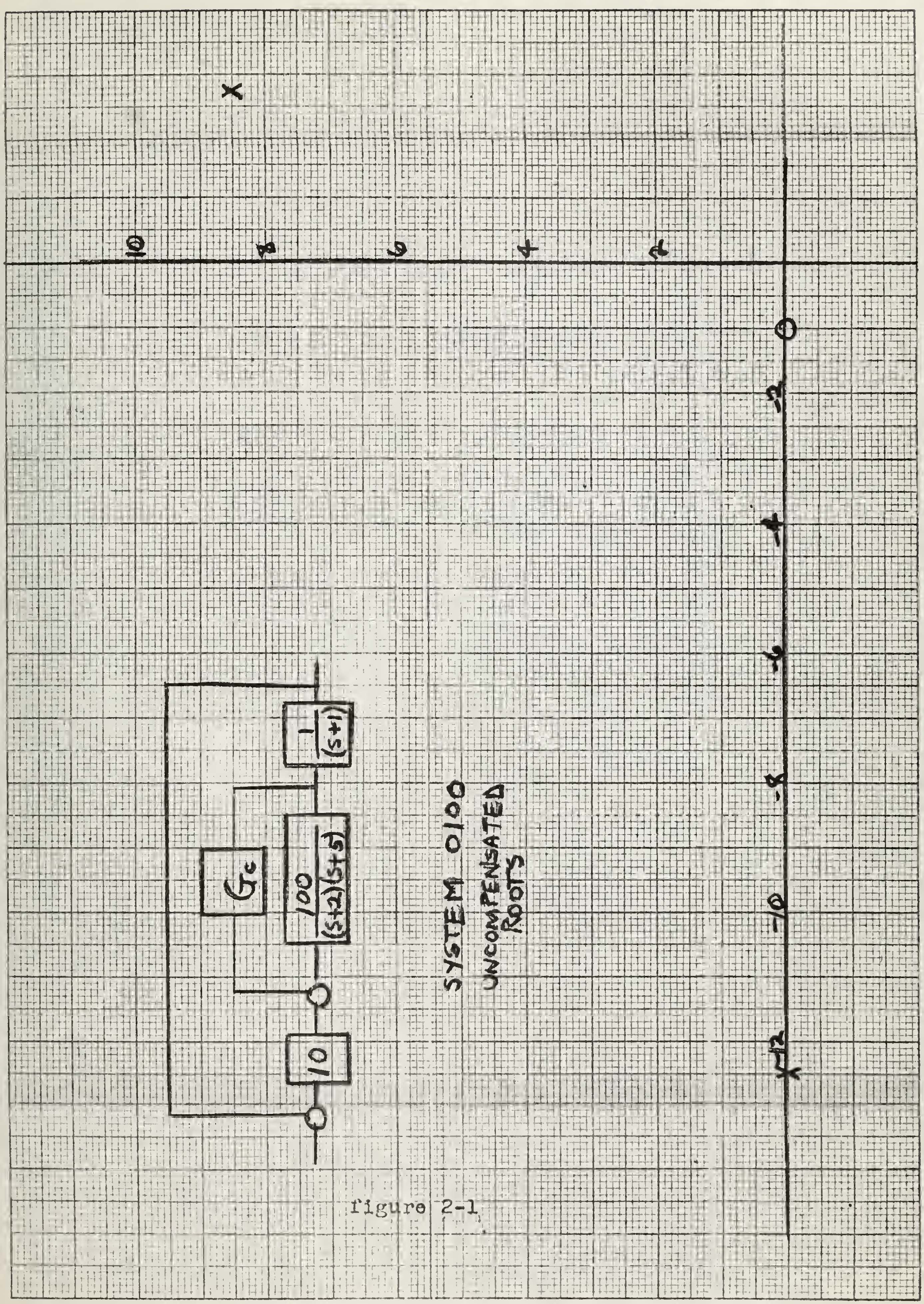

SYSTEM 0100
UNCOMPENSATED ROOTS
Gc
10
100/(s+2)(s+5)
1/(s+1)
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12

figure 2-1

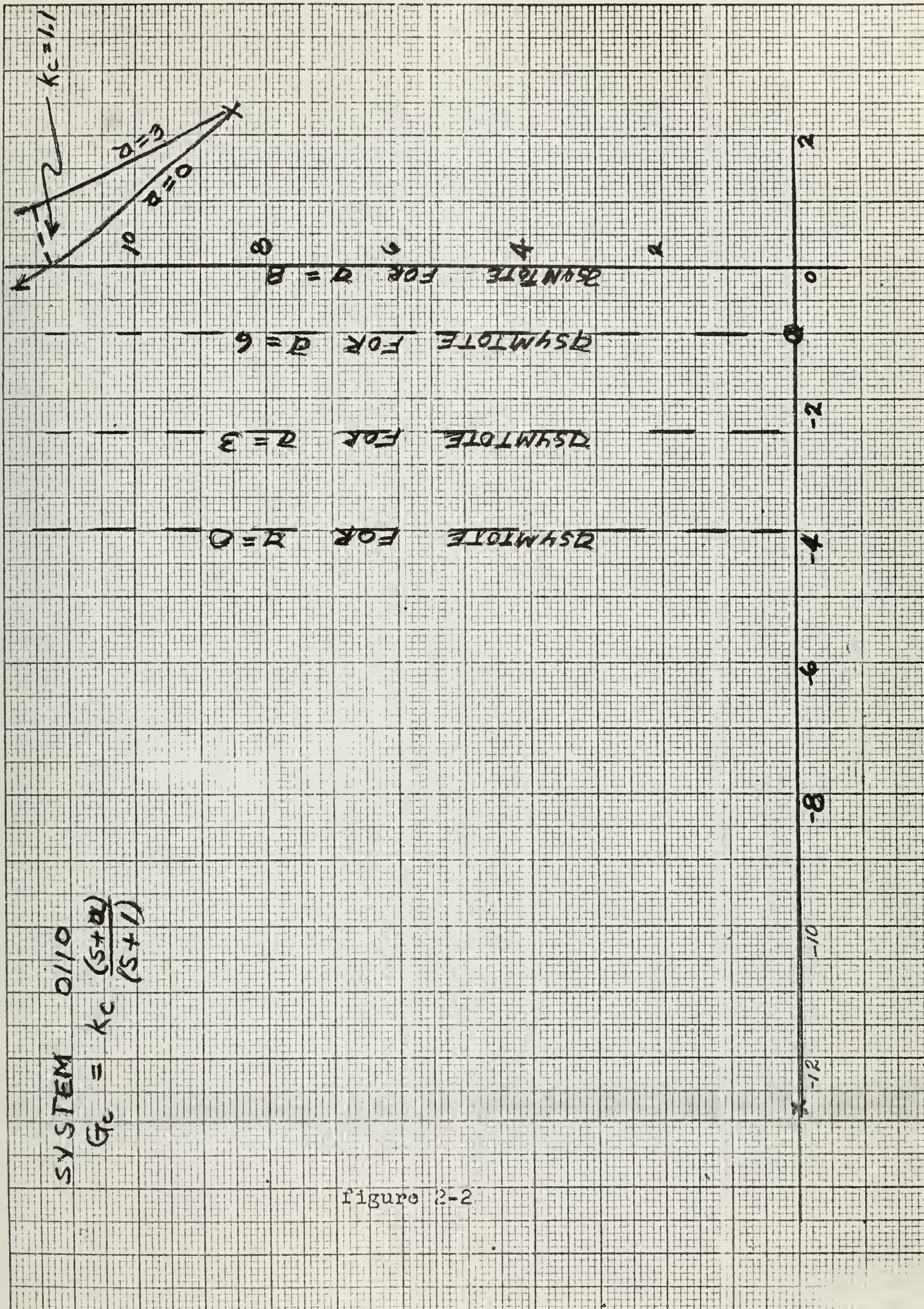

SYSTEM 0110
$G_c = K_c \frac{(s+a)}{(s+1)}$
$K_c = 1.1$
ASYMTOTE FOR a = 8
ASYMTOTE FOR a = 6
ASYMTOTE FOR a = 3
ASYMTOTE FOR a = 0

figure 2-2

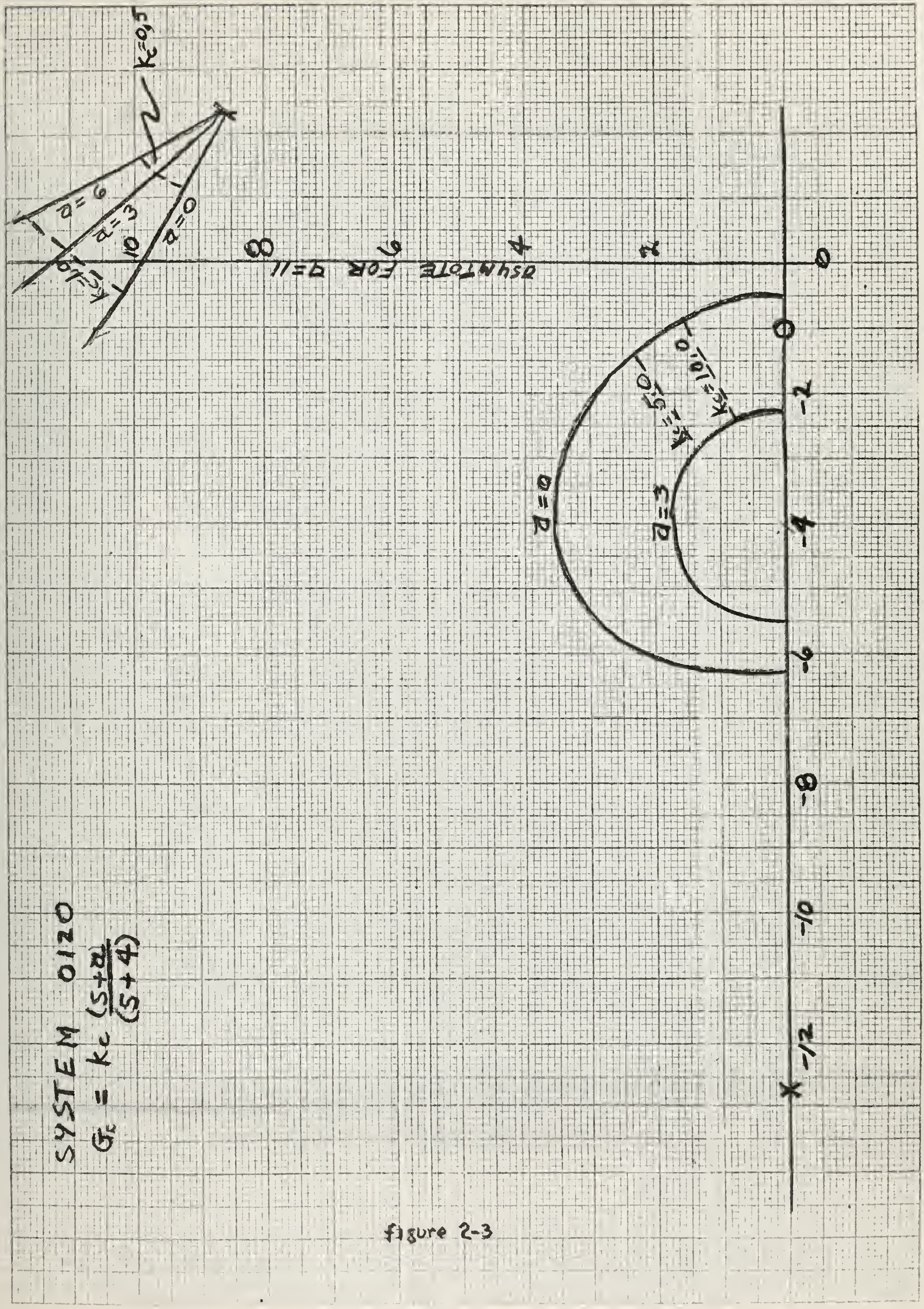

SYSTEM 0120
$G_c = K_c \frac{(s+a)}{(s+4)}$
asymptote for a=11
$K_c=0.5$
a=6
a=3
a=0
$K_c=1.0$
10
8
6
4
2
0
a=0
a=3
$K_c=5.0$
$K_c=10.0$
-2
-4
-6
-8
-10
-12

figure 2-3

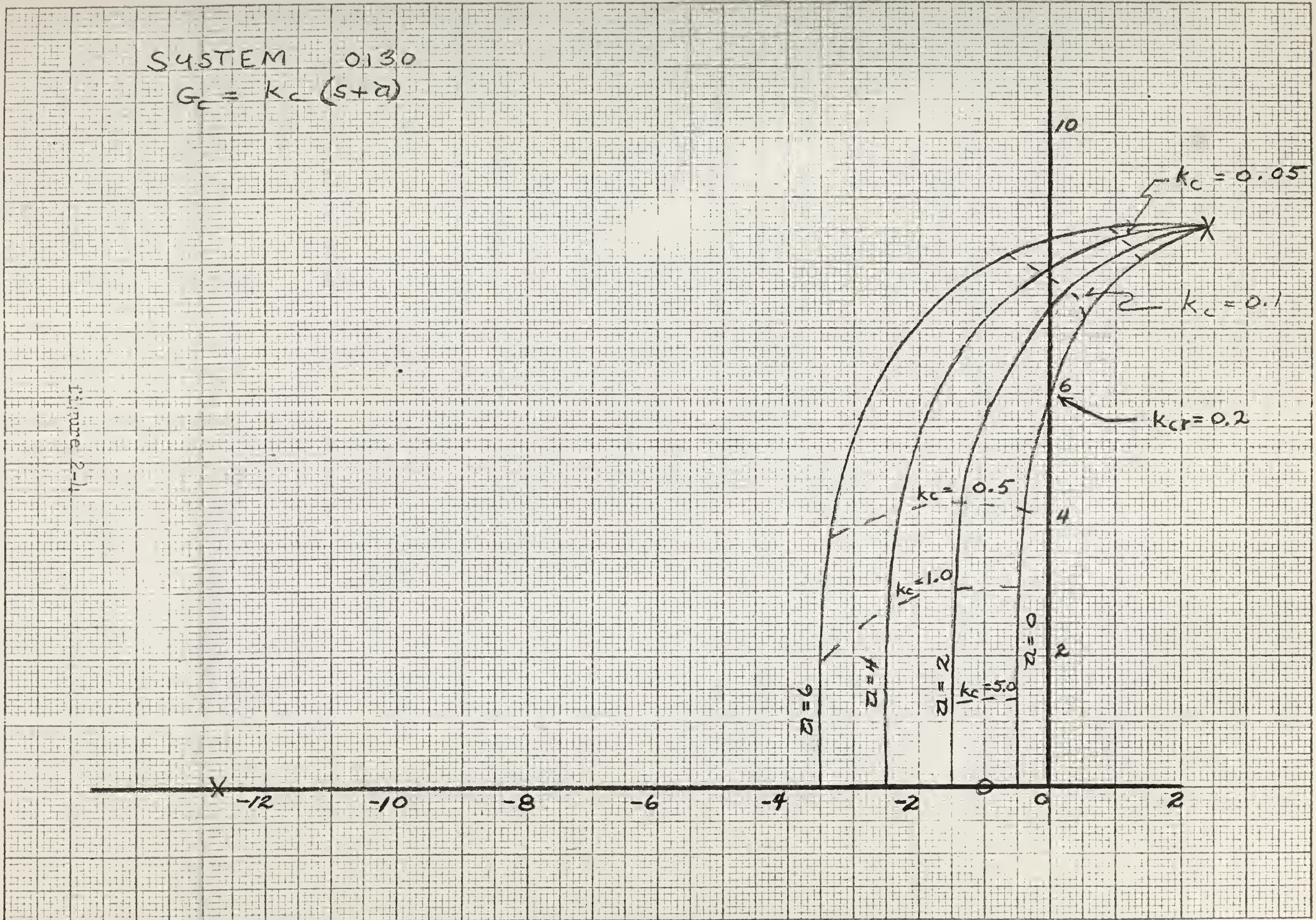

SYSTEM 0130
$G_c = k_c(s+a)$
$k_c = 0.05$
$k_c = 0.1$
$k_{cr} = 0.2$
$k_c = 0.5$
$k_c = 1.0$
$k_c = 5.0$
$a = 6$
$a = 4$
$a = 2$
$a = 0$
10
6
4
2
-12
-10
-8
-6
-4
-2
0
2

Figure 2-4

the gain beyond this value will increase ζ and decrease ω_n. The bandwidth, however, is relatively limited for a ζ between .4 and .7.

(3) First derivative plus proportional feedback.

This was, by far, the more flexible compensator investigated. The loci, with a variable, were similar to the locus of pure first derivative feedback but with increased ω_n. They are shown in figure 2-4. This gives a much wider available bandwidth. In general, increasing k_c increases ζ and decreases ω_n. Increasing a increases ω_n and increases ζ.

(4) "50" compensator.

The loci for this compensator are shown in figure 2-5. These loci are very similar to the first derivative plus proportional feedback curves, but are much more severely limited in the range of bandwidths obtainable. However, any ζ may be obtained by increasing k_c, provided the smaller bandwidth is acceptable.

C. Partially satisfactory compensators.

(1) Lag network.

These loci are shown in figures 2-2 and 2-3 for $a > 1$ and $a > 4$ respectively. The stabilizing ability of these compensators, however, is very limited to values of a not much greater than the compensators pole. For example, the maximum limit of a in figure 2-2 is 8. For $a \geq 8$, the system is completely unstable. In addition, there is a minimum value of k_{cr} which must be met to make the system stable with other values of a. The compensated system is also severely limited to small values of ζ. In general, this is not a very satisfactory compensator.

D. Unsatisfactory compensators.

The following compensators were considered completely unsatisfactory

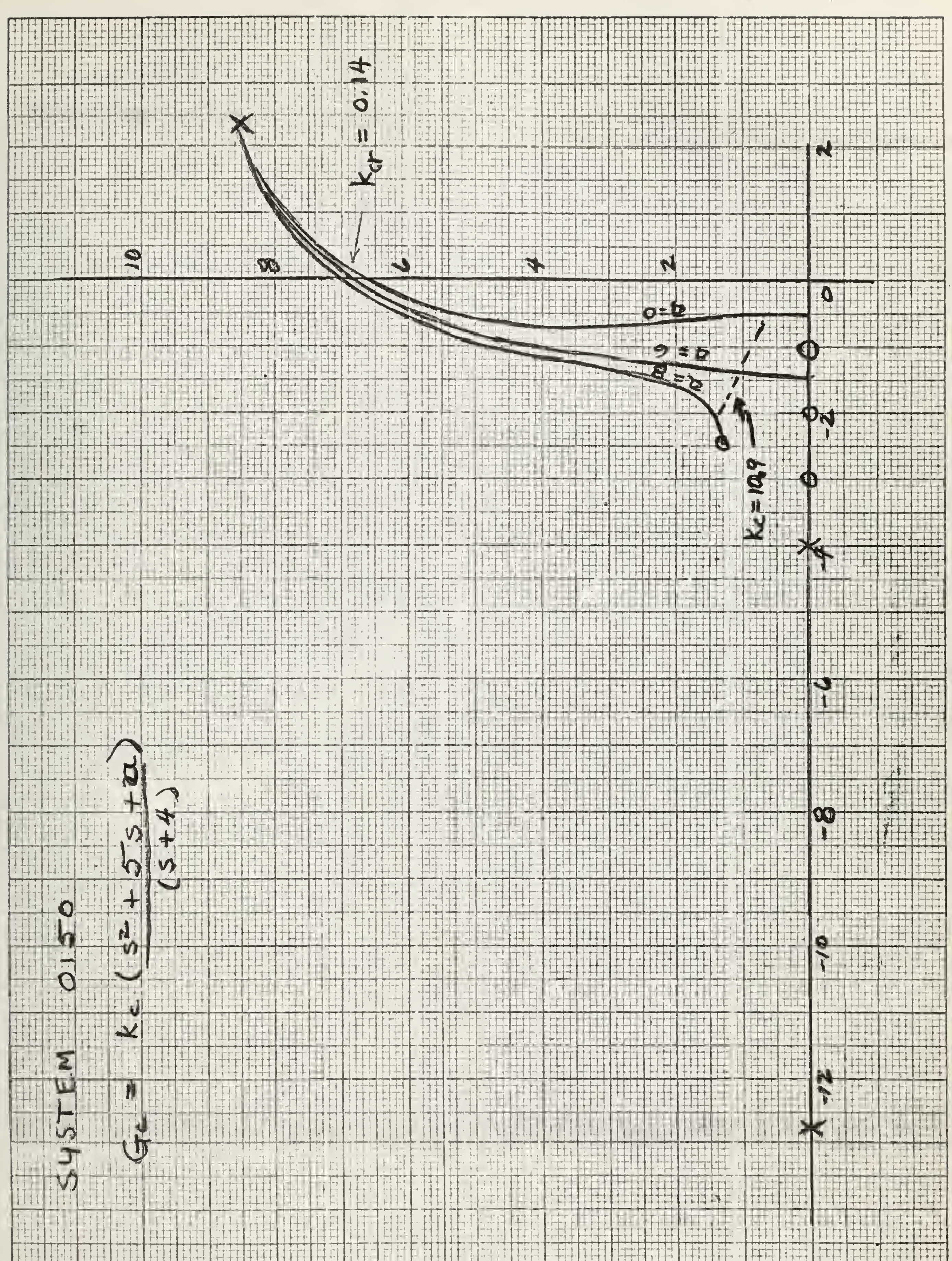

SYSTEM 0150
$G_c = K_c \frac{(s^2+5s+a)}{(s+4)}$
$K_{cr} = 0.14$
$K_c = 10.9$
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12

figure 2-5

due to the [illegible] stable [illegible]:

(1) Second derivative feedback (figure 2-6 with $\underline{g} = 0$).

(2) Second derivative plus proportional feedback (figure 2-6 with $\underline{g} \neq 0$).

(3) "[illegible]" compensator (figure 2-7).

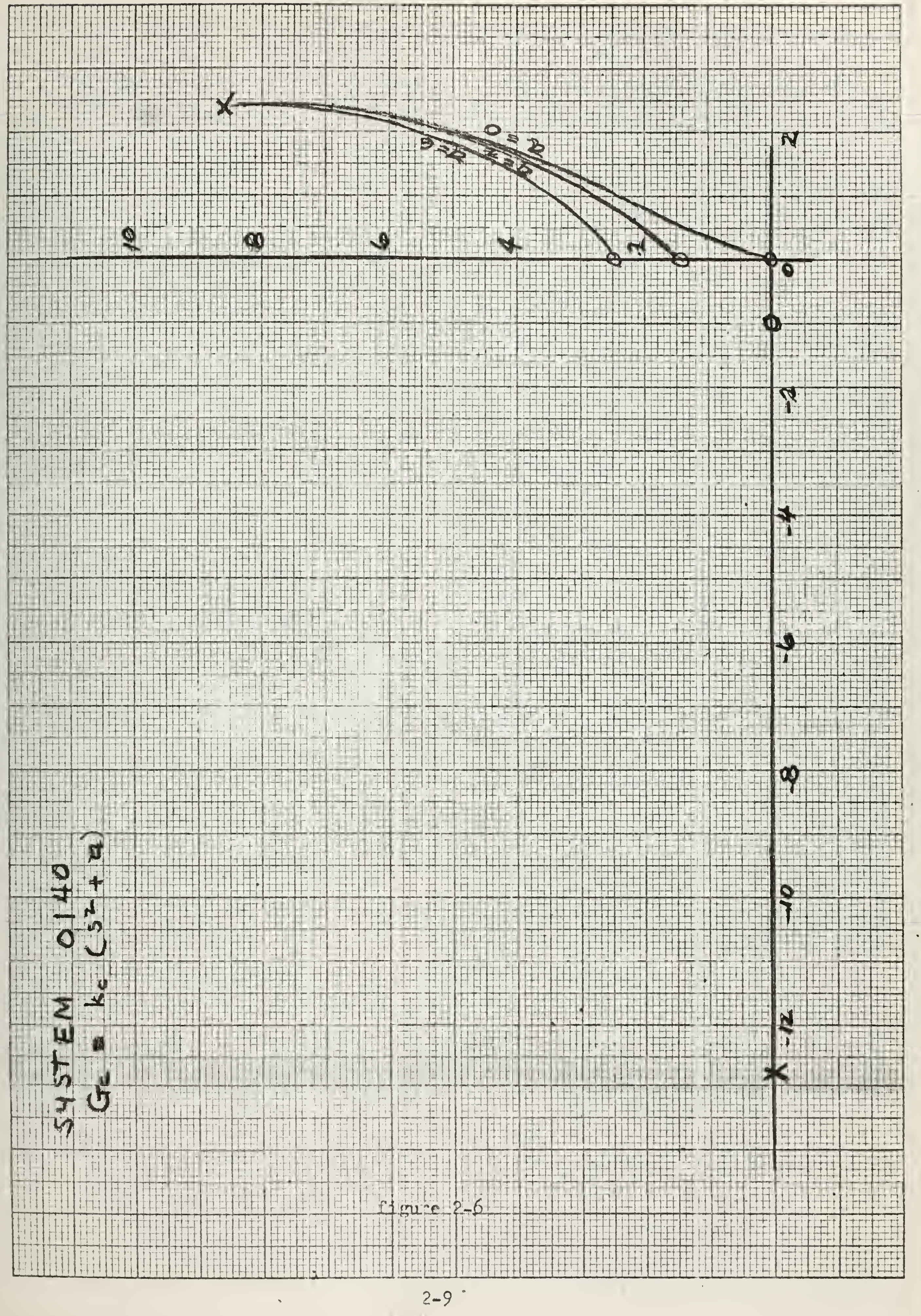


figure 2-6

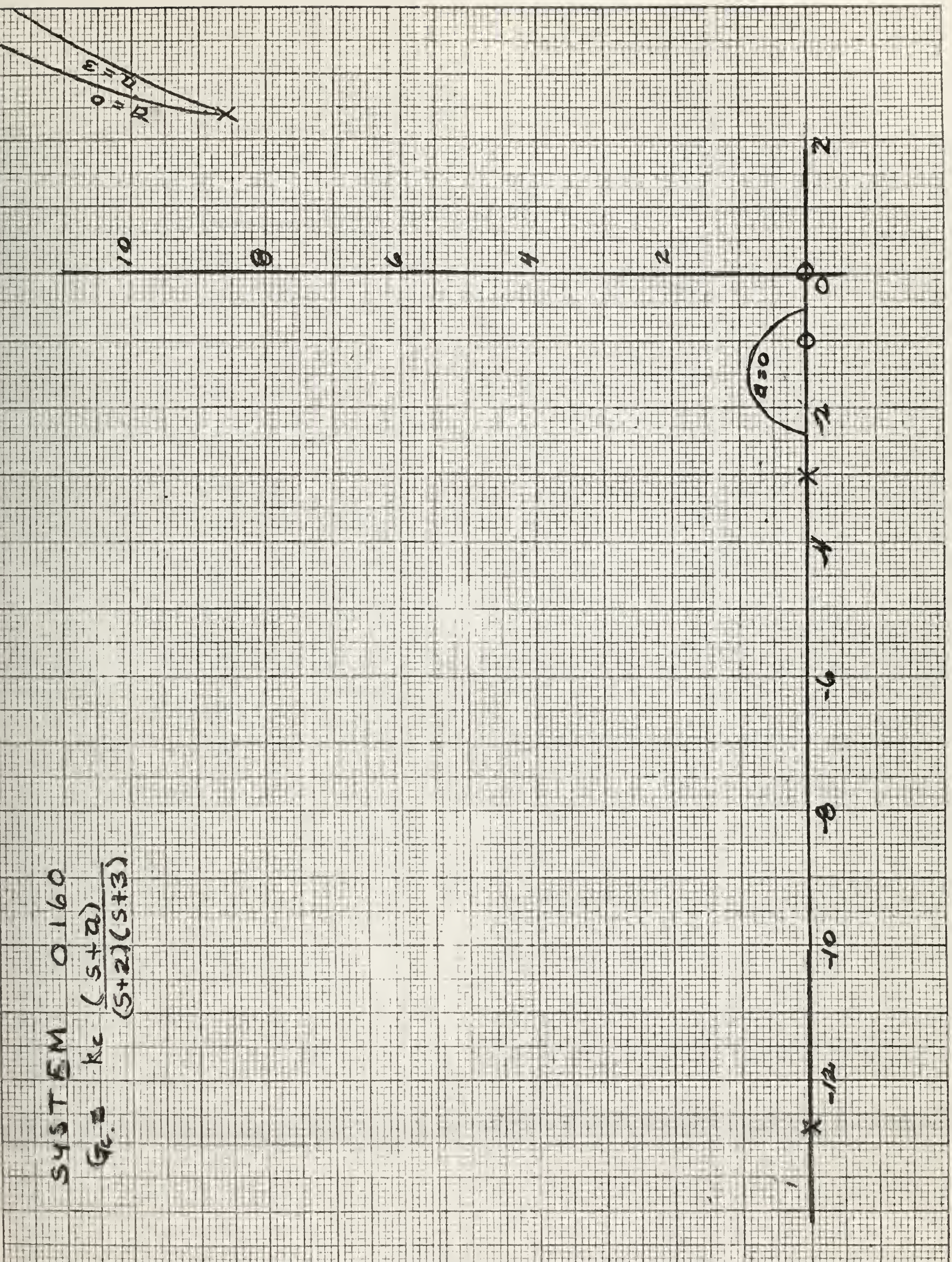


figure 2-7

[illegible]	[illegible]	c
10	[illegible]	1.0[illegible]
1[illegible]	[illegible]	1.[illegible]
[illegible]	[illegible]	0.[illegible]
[illegible]	[illegible]	1.[illegible]
[illegible]		1.[illegible]
[illegible]	[illegible]	0.19
[illegible]	[illegible]	[illegible].11[illegible]
[illegible]	[illegible]	. 85
[illegible]	–	unstable
[illegible]	[illegible]	[illegible].1[illegible]
[illegible]	[illegible]	[illegible].1[illegible]
[illegible]	–	unstable

3. Group II - type one system with third order motor function.

A. General.

The only system investigated which falls into this group is system 3800. The block diagram for this system is illustrated in figure 3-1. Also included in figure 3-1 are the roots of this system.

Purposely, a gain of 100 was selected for the uncompensated system in order to position the roots to insure initial instability. Thus stability is the primary objective in compensating this system; while the ability to vary the ζ and ω_n of the stable roots is also to be considered.

B. Completely satisfactory compensators.

Two of the compensators investigated were completely successful in stabilizing the system. A brief analysis of the effects produced by these compensators follows:

(1) "30" compensator with a not equal to zero.

The use of a combination of first derivative and proportional feedback provides effective compensation of the 3800 system. Not only does its influence stabilize the system, but also it presents the designer with a relatively favorable choice of root locations.

As shown in figure 3-2, stability occurs in the compensated system for every value of a, the proportional feedback component of the compensator. However, in order for stability to occur the compensator's gain, k_c, must be greater than a minimum value, k_{cr}. These values of k_{cr}, which vary with a, are listed in table 3-1.

The values of the design parameters ζ and ω_n that are available to the designer through use of this compensator depend to a great extent on the variable a. If ζ is maintained constant, ω_n, can be increased by increasing a. On the other hand if k_c is maintained

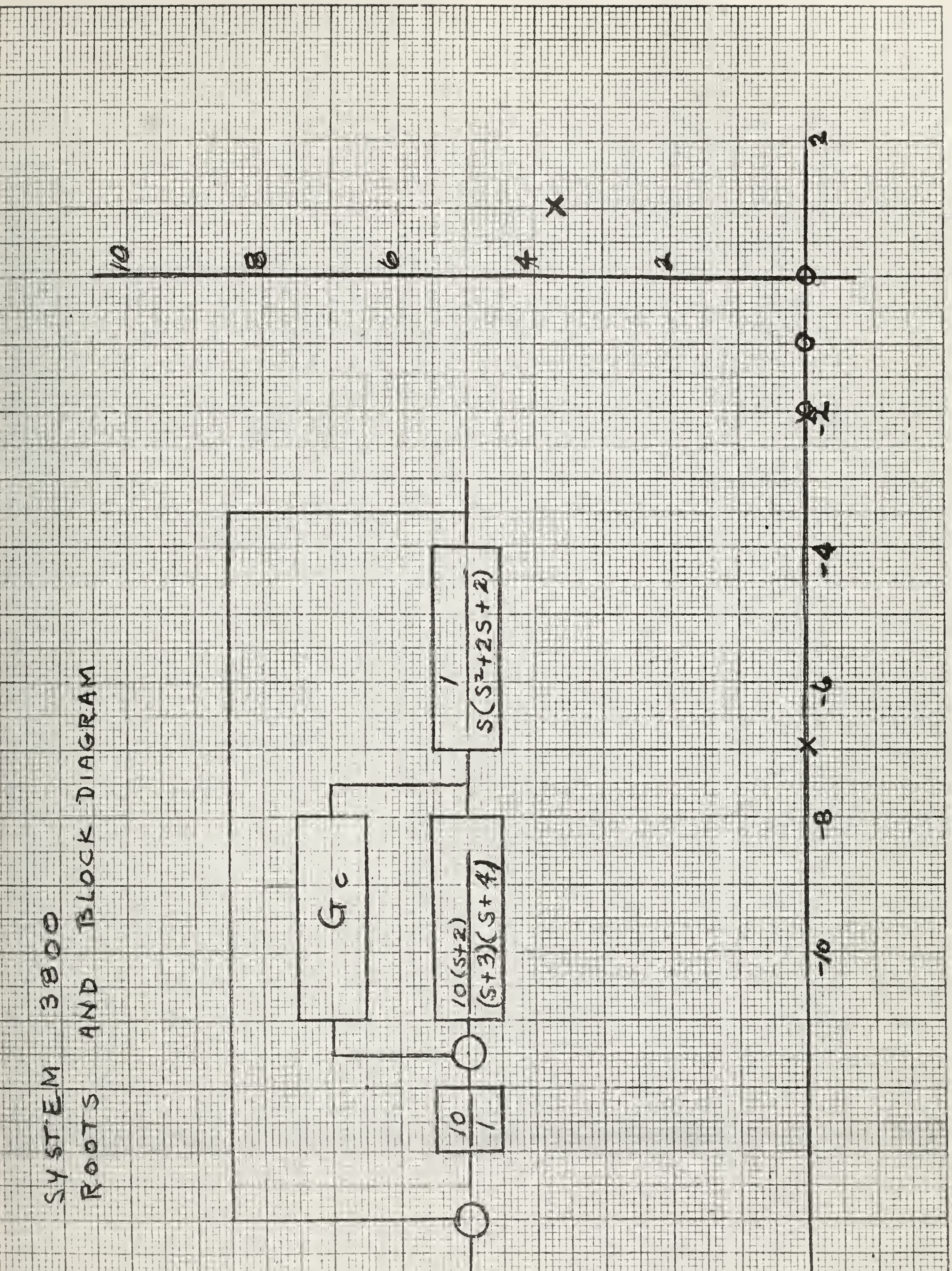

SYSTEM 3800
ROOTS AND BLOCK DIAGRAM
10/1
Gc
10(S+2)/(S+3)(S+4)
1/S(S²+2S+2)
10
8
6
4
2
-10
-8
-6
-4
-2
2

figure 3-1

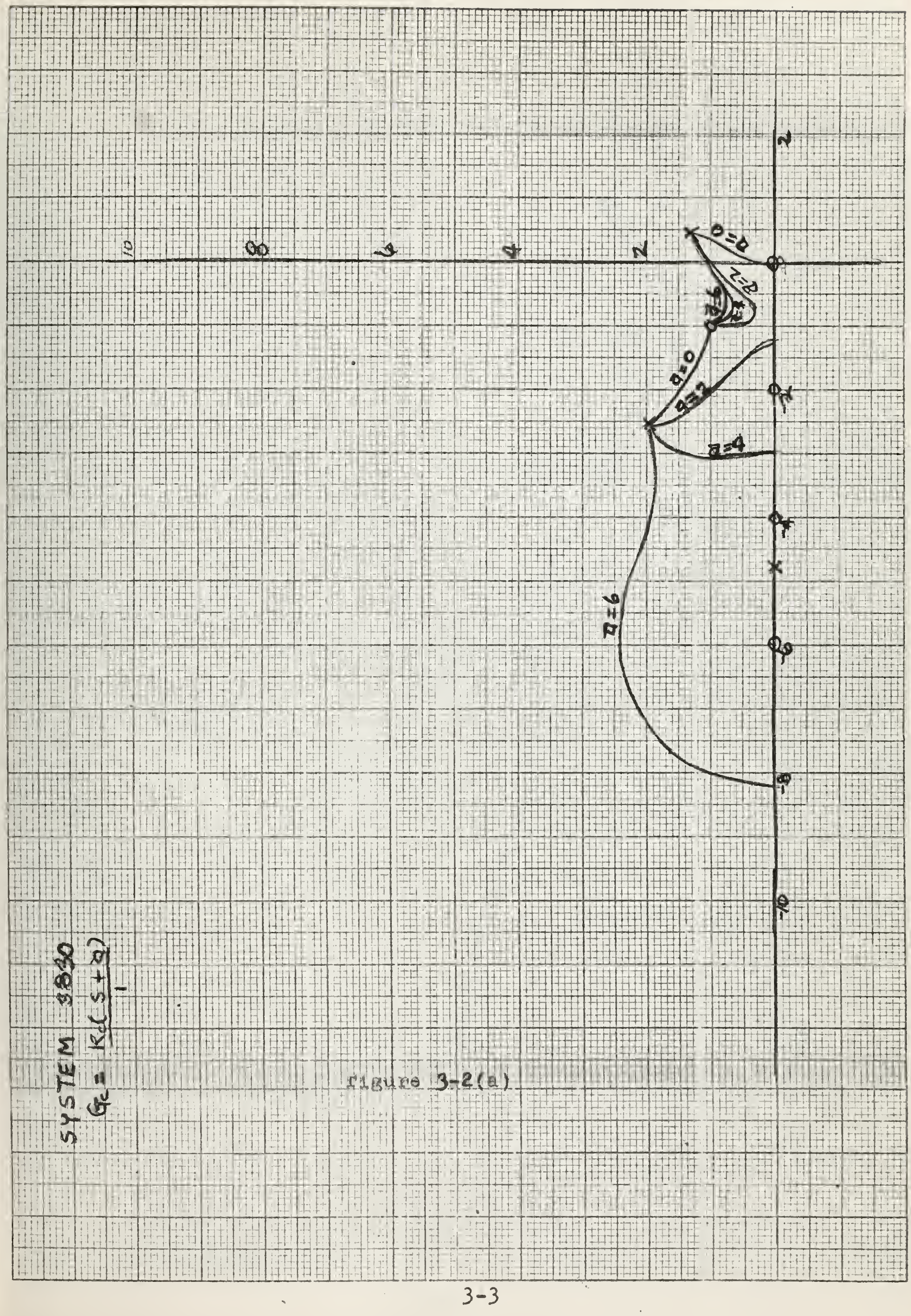
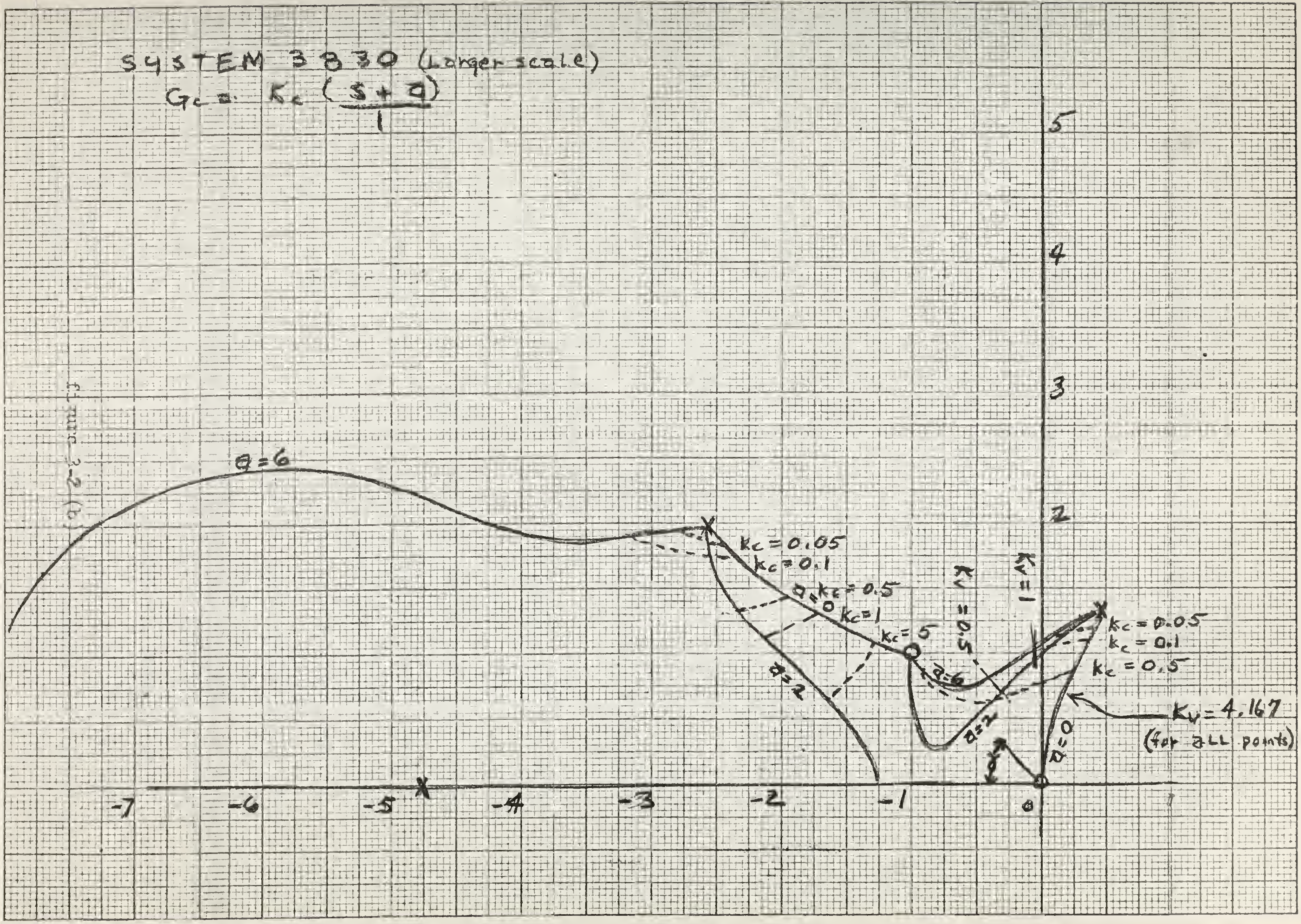


figure 3-2(a)


SYSTEM 3830 (Larger scale)
Gc = Kc (s + a) / 1
a = 6
a = 2
a = 0
Kc = 0.05
Kc = 0.1
a = 0 Kc = 0.5
Kc = 1
Kc = 5
Kc = 0.5
Kv = 0.5
Kv = 1
Kv = 4.167
(for all points)
-7
-6
-5
-4
-3
-2
-1
0
2
3
4
5

constant, ζ can be decreased from 1.0 to a minimum by increasing a. This minimum value of ζ is equal to $\cos\gamma$, where γ is shown in figure 3-2.

Also ζ and ω_n depend on the value of k_c. By increasing k_c from k_{cr} to infinity, while maintaining a constant, ζ will increase from zero to $\cos\gamma$, or even greater values of ζ if a is small. Likewise, ω_n also may increase with increasing k_c; or on the other hand, it may decrease depending on the specific value of a used.

(2) "50" compensator.

Effective compensation is obtained by use of the "50" compensator. In addition to providing stability to the basic system, use of this compensator allows the designer a reasonable degree of flexibility in choosing root locations.

As shown in figure 3-3, stability occurs for every value of a provided that k_c is greater than a minimum, k_{cr}. Values of k_{cr}, which vary slightly with a, are listed in table 3-1.

A comparison of figures 3-2 and 3-3 reveals the similarity between the predominating sections of the root loci. Thus it is reasonable to believe that the flexibility provided by this compensator is not very different from that provided by the "30" compensator. Consequently, because of this small difference coupled with the fact that the flexibility of the "30" compensator has been explained in detail, a further discussion of the "50" compensator is not necessary.

C. Partially satisfactory compensators.

Four of the compensators investigated are considered to provide compensation which is only partially satisfactory. This is due to either one or both of two reasons depending on the system. In three of the compensated systems stability does not occur for all values of

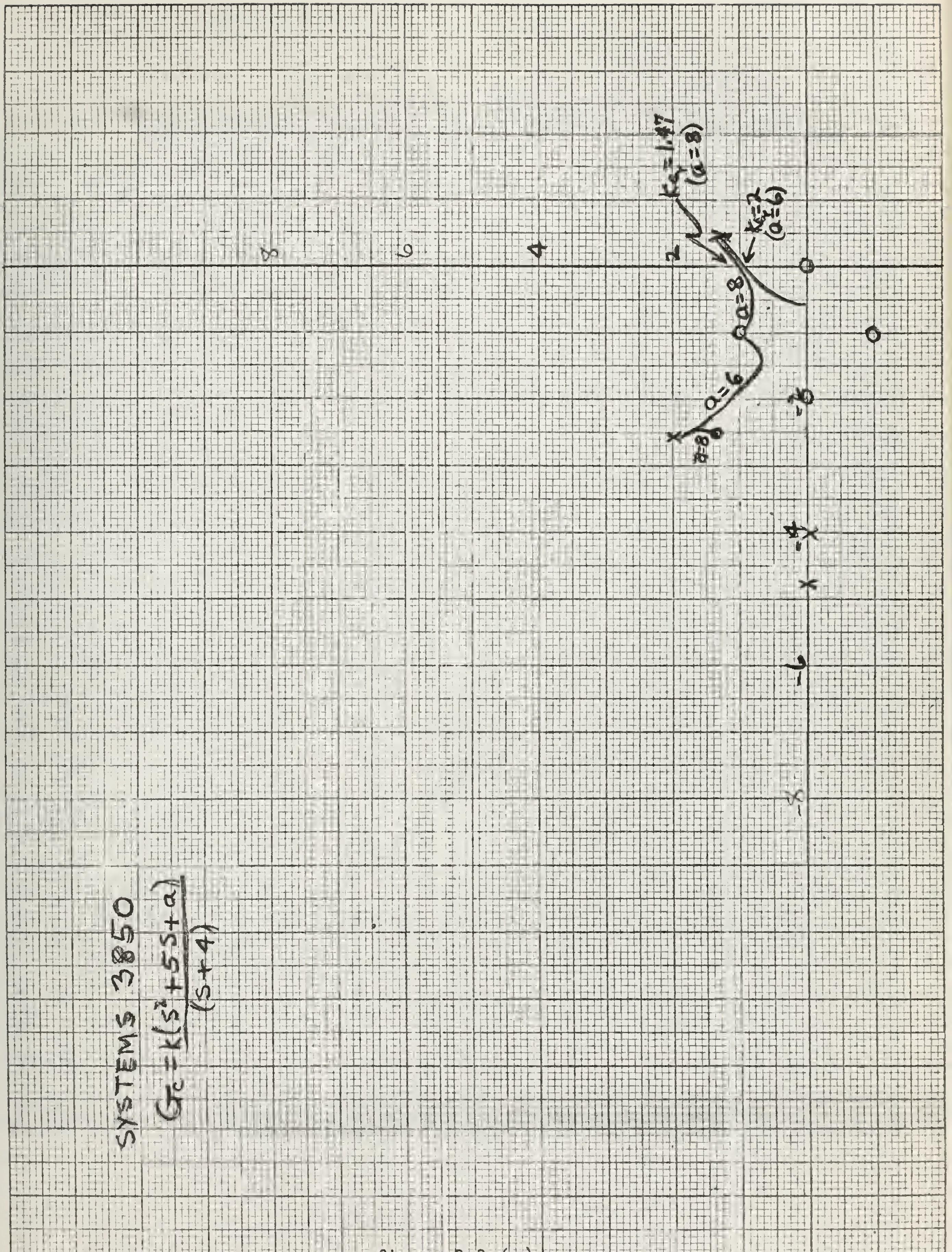
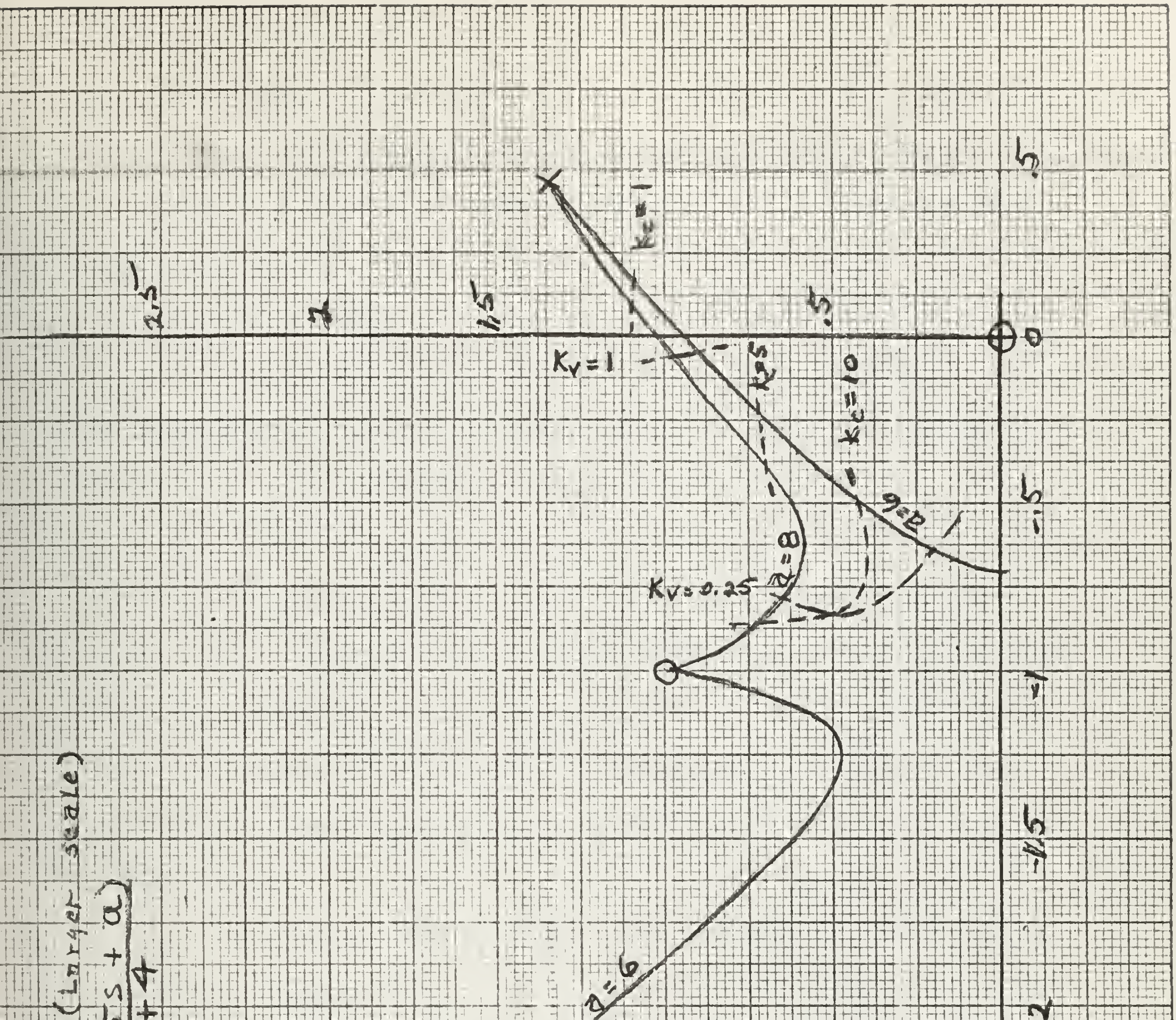

SYSTEMS 3850
$G_c = \frac{k(s^2+5s+a)}{(s+4)}$
8
6
4
2
-2
-4
-6
-8
a=8
a=6
a=8
K_{c_1}=1.87 (a=8)
K_{c_1}=2 (a=6)

figure 3-3 (a)



SYSTEM 3850 (Larger scale)
$G_c = \frac{K_c(s^2+5s+a)}{s+4}$
a=8
a=6
a=8
a=2
$K_V=1$
$K_V=0.25$
$k_c=1$
$k_c=5$
$k_c=10$
2.5
2
1.5
.5
.5
0
-.5
-1
-1.5
-2
-2.5
-3

figure 3-3.(b)

a. In the remaining system stability, in addition to being influenced by a, occurs only when k_c is within a finite range of values.

Further discussion of these compensators will be conducted individually below.

(1) Lag network.

The lag network can be useful in compensating the 3800 system. It definitely will induce stability. However, the flexibility obtained through use of this compensator is not too good.

As shown in figures 3-4 and 3-5 for $a > 1$ and 4 respectively, the lag network stabilizes the system provided a does not exceed an upper limit. This upper limit is that value of a which causes the root locus to be asymptotic to the imaginary axis. If a is less than this limiting value, stability is not assured unless k_c is greater than a minimum gain, k_{cr}. Values of k_{cr}, which vary with a, are shown in table 3-1.

It is also apparent from figures 3-4 and 3-5 that the predominating sections of the complex root loci do not vary to any great extent as a changes. In addition, these particular sections of root loci terminate on a complex zero. Therefore, the designer must not expect very much flexibility for meeting specifications by varying a or k_c if this compensator is used.

The maximum value of ζ provided by this compensator is approximately 0.7 for this particular system. Of course values of smaller than this may be obtained by decreasing k_c.

The most effective way of varying ω_n in this compensated system is by varying ζ. An increase in ζ from zero to the maximum possible value could cause ω_n to either increase or decrease depending on the difference in size between a and the compensator's pole.

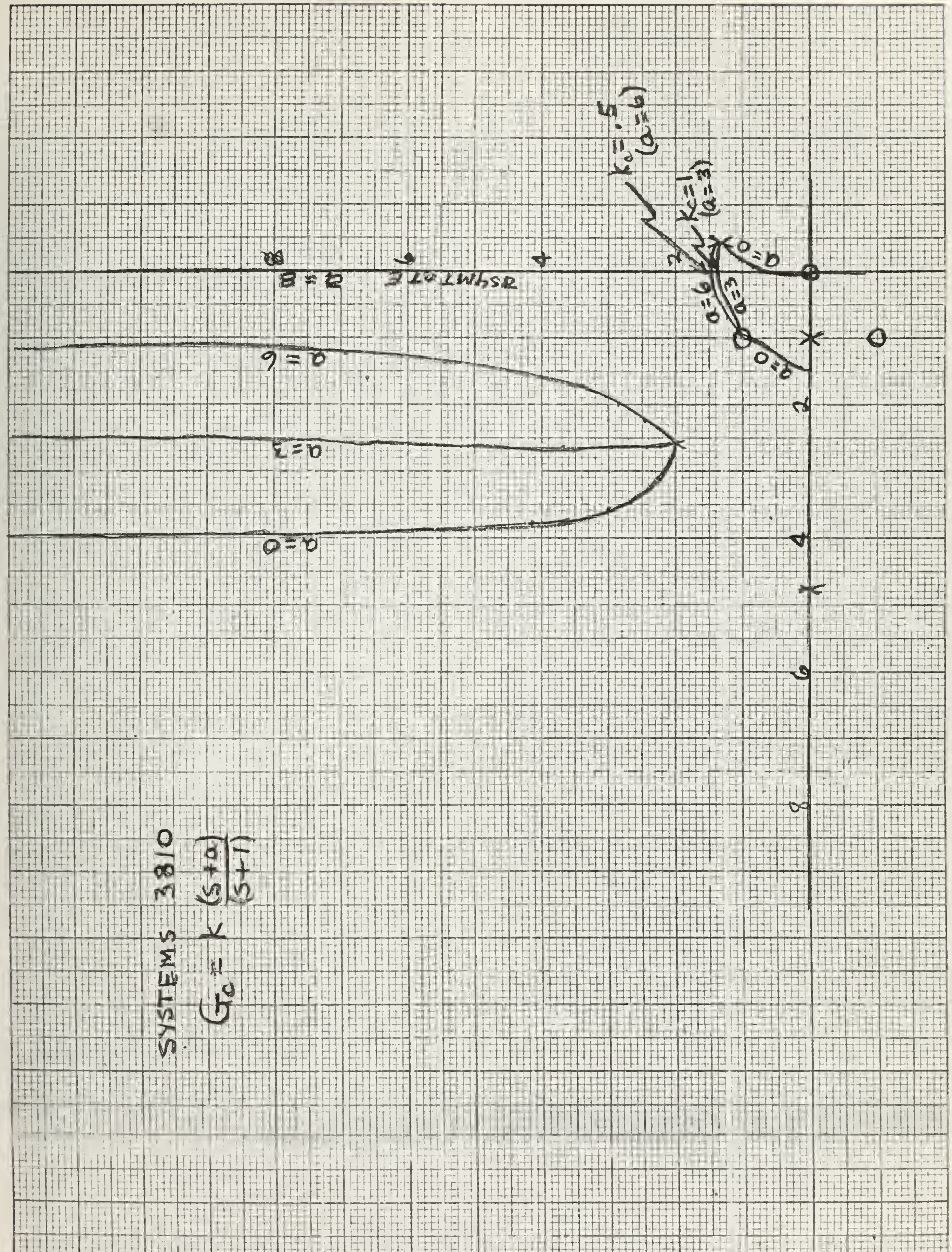
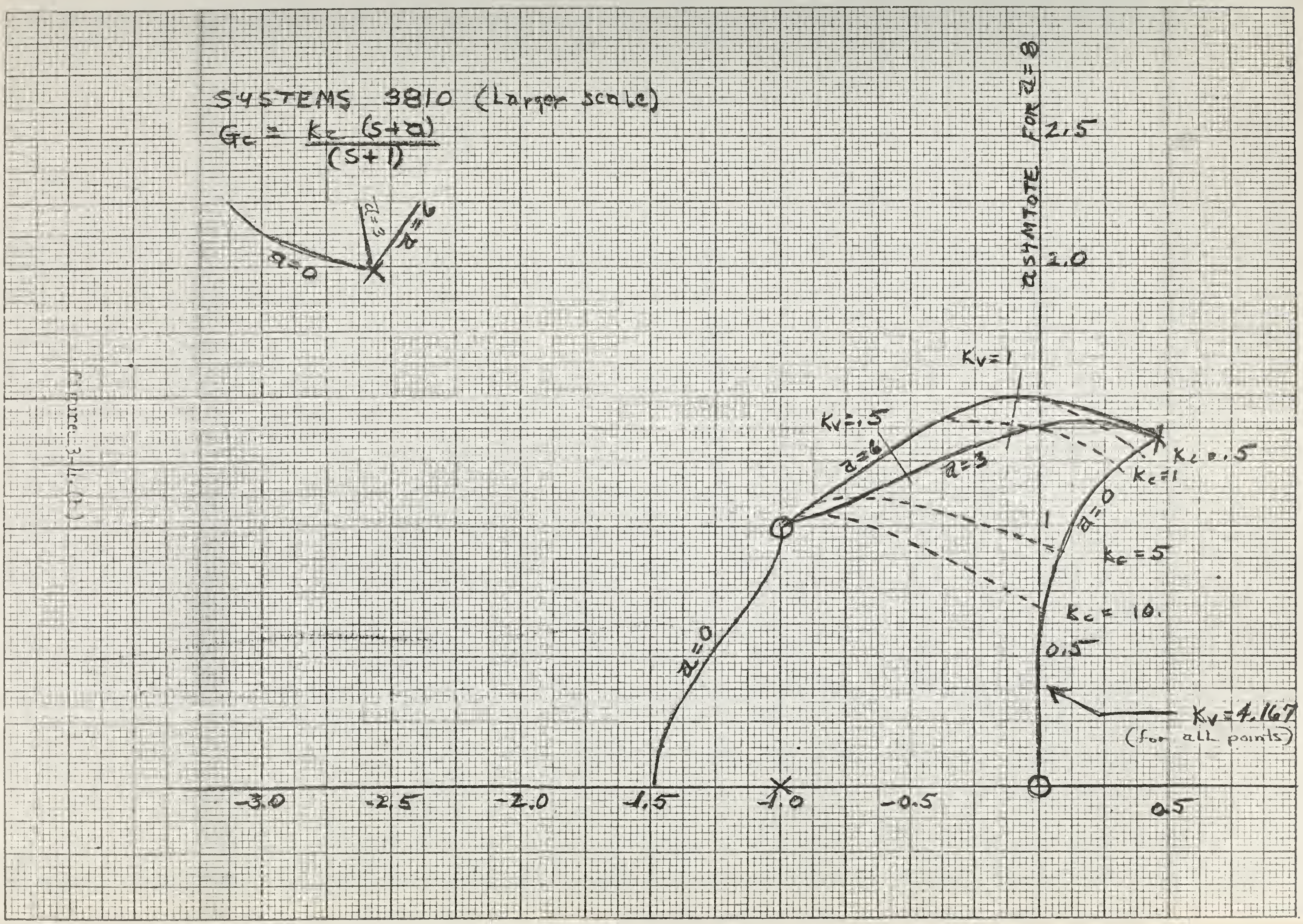

SYSTEMS 3810
$G_c = K \frac{(s+a)}{(s+1)}$
asymptote
a = 0
a = 3
a = 6
$K_c = .5$ (a=6)
$K_c = 1$ (a=3)
2
4
6
8

figure 3-4 (a)



SYSTEMS 3810 (Larger scale)
$G_c = \frac{K_c\,(S+a)}{(S+1)}$
a=0
a=3
a=6
ASYMTOTE FOR a=8
2.5
1.0
Kv=1
Kv=.5
a=6
a=3
a=0
Kc=.5
Kc=1
Kc=5
Kc=10.
0.5
a=0
Kv=4.167
(for all points)
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.5

figure 3-4.(b)

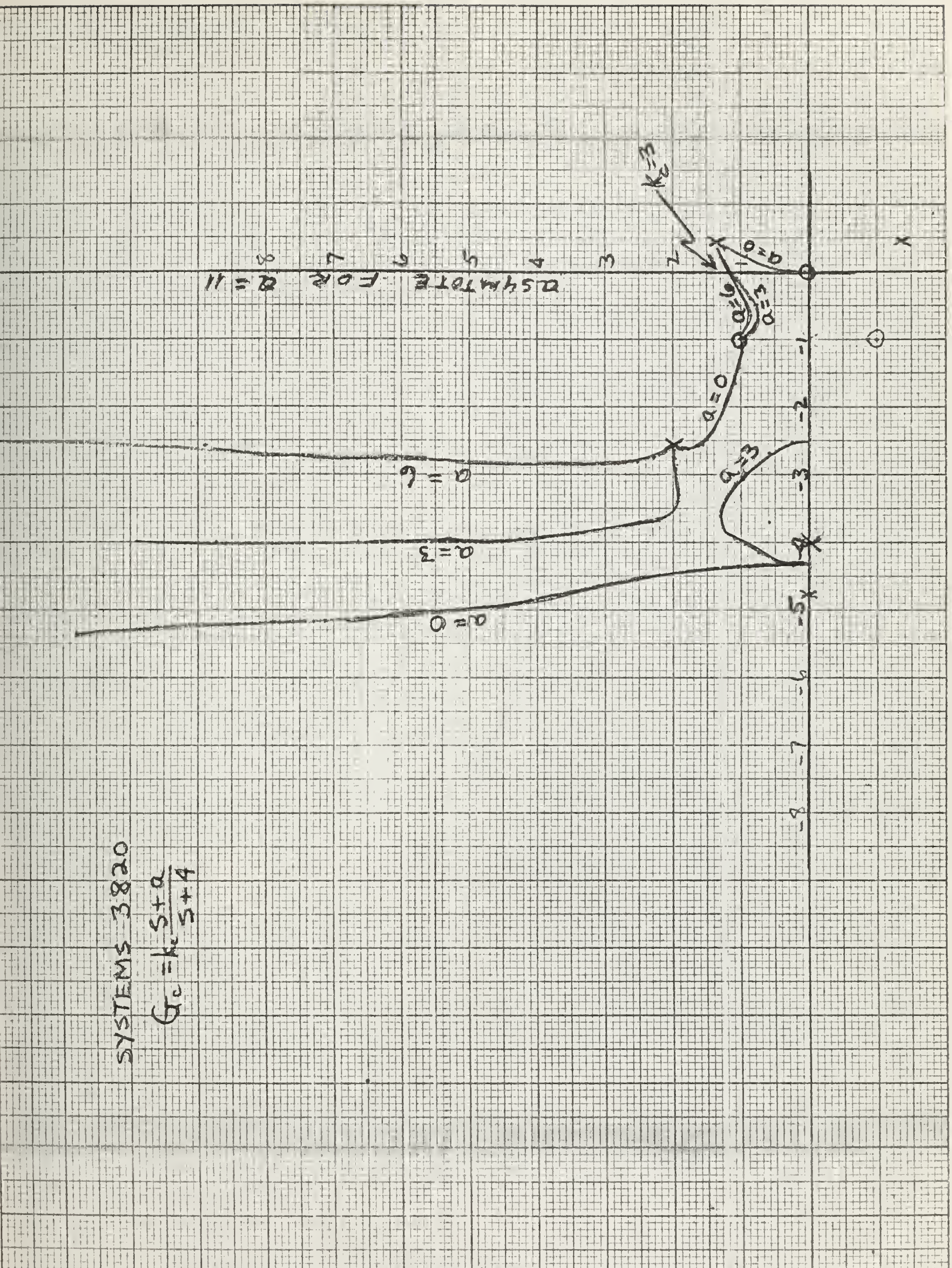


figure 3-5 (a)

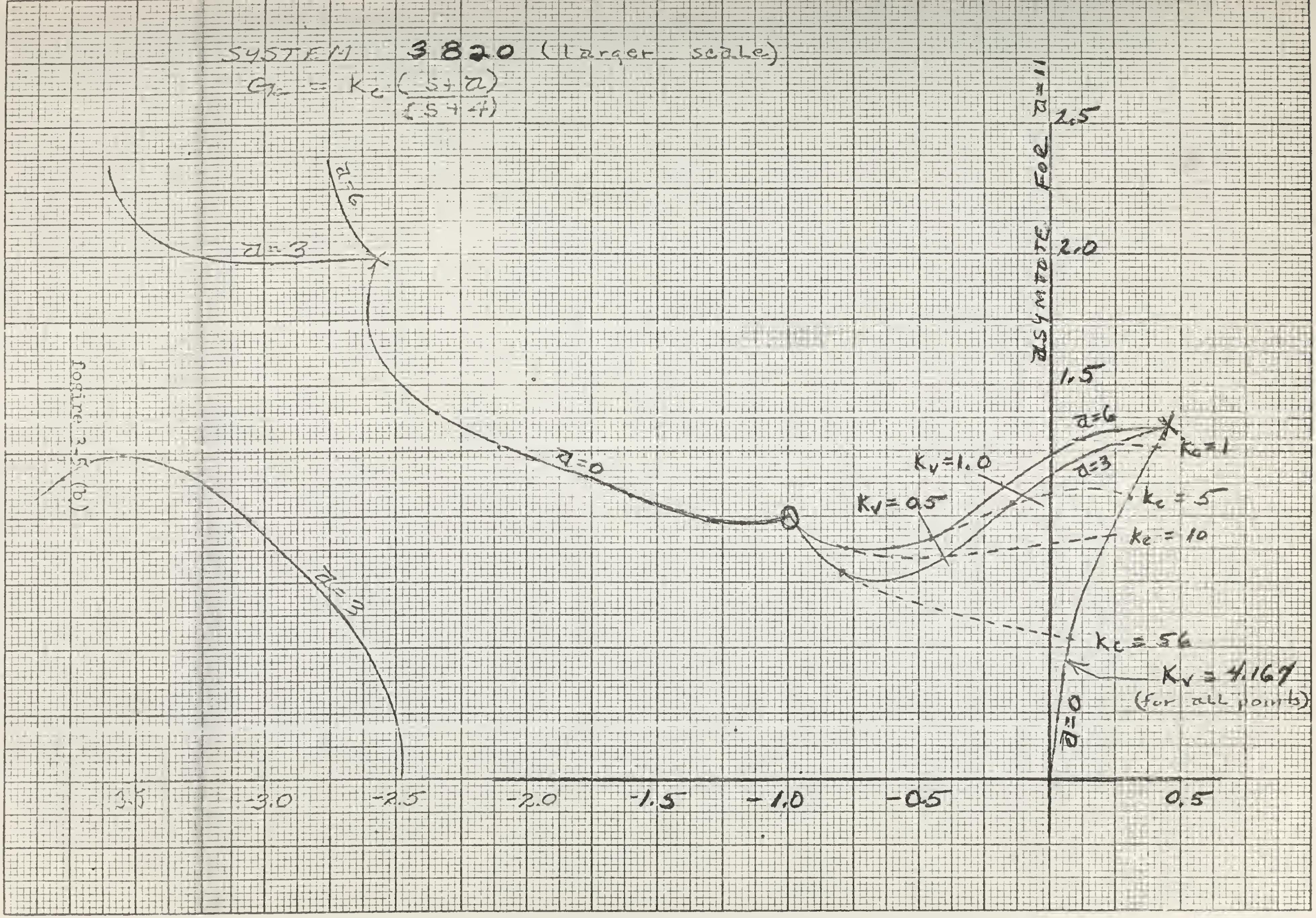

SYSTEM 3820 (larger scale)
$G_c = K_c \frac{(s+a)}{(s+4)}$
ASYMTOTE FOR a=11
a=6
a=3
a=0
a=3
a=6
a=3
a=0
K_v=1.0
K_v=0.5
k_c=1
k_c=5
k_c=10
k_c=56
K_v = 4.167
(for all points)
2.5
2.0
1.5
3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.5

Figure 3-5 (b)

However, for either case the total variation in ω_n will not be pronounced.

(2) Lead network.

There does not appear to be much difference between the effectiveness of the lead and lag networks. As shown in figures 3-4 and 3-5, the root loci of these two compensators supplement each other. Thus the flexibility available to the designer is approximately the same. However, there is a difference with respect to the range of a for which the lead network will cause stability to occur.

When a of the lead network is equal to zero, stability in the compensated system will not occur. For all other values of a this compensator does produce stability provided the compensator's gain, k_c, is greater than a minimum, k_{cr}. These values of k_{cr} vary with a and are listed in table 3-1.

(3) "60" compensator.

The effectiveness of the "60" compensator is more so limited than that of the two just previously discussed. This is due to the fact that the restrictions on a and k_c, for stability, are more stringent.

In order that stability might occur in the compensated system there are two conditions which must be satisfied. First, a must not be zero. The second condition is that the value of k_c must be within finite limits. As shown in figure 3-6, these limits depend on the value of a in that as a increases the interval between them decreases. The limiting values of k_c for those values of a investigated are listed in table 3-1.

Although a does influence strongly the stabilizing capability of the "60" compensator, it seems to have only a small effect on

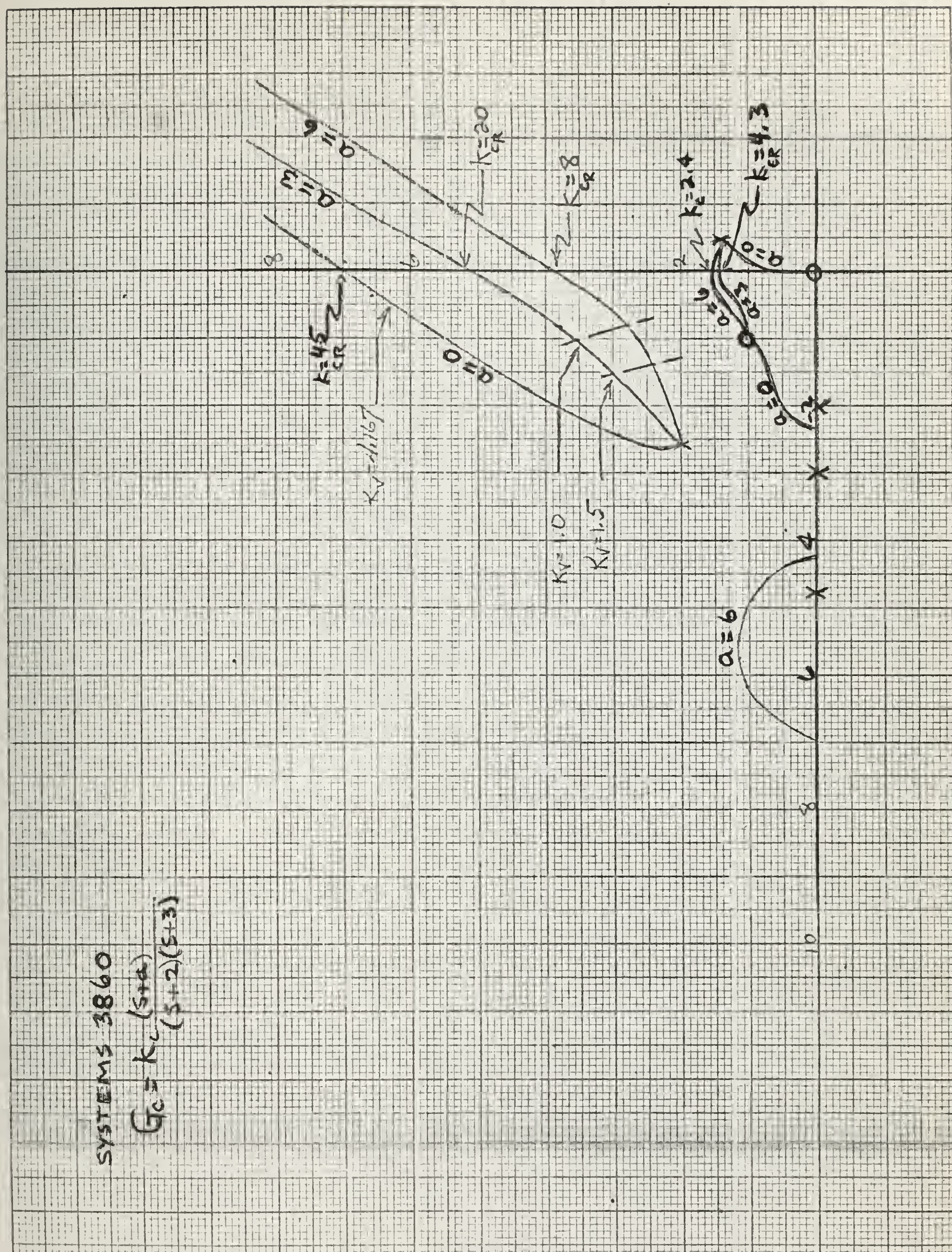
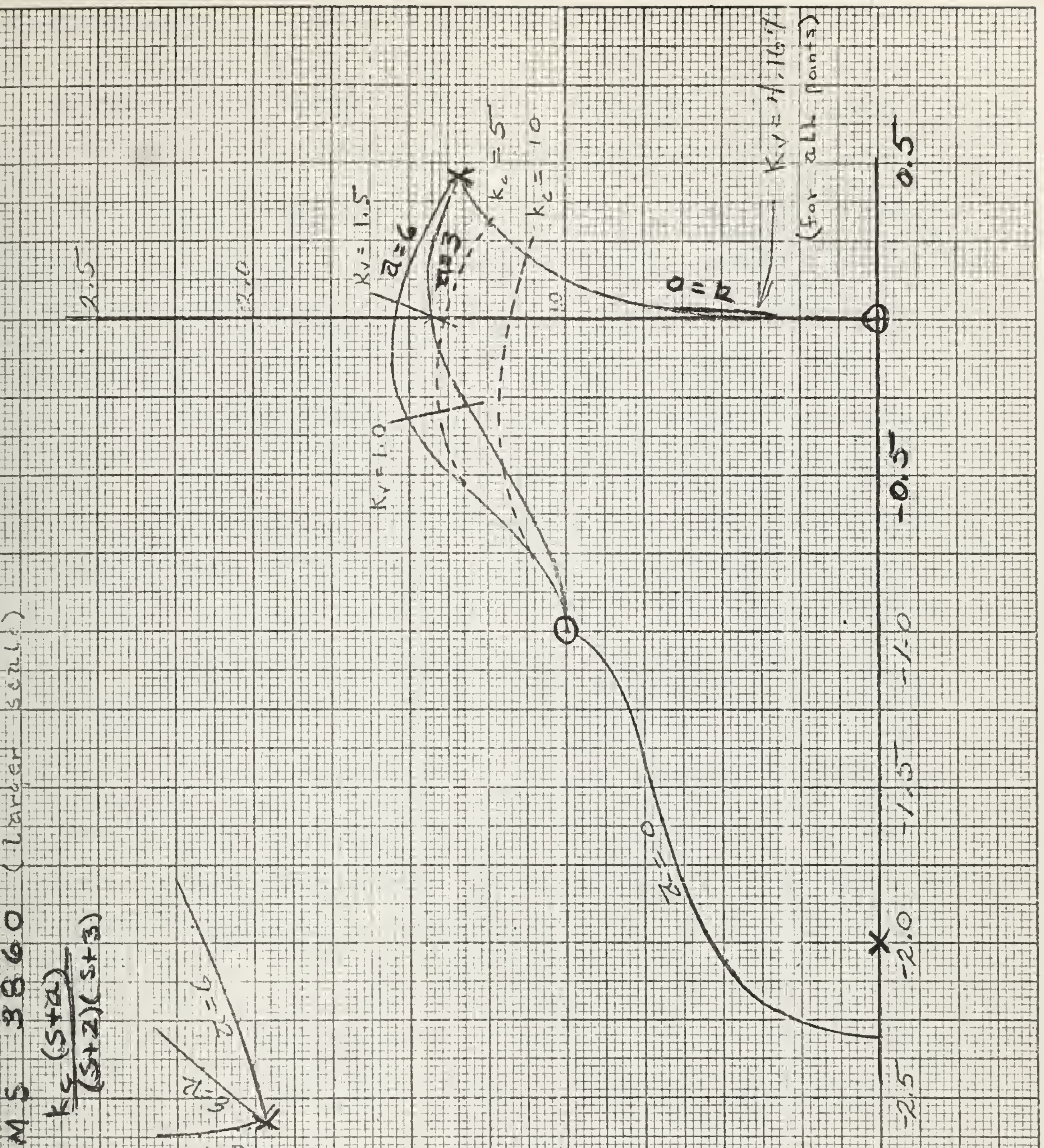

SYSTEMS 3860
Gc = Kc (S+a) / (S+2)(S+3)
a=6
a=3
a=0
K=20 CR
K=8 CR
K=45 CR
Kv=1.167
Kv=1.0
Kv=1.5
Kc=2.4
K=4.3 CR
a=6
2
4
6
8
10

figure 3-6 (a)



SYSTEMS 3860 (Larger scale)
G_c = k_c (s+a) / (s+2)(s+3)
a=6
a=3
a=0
a=6
a=3
a=0
Kv = 1.5
Kv = 1.0
k_c = 5
k_c = 10
Kv = 1.167
(for all points)
2.5
2.0
1.0
0.5
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5

figure 3-6 (b)

the predominating section of the complex root locus. This portion of the root locus does not seem to vary much. However, as figure 3-6 shows, it is so situated that it is possible to obtain roots within the range of 0.4 to 0.7 by varying k_c from k_{cr} (minimum gain limit) up to the maximum limit.

(4) Second derivative with proportional feedback.

The effectiveness of this compensator is strongly influenced by the value of a. For a less than a critical value this is a poor compensator: but, for a greater than critical the compensator's effectiveness is similar to that of other partially satisfactory compensators.

The root loci of figure 3-7 show the influence of a on this compensator's ability to stabilize a system. For a equal to zero stability does not occur at all: while, for a greater than zero but less than the critical value stability does occur although only to a small degree. However, for a greater than the critical, this compensator is capable of providing good stabilization. This critical value of a for the "64" system appears to lie between 2 and 4. For all values of a other than zero, stability only occurs if k_c is greater than a minimum, k_{cr}. The values of k_{cr} corresponding to the various values of a are listed in table 3-1.

In addition to stability, the flexibility provided by this compensator is strongly influenced by a also. For a less than the critical this compensator's worth is very small due to a lack of flexibility. On the other hand, for a greater than critical the flexibility is similar to that of the "60" compensator. However, even for this case the compensator's usefulness is somewhat limited to the large values of k_c which set [illegible] in the range 0.4 to 0.7 and restrict [illegible] to relatively

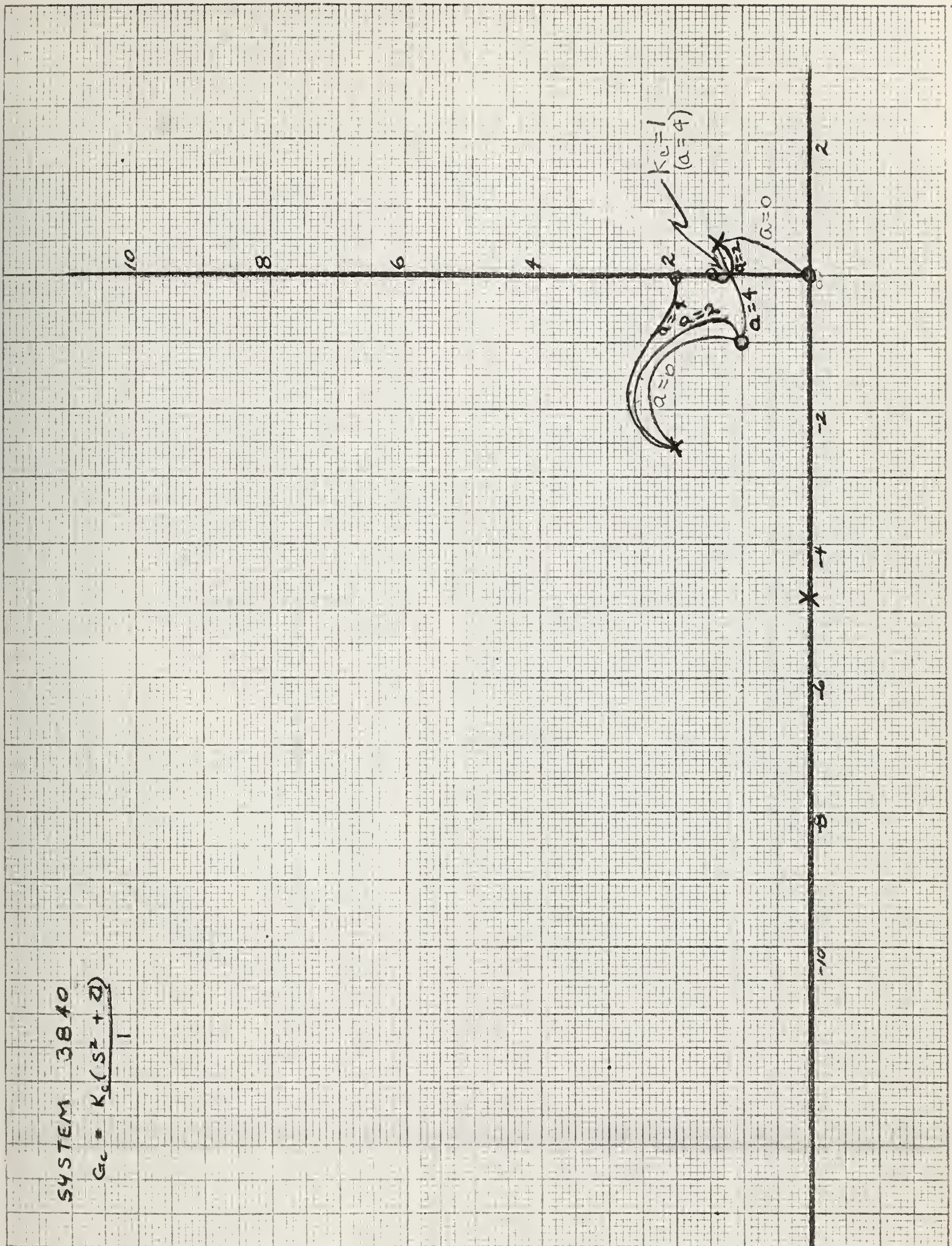
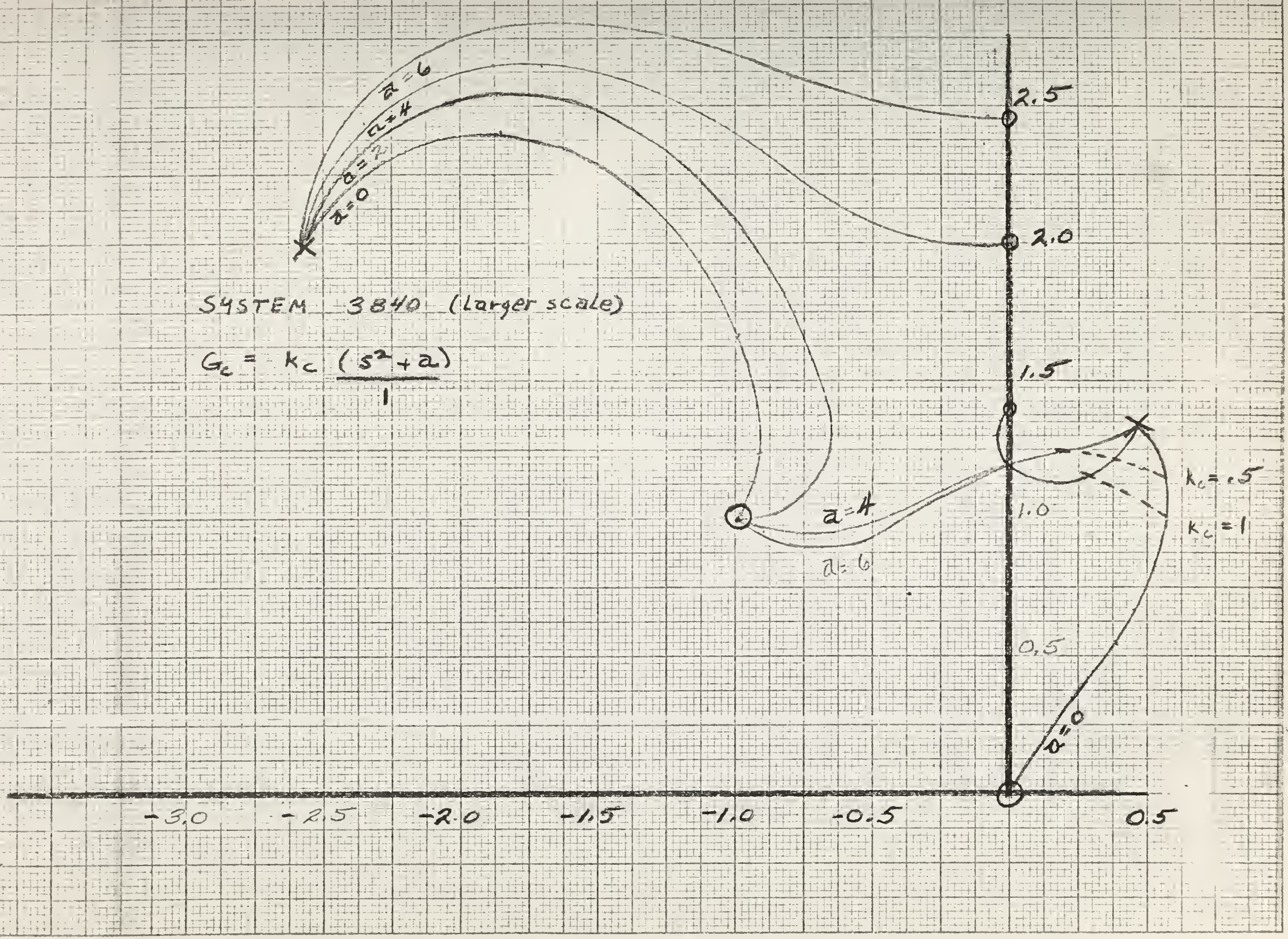


figure 3-7 (a)



SYSTEM 3840 (Larger scale)
$G_c = k_c \frac{(s^2+a)}{1}$
a=6
a=4
a=2
a=0
a=4
a=6
a=0
$k_c = .5$
$k_c = 1$
2.5
2.0
1.5
1.0
0.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.5

Figure 3-7 (b)

fixed, small values.

D. Completely unsatisfactory compensators.

The effectiveness of two of the compensators investigated is completely unsatisfactory. These two compensators consist of the first derivative and second derivative feedback. The root loci showing their effect are shown in figures 3-2 and 3-7 for a equal to zero.

F. Normalization.

Methods by which the root loci plotted in figures 3-2 to 3-7 may be correlated with those of any Group III system have not been investigated to any great extent.

TABLE 3-1

APPROXIMATE LIMITS OF STABILITY

Compensator	a	Lower limit		Upper limit	
		k_{cr}	K_v	k_{cr}	K_v
30	2	1.47	1.208		
30	4	0.709	1.239		
30	6	0.492	1.204		
50	6	2.116	1.143		
50	7	1.764	1.166		
50	8	1.47	1.208		
50	9	1.35	1.250		
10	3	1.021	1.173		
10	6	0.591	1.054		
20	3	3.657	1.268		
20	6	1.470	1.469		
60	3	4.389	1.473	20.000	0.450
60	6	2.540	1.337	8.500	0.530
40	2	4.389	0.501		
40	4	1.021	0.946		
40	6	0.492	1.204		

4. Group III - type one system with second order motor function and one excess zero in G_b.

A. General.

System 1400 is the only system in Group III which was investigated. The block diagram and the uncompensated roots of this system are shown in figure 4-1.

It is significant to note that the basic system has only real, stable roots. Therefore, the express purpose for compensating this system is to relocate these roots in the negative half of the complex plane. In view of this purpose, the flexibility that the compensator provides the designer will be given considerable attention in the brief analysis which is to be conducted below.

B. Completely satisfactory compensators.

Four of the compensators investigated are considered to be completely satisfactory in compensating the 1400 system. This is not meant to imply that these four are better than that compensator which is considered to be only partially satisfactory; but to the contrary, it only indicates that for each compensator two conditions are met. These conditions are:

1. stabilization is possible for any value of a used

2. stabilization always occurs when k_c is greater than k_{cr}. However, in some cases k_{cr} is 0 because of the fact that the entire root locus lies to the left of the imaginary axis.

Because of its effect on stability, the value of k_{cr} is of interest. By definition k_{cr} is the minimum compensator gain, k_c, possible that a stable system may have. The values that k_{cr} assumes depends on a. Therefore, these values along with their corresponding value of a are listed in table 4-1.

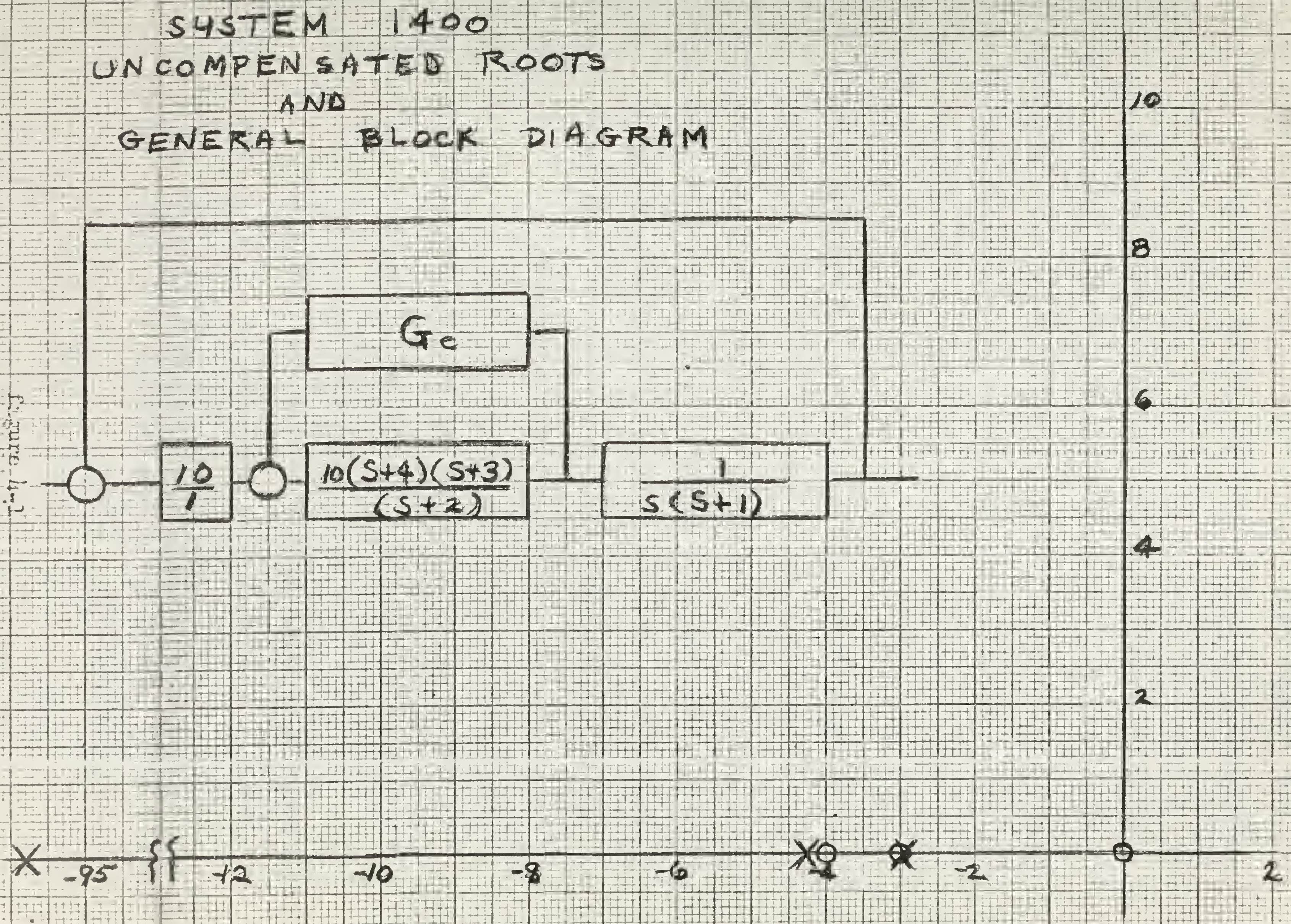

SYSTEM 1400
UNCOMPENSATED ROOTS
AND
GENERAL BLOCK DIAGRAM
G_c
$\frac{10}{1}$
$\frac{10(S+4)(S+3)}{(S+2)}$
$\frac{1}{S(S+1)}$
10
8
6
4
2
-95
-12
-10
-8
-6
-4
-2
2

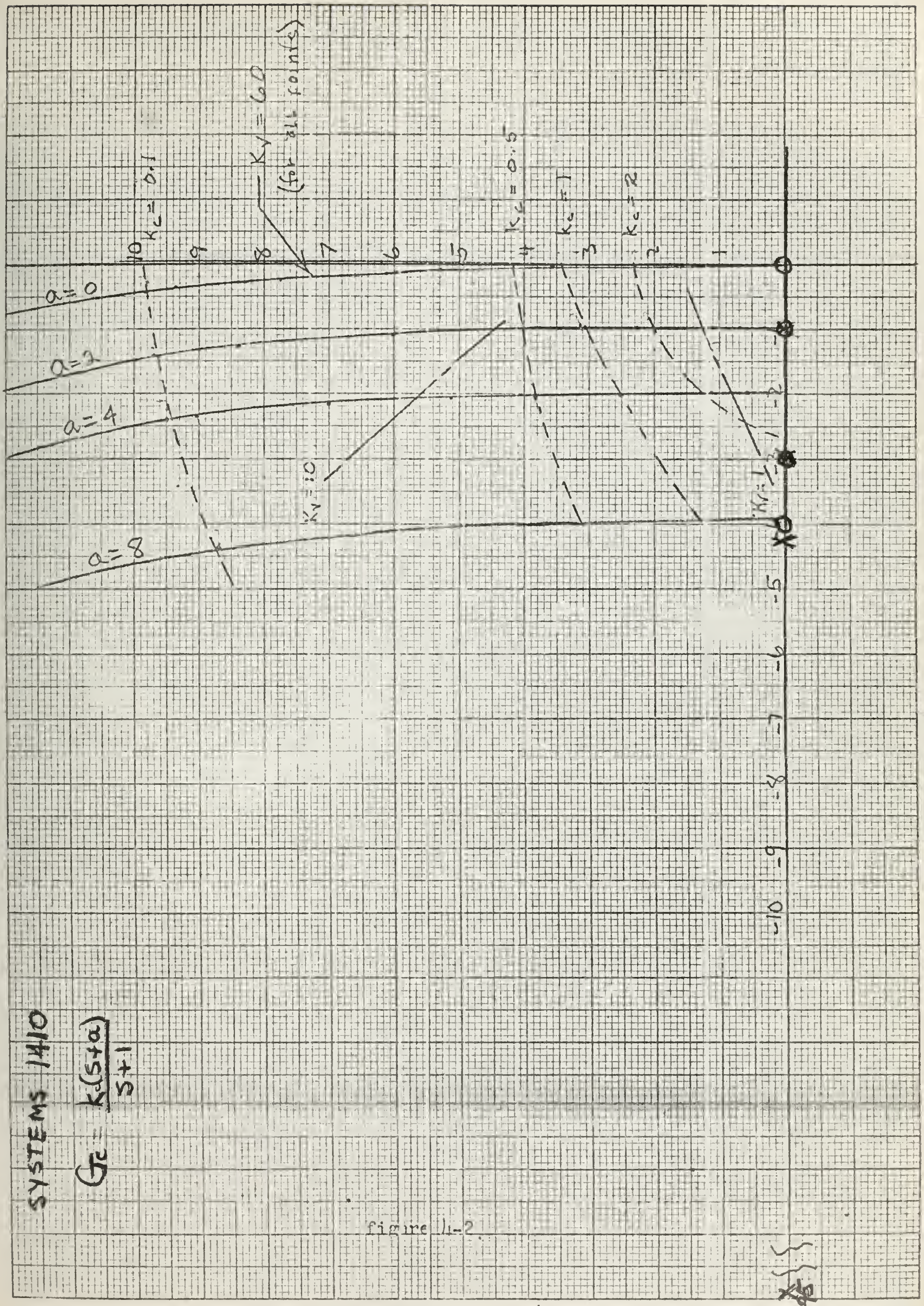
figure 4-1

Each of the four compensators differ somewhat in their effect on the basic system, and consequently in their effectiveness. Therefore, in view of their individuality, each will be briefly discussed below.

(1) Lag network.

The effectiveness of the lag network in compensating the Group III systems appears to exceed that of any other compensator investigated. The primary reason for this is the exceptional flexibility that it provides the designer. First of all, this system is stable regardless of the value of $\underline{a}$ or k_c; consequently, there is no restriction on the values which can be assumed by either $\underline{a}$ or k_c. In addition, the shape and orientation of the dominating section of the complex root loci is quite favorable, particularly for system 1410. As shown by the root loci of figure 4-2, the dominating complex curves are nearly vertical, straight and capable of being moved to intersect any place along the negative real axis which is less than s equal -95. Obviously this would provide nearly unlimited flexibility in allowing the designer to choose from a wide range of ζ and ω_n. On the other hand, the root loci of system 1420 as shown in figure 4-3 are not endowed with quite as many of the favorable characteristics as was 1410, but nevertheless, it still provides a large degree of flexibility.

Now that the extent to which ζ and ω_n may be varied has been indicated, it is also of interest to observe how this variation may be accomplished using k_c and $\underline{a}$. In particular, for either the 1410 or 1420 system ζ may be varied in either of two ways: increasing k_c while holding $\underline{a}$ constant, or increasing $\underline{a}$ while holding k_c constant. The former method causes ζ to vary from a small value (which in itself increases with $\underline{a}$) to 1; whereas, the latter method causes a much



SYSTEMS 1410
$G_c = \frac{K_c(s+a)}{s+1}$
a = 0
a = 2
a = 4
a = 8
$K_c = 0.1$
$K_c = 0.5$
$K_c = 1$
$K_c = 2$
$K_v = 60$ (for all points)
$K_v = 10$
$K_v = 1$

figure 4-2

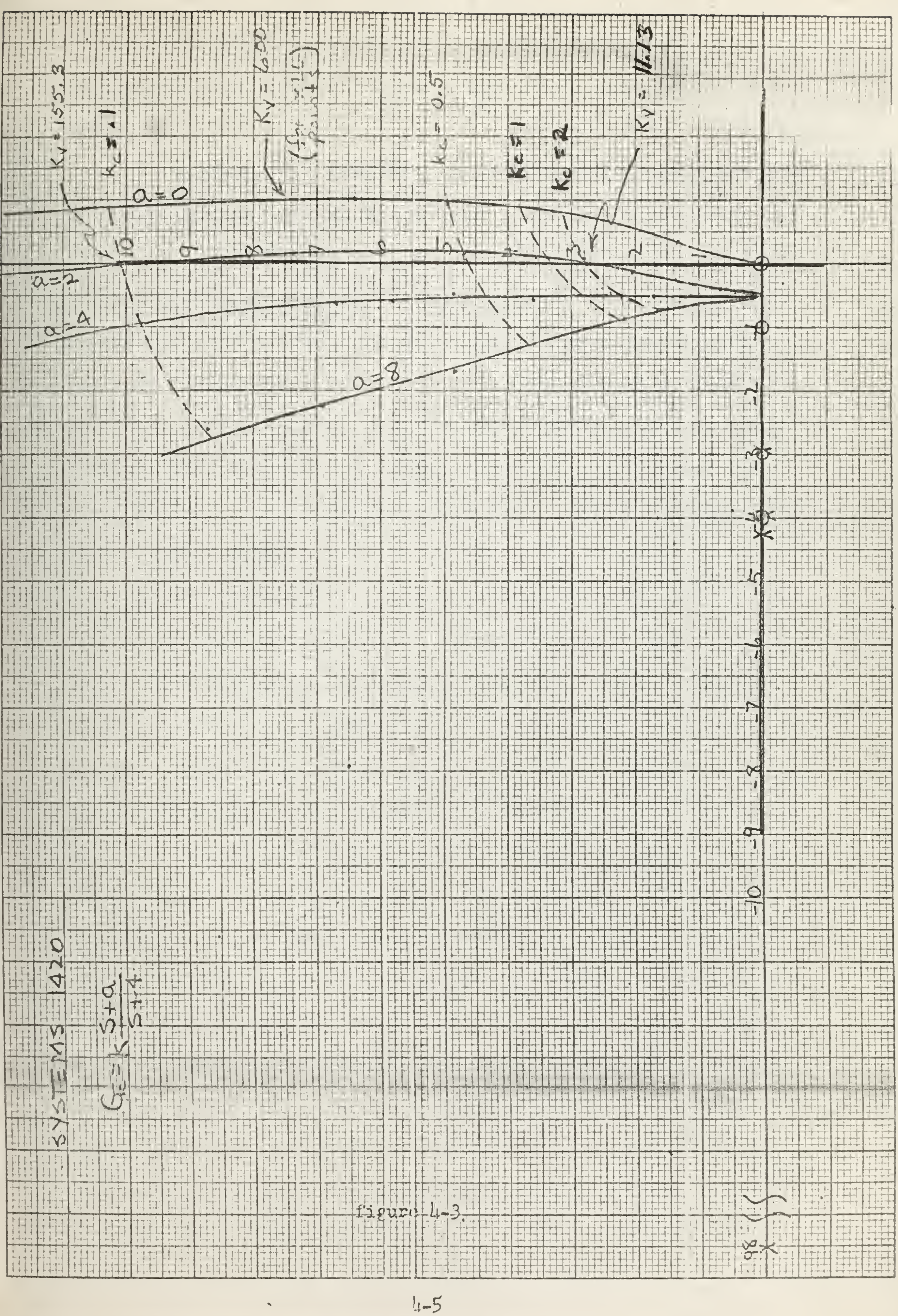

SYSTEMS 1420
Gc = k (S+a)/(S+4)
Kv = 155.3
Kc = .1
Kv = 6.80
Kc = 0.5
Kc = 1
Kc = 2
Kv = 11.13
a=0
a=2
a=4
a=8
-10
-9
-8
-7
-6
-5
-4
-3
-2

figure 4-3.

smaller variation in ζ. Likewise, ω_n can be varied simply by varying a.

In view of the fact that pole-zero cancellation has occurred in system 1410, it seems at first glance that the applicable root loci plots should be considered to represent only a special case. However, in spite of this observation, the significance of figure 4-2 is still noteworthy. If the pole of the compensator had been less than that of the motor function (this latter pole causes the zero of the root locus), then the resulting root loci would be similar in appearance to those of figure 4-2. Thus this figure does represent to some extent the root loci for the case where the compensator's pole is less than that of the motor function.

(2) "60" compensator.

The effectiveness of the "60" compensator is much less than that of the lag network. As shown in figure 4-4, except for k_c equal to infinity when a is 0, this compensator, regardless of the values of a or k_c, does not cause instability; yet, at the same time it does not provide favorable flexibility in the choice of root locations. One reason for this is the fact that the range of ζ available is highly dependent on the value of a. For a equal to 0 all values of ζ are possible; whereas, when a is greater than 0, ζ can not get smaller than the minimum which is established by the complex root locus. (This minimum ζ can be ascertained to be cos γ, where γ is the angle that a line from the origin, tangent to the complex root locus curve, makes with the negative real axis). Thus for a large enough it is not possible to locate roots such that their ζ will be in the desirable 0.4 to 0.8 range, which definitely is not favorable. This value of a for the 1460 system is approximately 3.

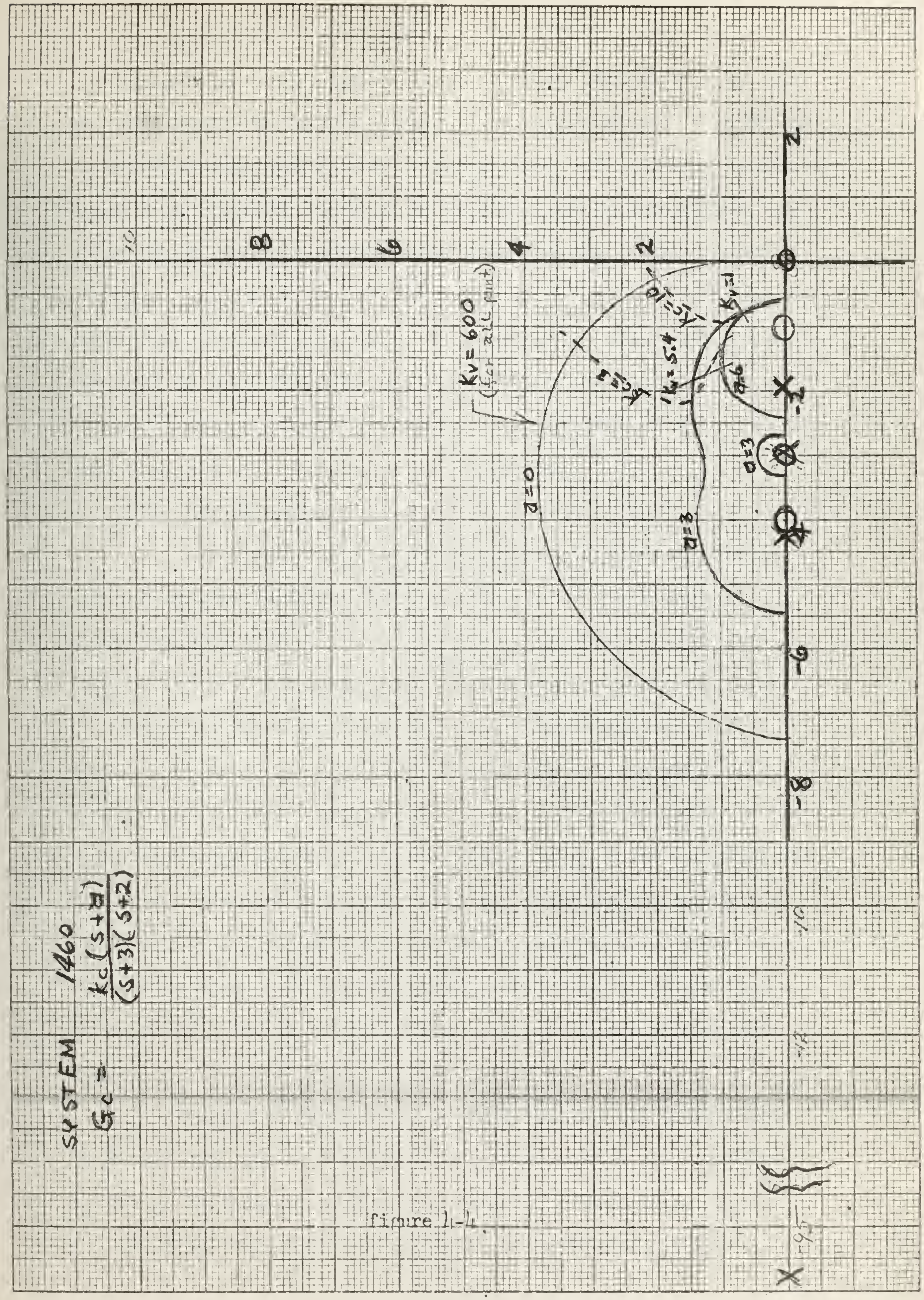

SYSTEM 1460
Gc = kc(s+a) / (s+3)(s+2)
Kv = 600
(for all pnt)
a=0
a=3
a=3
a=6
Kc=3
Kc=10
Kv=5.4
Kv=1
8
6
4
2
-8
-6
-4
-2
2
-10
-12
-95

figure 4-4

Subject to the limitation discussed above, changes in ω_n and ζ are possible by varying a and k_c. As previously mentioned ζ can be caused to vary from 0 to 1 by increasing k_c and holding a constantly equal to 0. However, for this compensator, more so than others, any change in ζ causes a corresponding change in ω_n. In particular when a is equal to 0, ω_n decreases to 0 as ζ approaches 0. On the other hand for a not equal to 0, ζ can only be varied from its minimum value to 1 by increasing k_c. Likewise, as was the case when a equaled 0, ω_n is highly dependent on the variation of ζ for this case also, and will decrease as ζ decreases.

(3) "30" compensator for a not equal to 0.

The effectiveness of the combination of first derivative and proportional feedback ("30" compensator with a not equal to 0), while less than that of the lag network, is still somewhat better than that of the "60" compensator. The favorable aspect for this compensator as shown in figure 4-5, is the fact that there is no limitation placed on the values of ζ obtainable. Its unfavorable aspects are the small range of ω_n which is available for ζ in the 0.4 to 0.8 range and the limitation placed on k_c by the necessity for stability. Although this compensator can stabilize the system regardless of the value of a (except for a equal to 0), it can only do so if k_c is greater than the lower limits, k_{cr}, which are listed in table 4-1. Thus k_c is limited to values which are greater than k_{cr}.

Using this compensator, roots having various combinations of ζ and ω_n may be selected by carefully varying the variables k_c and a. In particular, ζ may be varied in either of two ways: increasing k_c while a remains constant, or increasing a while k_c

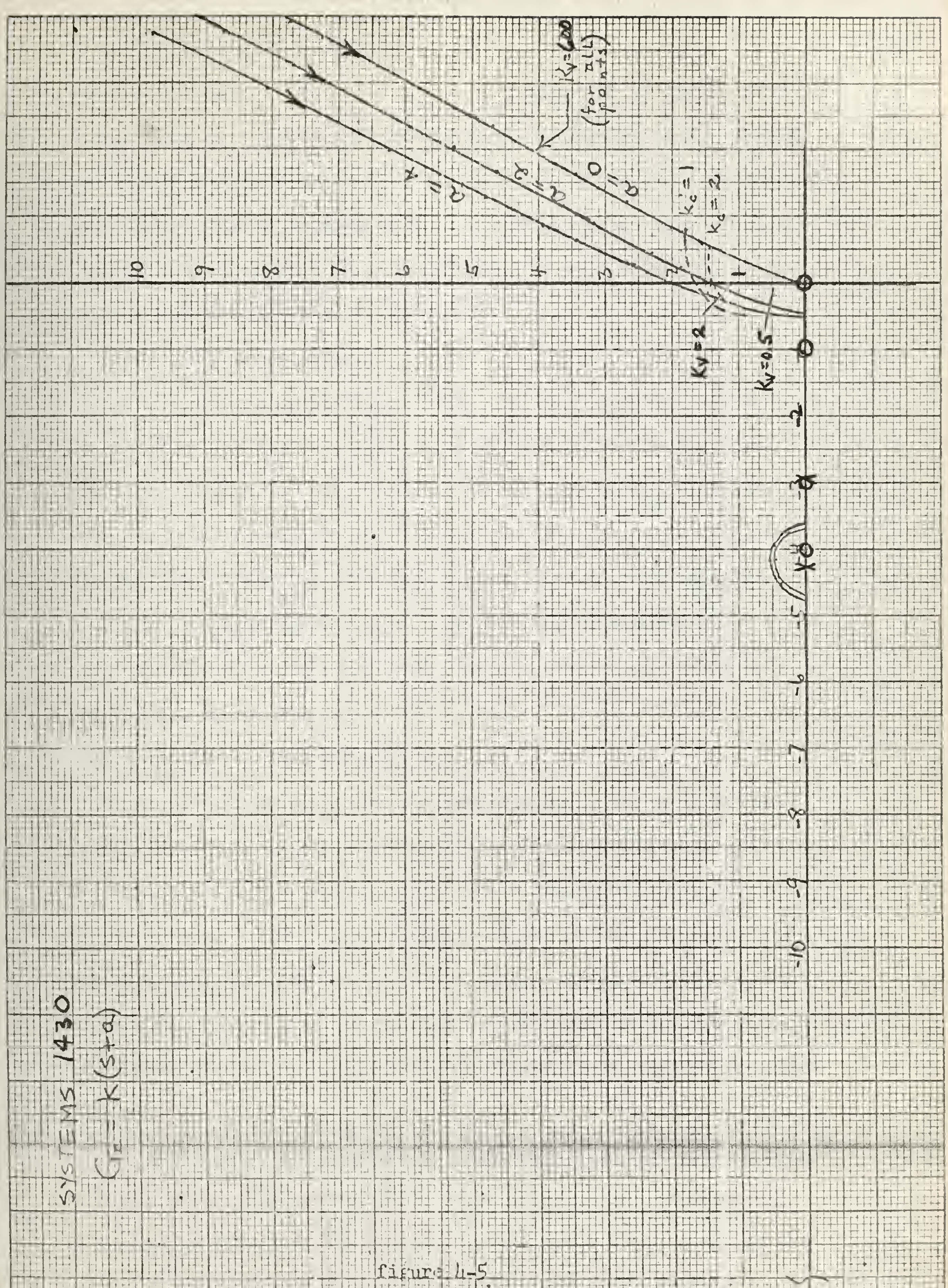


figure 4-5

remains constant. The former method will cause ζ to vary from 0 to 1 depending on the amount of change which is applied to k_c; whereas, the latter method as shown by the constant k_c contours in figure 4-5 only produces small changes in ζ. Likewise, in the case of ω_n changes will occur when ζ increases if a is held constant, or in other words when k_c only is increased.

(4) "50" compensator.

As shown in figure 4-6, the effect of the "50" compensator on the Group III system is very similar to that of the "30" compensator. Except for differences in the values of k_{cr} as listed in table 4-1. the stability requirements are the same. Likewise, the flexibility provided by these two compensators is quite similar. Therefore, because the difference between the effectiveness of these two compensators is insignificant, further discussion of the "50" compensator is unwarranted.

C. Partially satisfactory compensators.

Only one of the compensators investigated - the lead network- is considered to be partially satisfactory. Primarily this implies that the stability of the compensated systems depends not only on the lower gain limits, k_{cr}, as was the case for the "30" and "50" compensators. but also on upper gain limits and a. In addition the above also implies that the flexibility provided by this compensator is more so limited than for those considered to be completely satisfactory.

Nevertheless, a comparison of figures 4-2 and 4-3 reveals that there is also one other factor to be taken into account when considering the effectiveness of the lead network. This factor is the ratio of the compensator's pole to that of the motor function. If

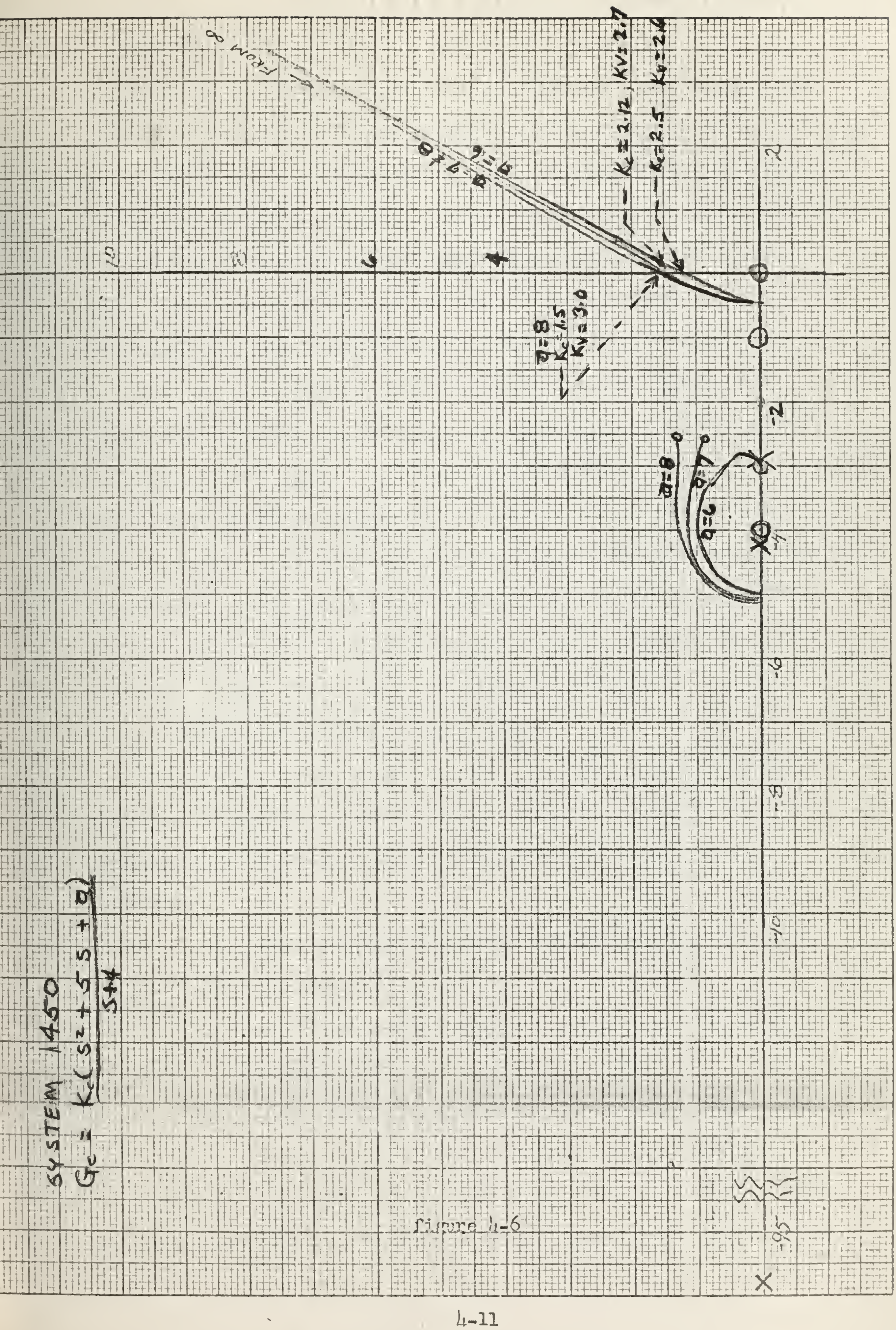

SYSTEM 1450
G_c = K_c(s^2 + 5s + a) / (s+4)
a=8
K_c=1.5
K_v=3.0
K_c=2.12 K_v=2.7
K_c=2.5 K_v=2.4
FROM 8
a=8
a=7
a=6

figure 4-6

this ratio is less than or equal to 1 then the root loci of figure 4- apply. For this case stability occurs for all values of $\underline{a}$ and k_c except for k_c equal to infinity when $\underline{a}$ equals 0. Therefore, the effect of this compensator is similar to that of the lag network and actually the root loci of the former supplement those of the latter. On the other hand, if the ratio is greater than 1, then the root loci of figure 4-3 and 4- and the limitations on stability are more complex. In this case, if $\underline{a}$ equals 0, instability persists for values of k_c greater than k_{cr}, but if $\underline{a}$ is nearly as large as the magnitude of the compensator's pole ($\underline{a}$ equal to or greater than three for the 1420 system) then the effectiveness of this compensator is quite similar to that of the lag network, and actually the two sets of root loci are supplementary. However, if $\underline{a}$ is much smaller than the pole, stability will be acquired only if k_c is restricted to values between the upper and lower limits. These limits are listed in table 4-1.

D. Completely unsatisfactory compensators.

Three of the compensators investigated for this group are considered to be completely unsatisfactory in view of the fact that they cause instability to occur in the basic system. These are the following:

(1) first derivative feedback - effect shown by root loci of figure 4-6

(2) second derivative feedback - effect shown by root loci of figure 4-7

(3) combination of second derivative and proportional feedback - effect shown by the root loci of figure 4-7.

While it is somewhat obvious from figures 4-6 and 4-7 that first and second derivative feedback are unsatisfactory, it is not so apparent

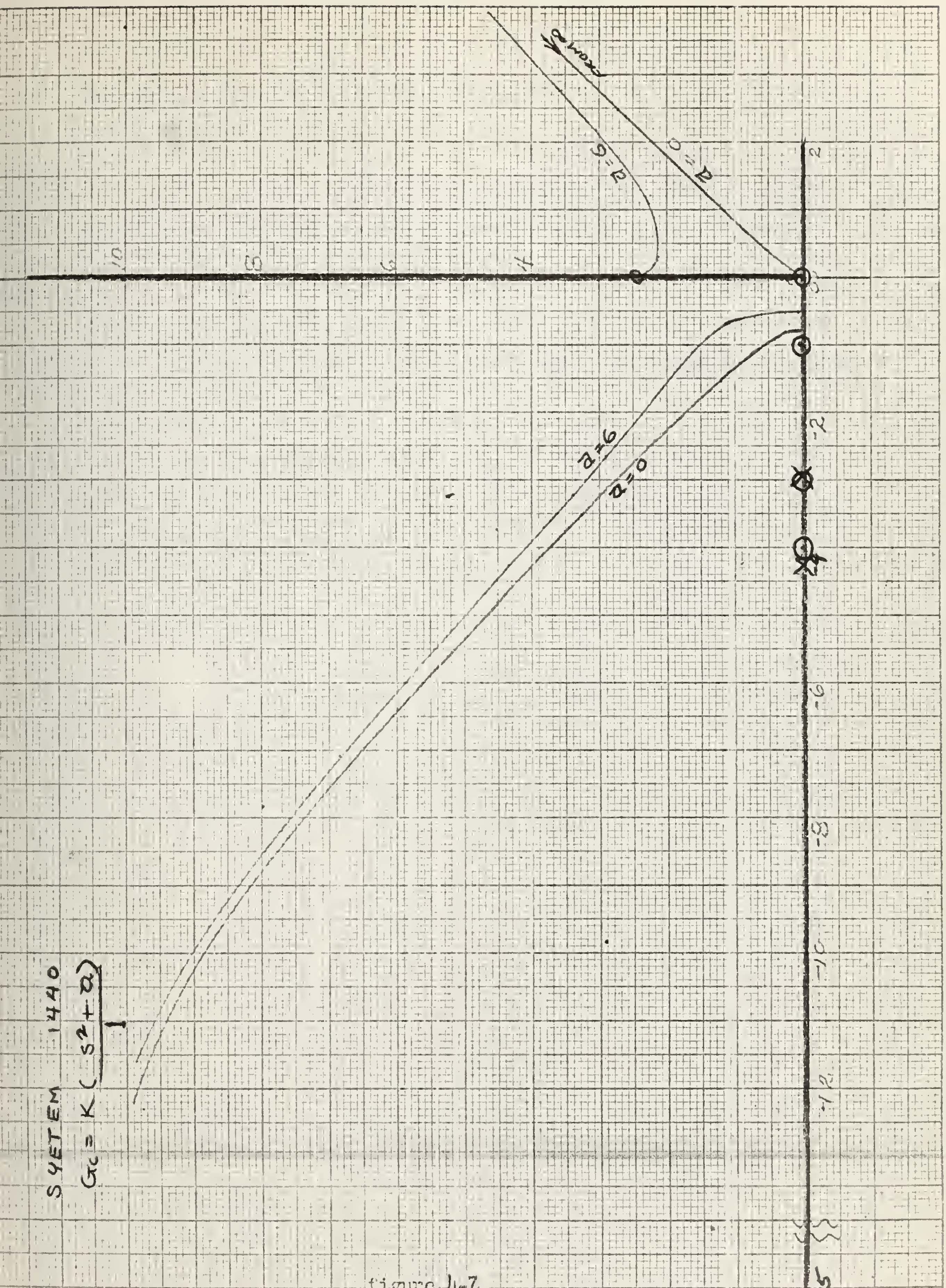


figure 4-7

for the case when a proportional feedback signal is also included. In this case it is possible for this proportional component, if large enough, to completely dominate as the compensating effect and thereby minimize the effect of the second derivative component. However, if this is the case, the compensation should be considered to consist of proportional feedback only which is not of interest here in view of the fact that it merely changes the gain of the open loop function.

4. Normalization.

Because investigation was limited to only one system for this group, it is not possible to draw any significant conclusions with respect to normalization.

TABLE 4-1

APPROXIMATE LIMITS OF STABILITY

Compensator	a	Lower limit k_{cr}	Lower limit k_v	Upper limit k_{cr}	Upper limit K_v
20	0	0.032	600.00	∞	600.000
20	2	0.095	155.364	1.764	11.129
30	2	1.683	2.957		
		.563	4.404		
50	6	2.540	2.613		
	7	2.116	2.688		
	8	1.764	2.822		
	9	1.470	3.009		

5. Group IV - type one system with second order motor function and zero excess poles in G_b.

a. General.

Three of the systems investigated fall into this group. They are systems 1000, 1200 and 1300. Figures 5-1, 5-2 and 5-3 illustrate the block diagram of these systems respectively. Also shown in these figures are the roots of the uncompensated systems. Two of the systems are stable: whereas, the 1300 system is unstable. Therefore, the use of compensation for the 1000 and 1200 systems is strictly for the purpose of relocating the roots to more desirable positions. On the other hand because the 1300 system is initially unstable, the compensator will be used to stabilize this system. Nevertheless, in each of these three systems the effect of the compensator is sufficiently similar so that analysis of the group in general is warranted.

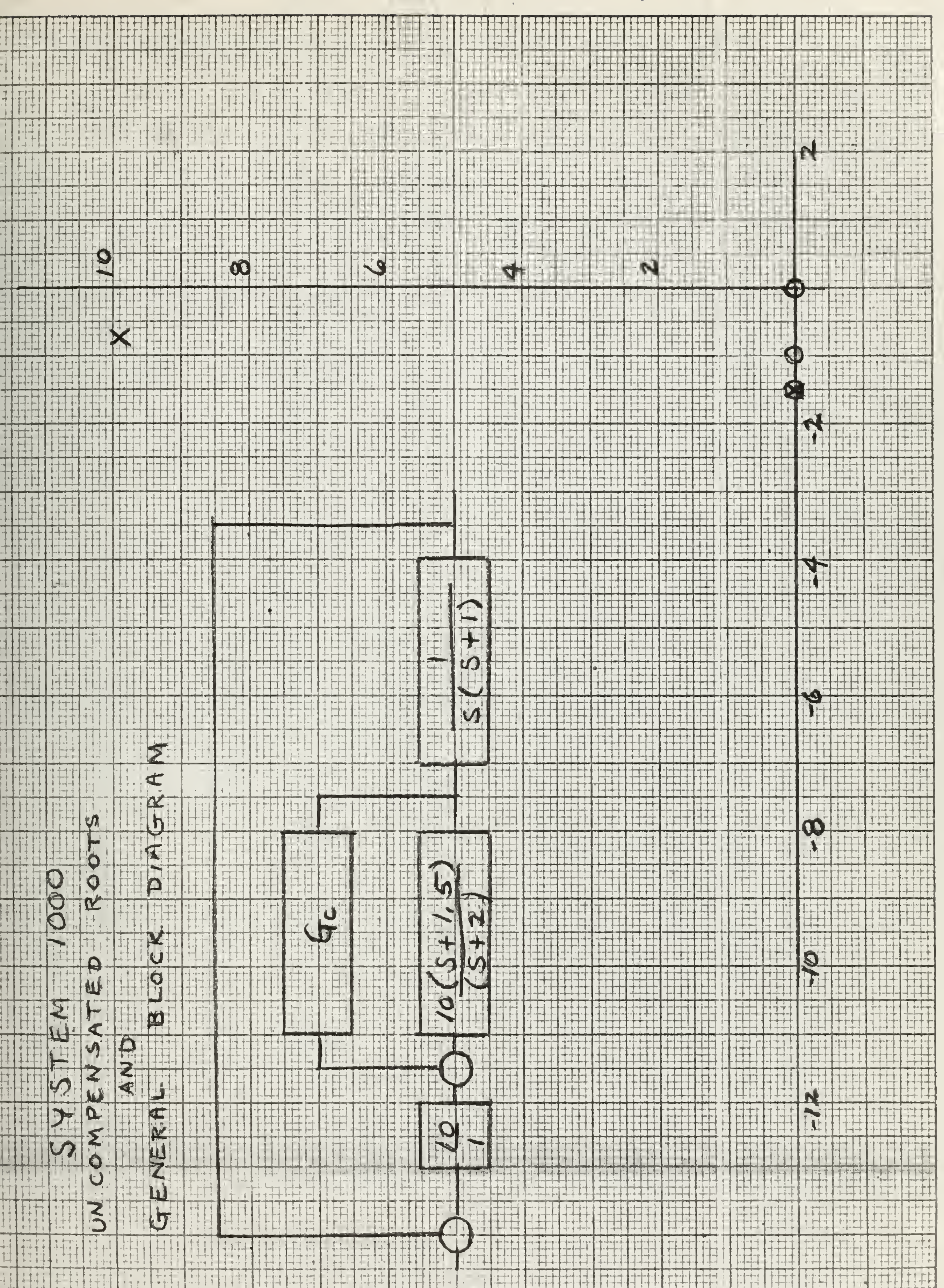
b. Completely satisfactory compensators.

Only three of the compensators investigated were completely satisfactory in producing stability or improving stability. Because one of these compensated systems, 1330, is more analogous to systems 1030 and 1230, whose compensation is considered to be only partially satisfactory, discussion of it will be deferred until later. The other two compensators will be briefly analyzed below.

(1) Lag network.

This compensator is quite effective in providing a favorable variation in the roots of the system. It lends considerable flexibility in that a wide range of combinations of the values of ζ and ω_n is made available.

The root loci for the compensated systems are shown in figures 5-4 to 5-9. In particular, figures 5-4 to 5-6 illustrate



SYSTEM 1000
UNCOMPENSATED ROOTS
AND
GENERAL BLOCK DIAGRAM
10/1
10(S+1.5)/(S+2)
1/S(S+1)
Gc
10
8
6
4
2
2
-2
-4
-6
-8
-10
-12

figure 5-1.

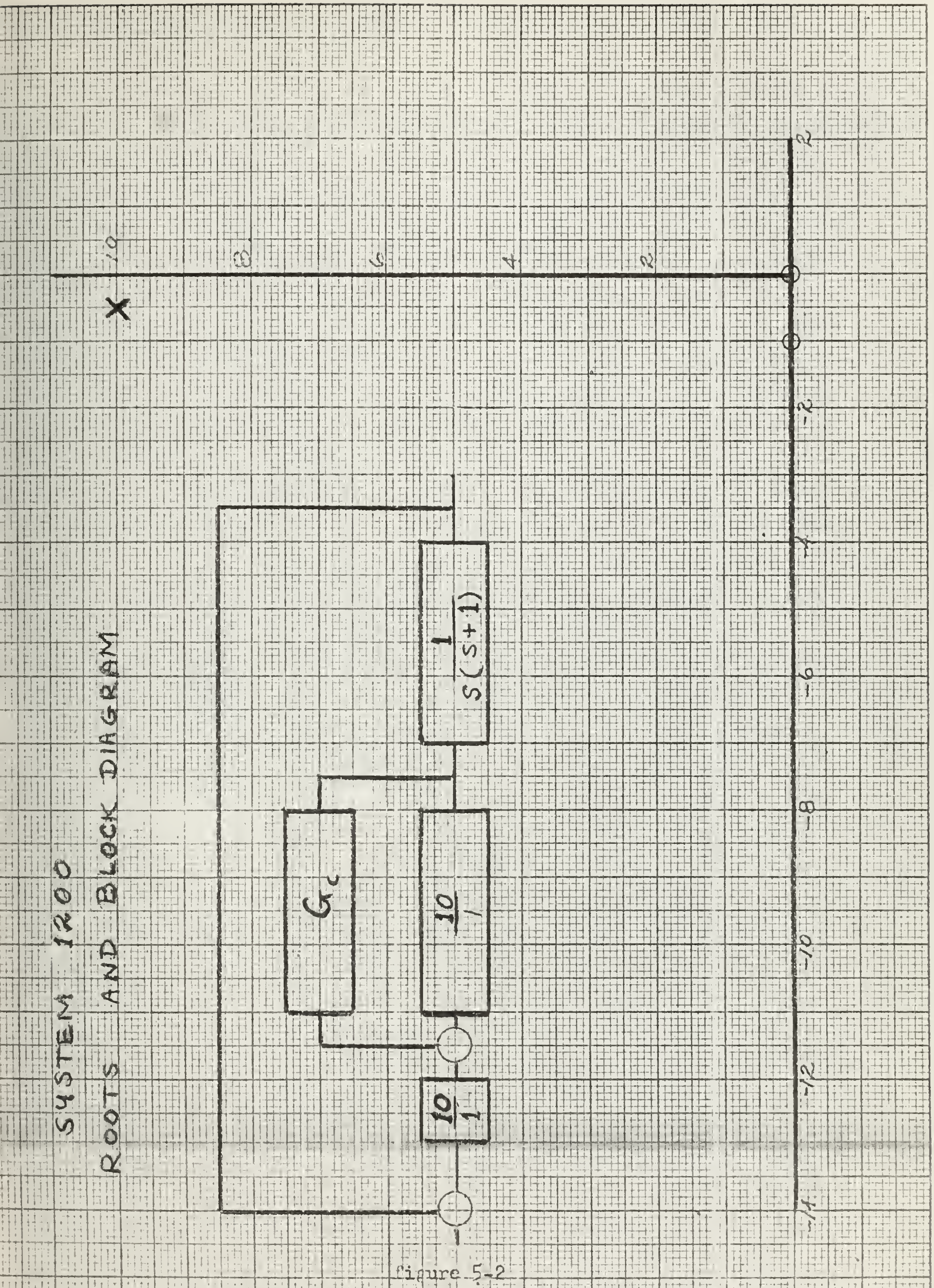

SYSTEM 1200
ROOTS AND BLOCK DIAGRAM
10/1
G_c
10/1
1/S(S+1)
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-14
2

figure 5-2

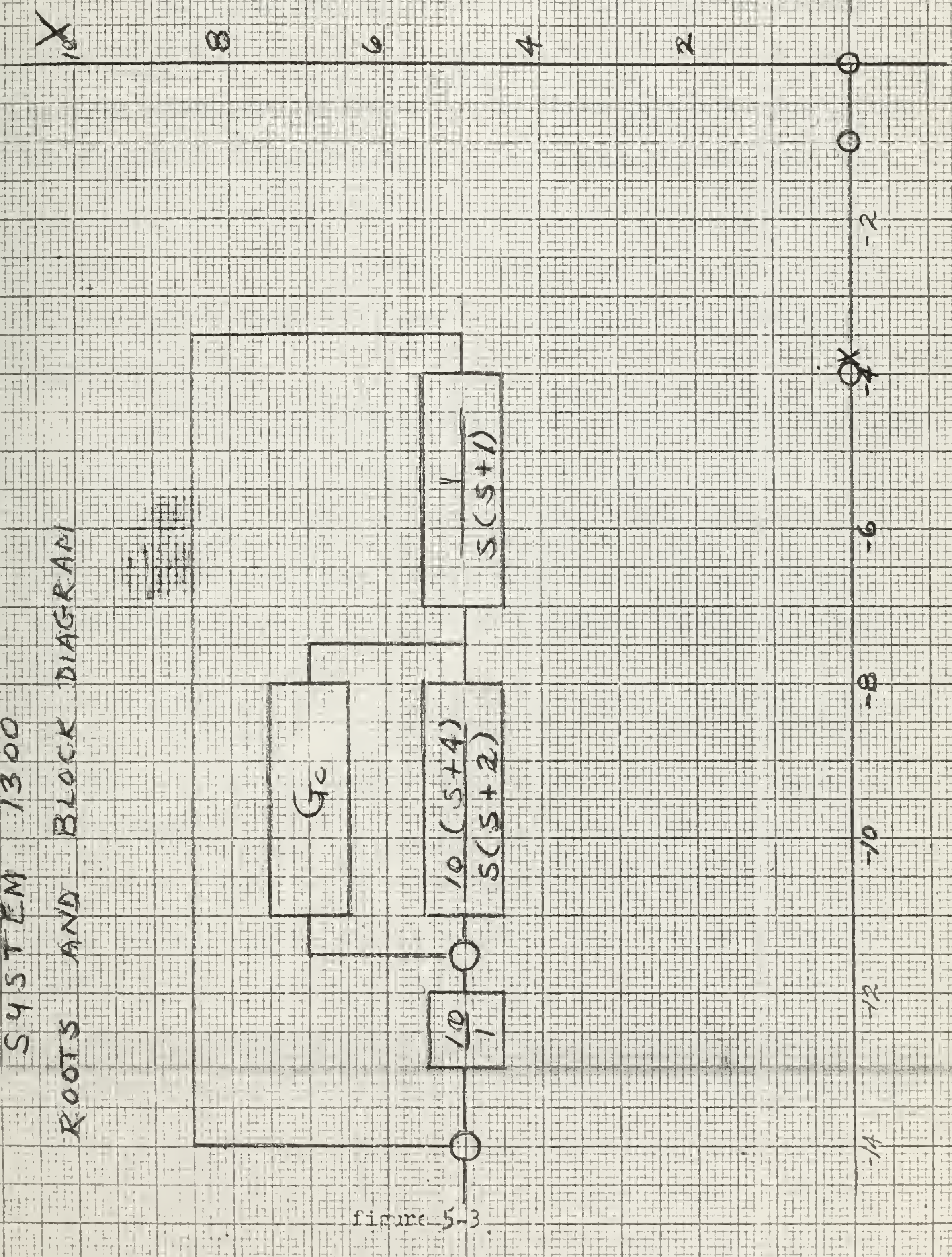

SYSTEM 1300
ROOTS AND BLOCK DIAGRAM
Gc
10 (S+4)
S(S+2)
1
S(S+1)
10
1
8
6
4
2
2
-2
-6
-8
-10
-12
-14

figure 5-3

the "10" compensator, which is the lag compensator for a greater than 1; while figures 5-7 to 5-9 illustrate the "20" compensator for a greater than 4.

The reason that the effect of the "10" and "20" lag compensators differ is due to the magnitude of the pole of G_c. For the "10" lag network the compensator's pole causes pole-zero cancellation to occur with the motor function's pole; whereas, for the "20" lag network the compensator's pole is greater than that of the motor function. If the pole of G_c had been made less than that of G_m then the root loci obtained would have been similar to those plotted in figures 5-4 to 5-6.

By use of this compensator the stability may be brought about in a basically unstable system such as shown by the root loci of figures 5-6 and 5-9. Provided k_c is made large enough, stability will occur for every value of a. These minimum values of k_c, or k_{cr}, depend to some extent on the value of a used and are listed in table 5-1.

For both the unstable and stable basic system, lag network compensation allows the designer much latitude in meeting design specifications. Whether the pole of the compensator is larger or smaller than that of the motor function, good flexibility will persist. However, the flexibility provided by the "10" lag network is somewhat more extensive than that provided by the "20" lag network.

For systems compensated by either of the lag network compensators the possible variation in ζ and ω_n is extensive. Depending on whether or not the basic system was stable, ζ may be varied from a small value or zero to 1.0 by increasing k_c and maintaining a constant, or by increasing a while maintaining k_c constant.

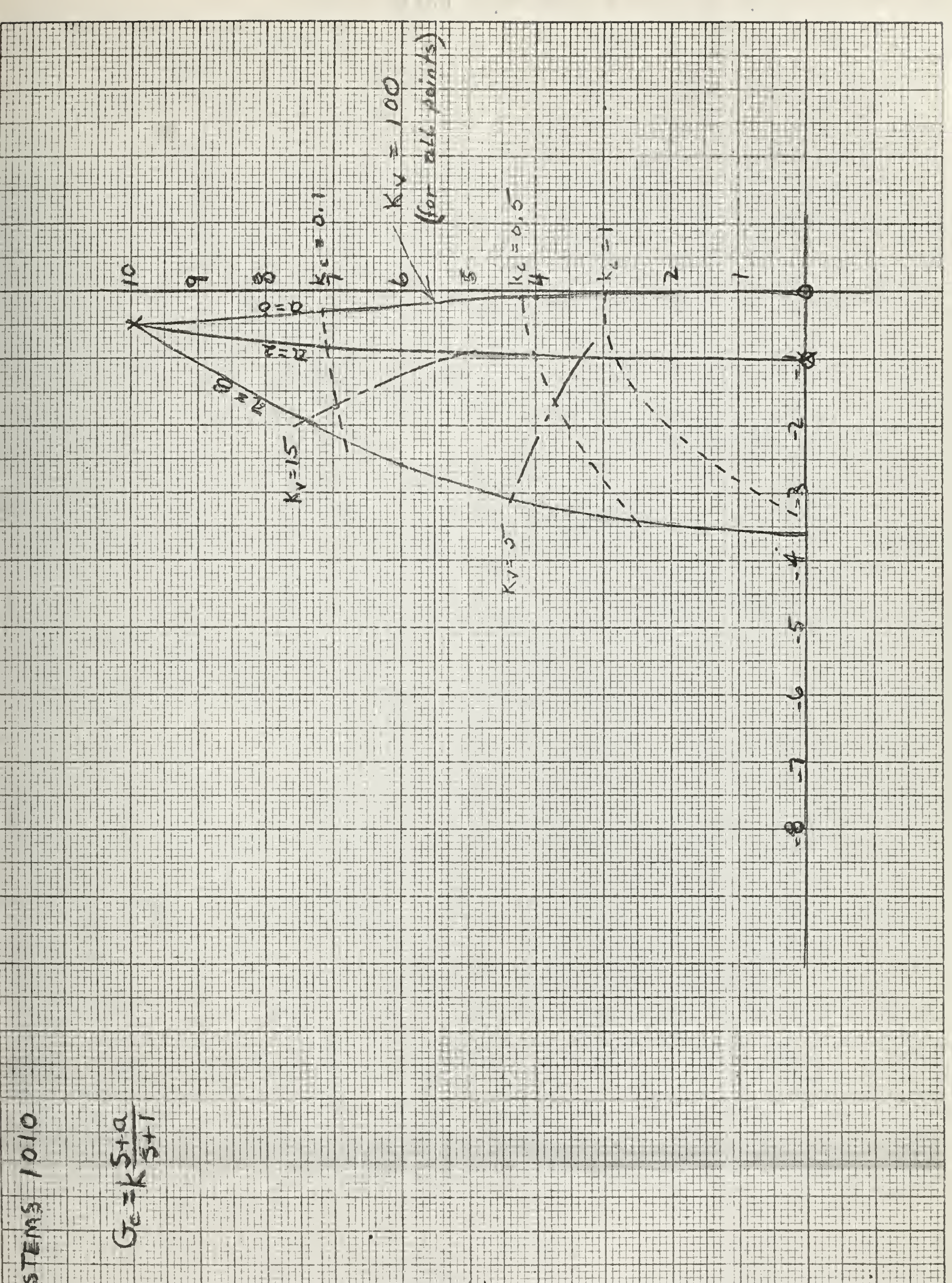

SYSTEMS 1010
$G_c = k\frac{s+a}{s+1}$
$K_v = 100$
(for all points)
$K_v = 15$
$K_v = 5$
$k_c = 0.1$
$k_c = 0.5$
$k_c = 1$
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8

figure 5-4

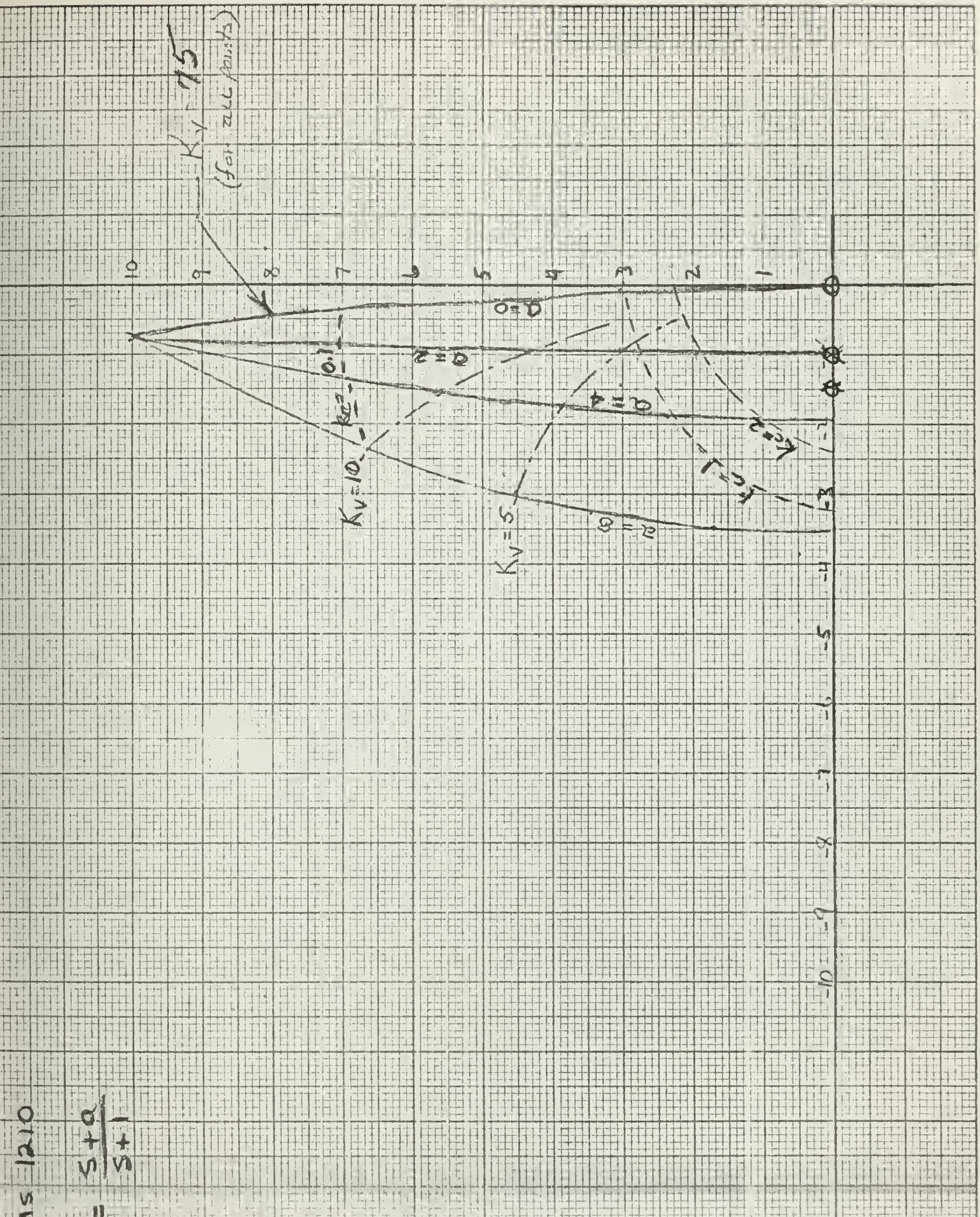
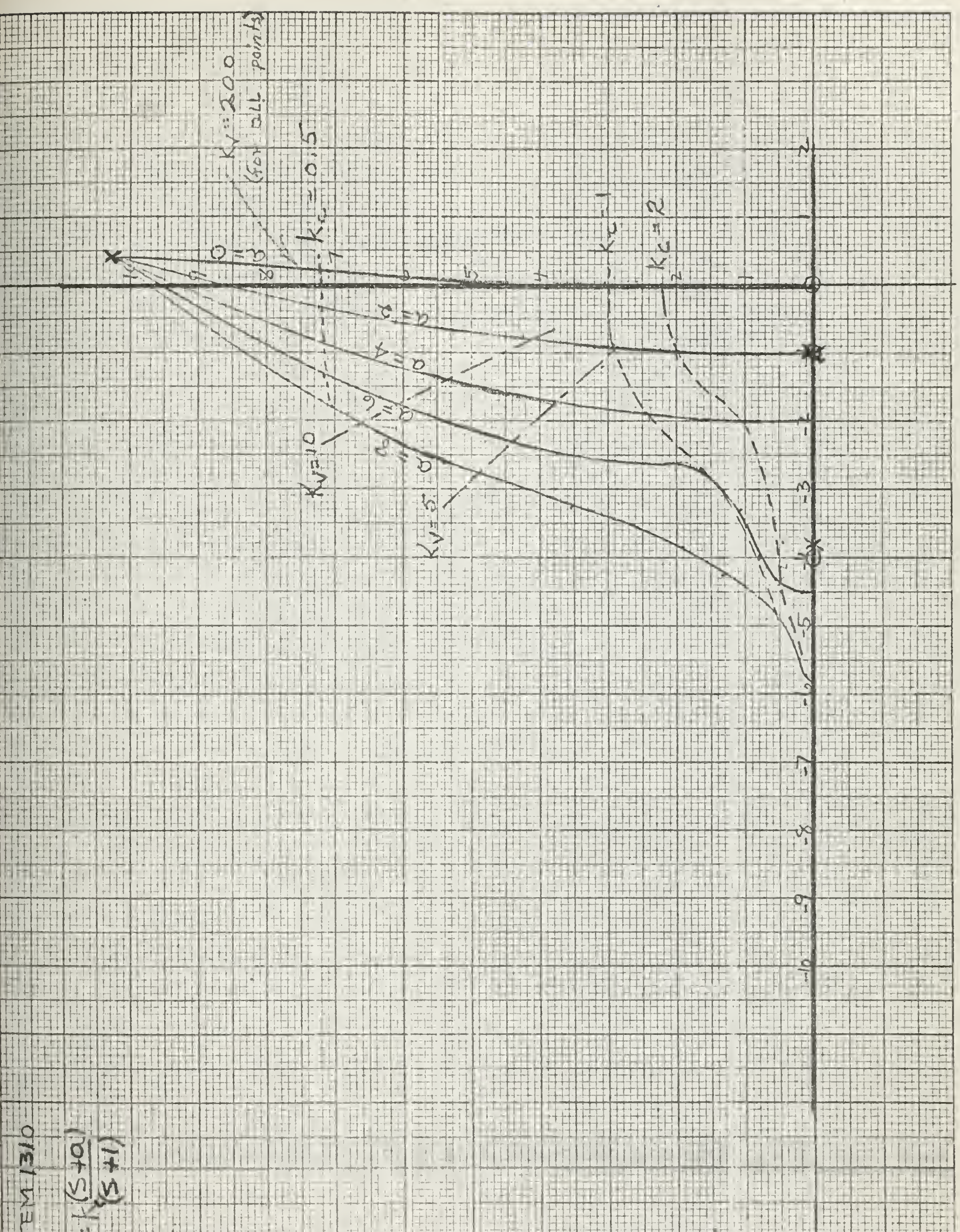

SYSTEMS 1210
$G_c = \frac{s+a}{s+1}$
$K_V = 15$ (for all points)
$K_V = 10$
$K_V = 5$
a=0
a=2
a=4
a=8

figure 5-5



SYSTEM 1310
$G_c = K_c \frac{(s+a)}{(s+1)}$
Kv = 200 (for all points)
Kc = 0.5
Kc = 1
Kc = 2
Kv = 10
Kv = 5

figure 5-6

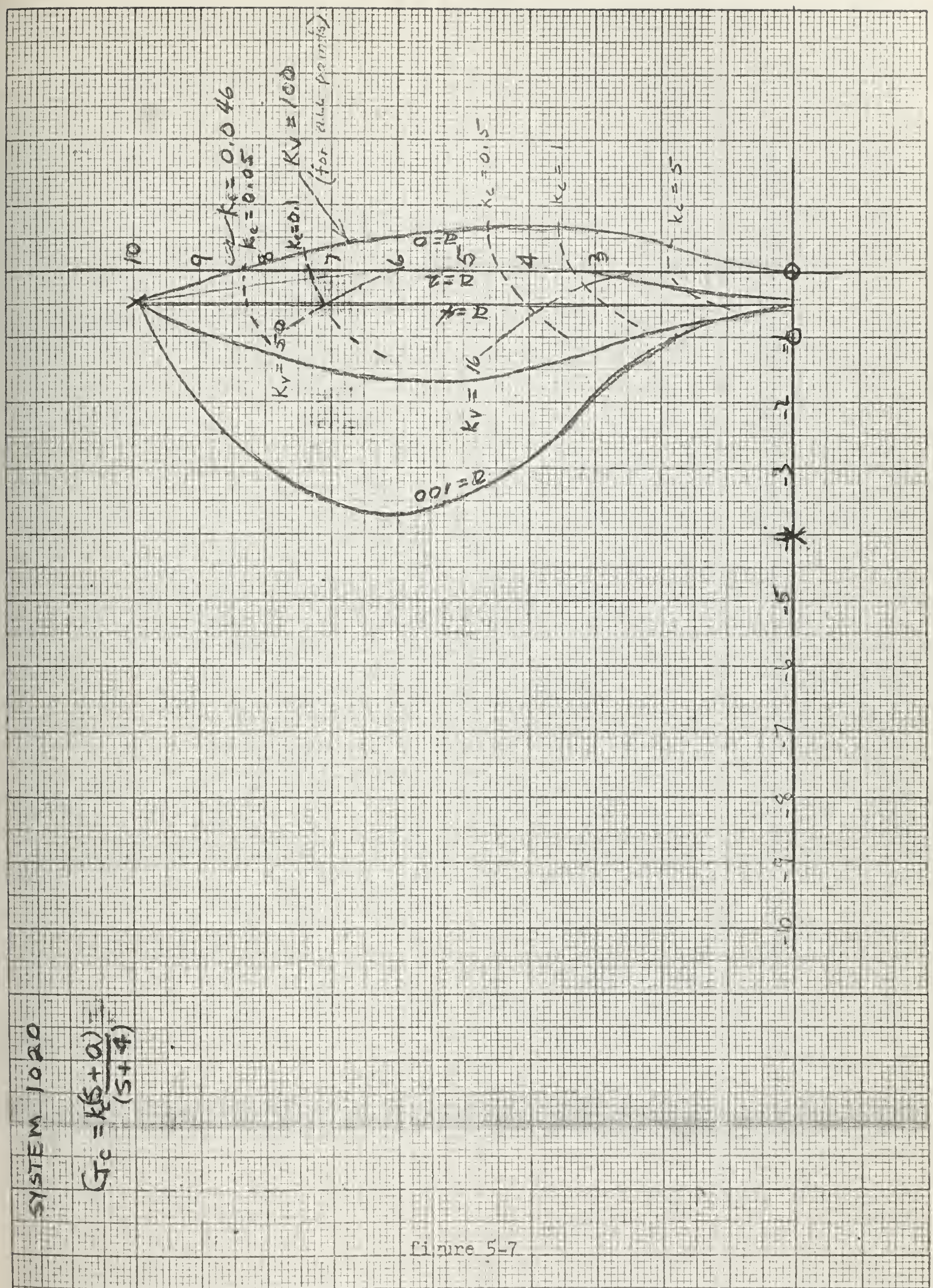
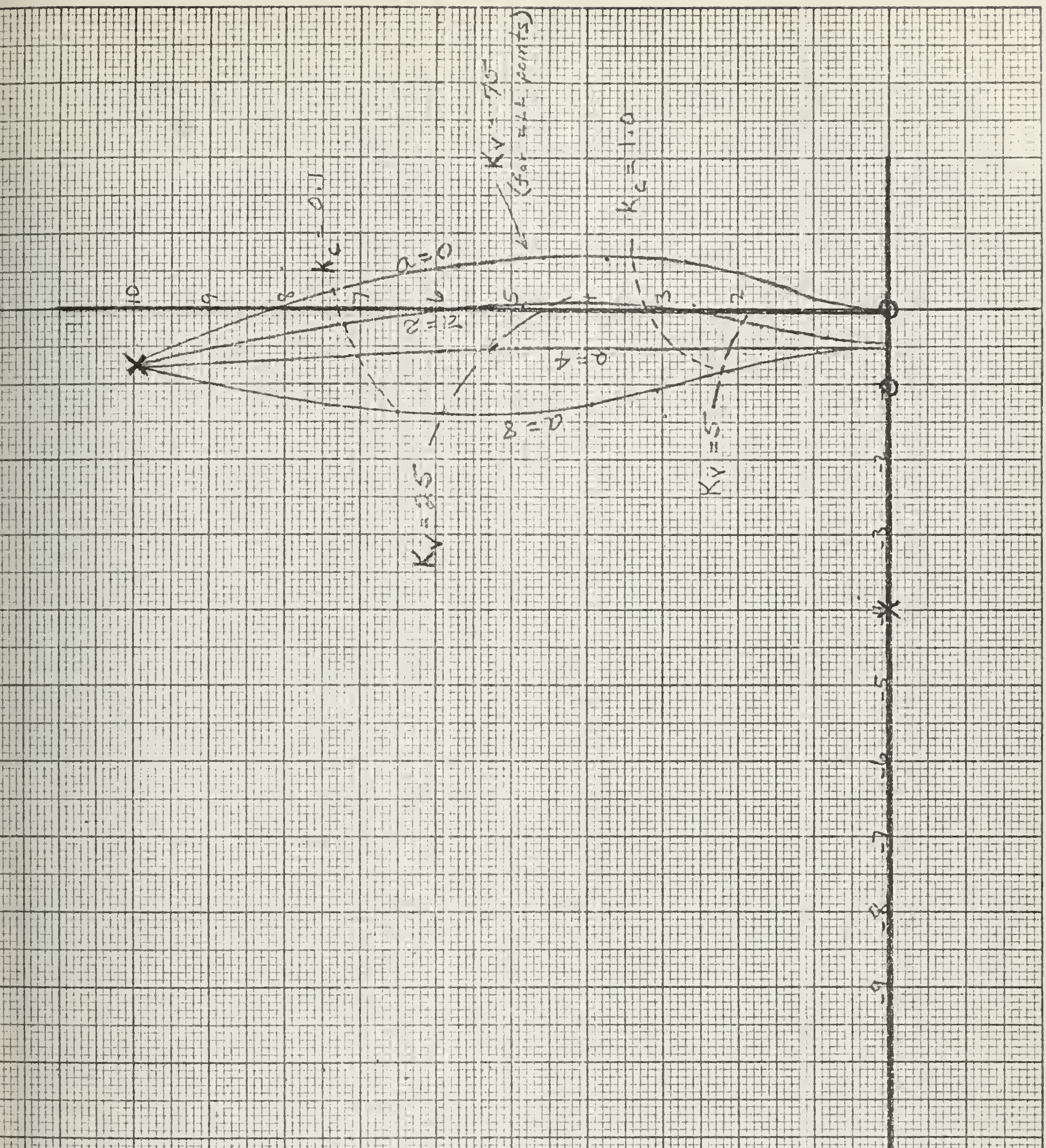

SYSTEM 1020
$G_c = \frac{k(s+a)}{(s+4)}$
Kv = 100
(for all points)
Kc = 0.046
Kc = 0.05
Kc = 0.1
Kc = 0.5
Kc = 1
Kc = 5
a = 0
a = 2
a = 4
a = 100
Kv = 50
Kv = 16

figure 5-7



SYSTEM 1220
$G_c = k\frac{(S+a)}{(S+4)}$
a=0
a=2
a=4
a=8
$K_c = 0.1$
$K_c = 1.0$
$K_V = 25$
$K_V = 5$
$K_V = 75$ (for all points)
10
9
8
7
6
5
4
3
2
-2
-3
-4
-5
-6
-7
-8
-9

figure 5-8

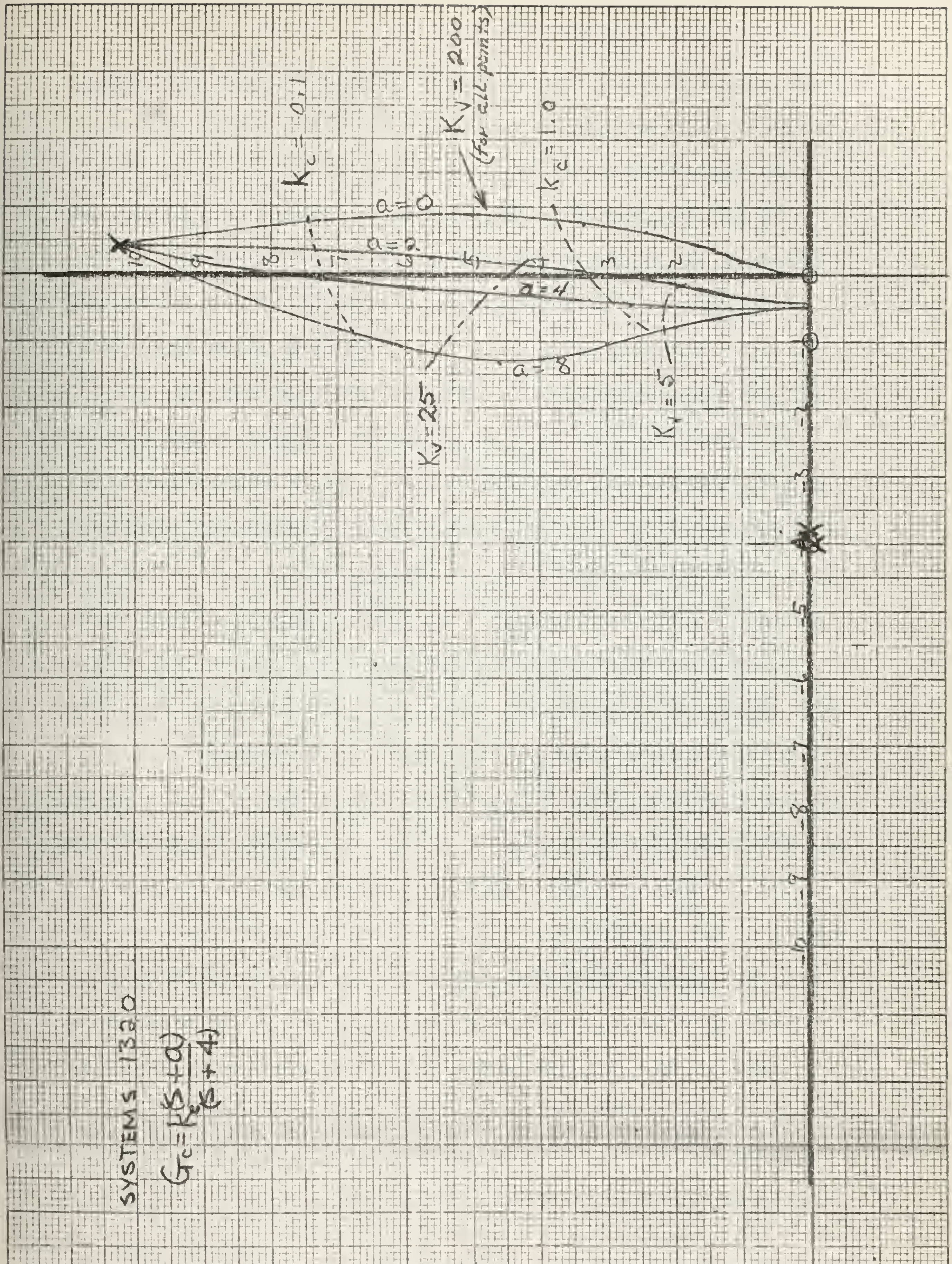

SYSTEMS 1320
$G_c = K_c \frac{(s+a)}{(s+4)}$
$K_v = 200$
(For all points)
$K_c = 0.1$
$K_c = 1.0$
$K_v = 25$
$K_v = 5$
a = 0
a = 2
a = 4
a = 8

figure 5-9

However, the variation possible for ζ is more sensitive to the particular compensator used. For ζ constant ω_n can be increased by increasing a. For the "10" compensator this increase is significant for all ζ but greatest for a large ζ. In contrast, for the "20" compensator this increase is much smaller for all ζ, varying from nearly zero for ζ large to a maximum for this compensator for in the range 0.7 to 0.4.

The primary difference between the effects produced on the three systems by the compensator consists of the lack of complete correlation between the root loci of the 1000, 1200 and 1300 systems. This incomplete correlation is in all probability due to the significant difference between the roots of the uncompensated systems. This supposition is supported by the fact that the correlation is greater between the two stable basic systems than between either of these systems and the unstable basic system. However, for ζ greater than 0.5 this correlation is very good among all three systems particularly for a small. But one exception to this does exist - the root loci for the 1310 system changes erratically to allow larger values of ω_n to be available for those values of a which would be favorable.

The fact that more deviation in the root loci does not occur is significant. This suggests that the relationship among the poles and zeros of the G_b function which influences the root loci the most is the excess of the number of poles over the number of zeros and not so much the actual size of these poles and zeros. Thus this fact indicates that the primary consideration to be made in attempting to normalize the plotted systems is the size of the poles and zeros of the motor function.

One other fact becomes significant in comparison of the root

loci. This is the fact that for the "20" compensator, correspondence was excellent for large values of ζ. Also noted was the fact that all the root loci, except that for $\underline{a}$ equal to zero, entered the negative real axis approximately half way between the zeros caused by the poles of the motor function. Thus, based on these facts it is conceivable that increasing or decreasing the distance between these zeros by some factor will also cause the root loci entrance point to move by the same factor. Such a change in the root loci would be reflected as a decrease or increase in ω_n for large values of ζ. To a certain degree this implies that normalization of this compensated system is possible by only considering the difference in the poles and zeros of the motor function.

(2) "60" compensator.

Of the two compensators in Group IV which are considered to be completely satisfactory, this one, by far, appears to be the most effective. Not only is it highly capable of producing stability, but also it provides exceptional flexibility to the designer in allowing him to select roots having a wide range of values of ζ and ω_n. The root loci of figure 5-10 is a good example of this compensator's ability to stabilize a system. In this system stabilization occurs almost immediately with k_c approximately the same for each value of $\underline{a}$. These values of k_{cr} are listed in table 5-1.

Because of the exceptional flexibility provided by this compensator, a wide choice in ζ and ω_n is available through variation of k_c and $\underline{a}$. As indicated in figures 5-10 to 5-12 any desired value of ζ ranging from that of the uncompensated system, or 0 for system 1060, to 1 may be obtained by increasing k_c from 0, or k_{cr}, while maintaining $\underline{a}$ constant. Likewise, desired values of ω_n may

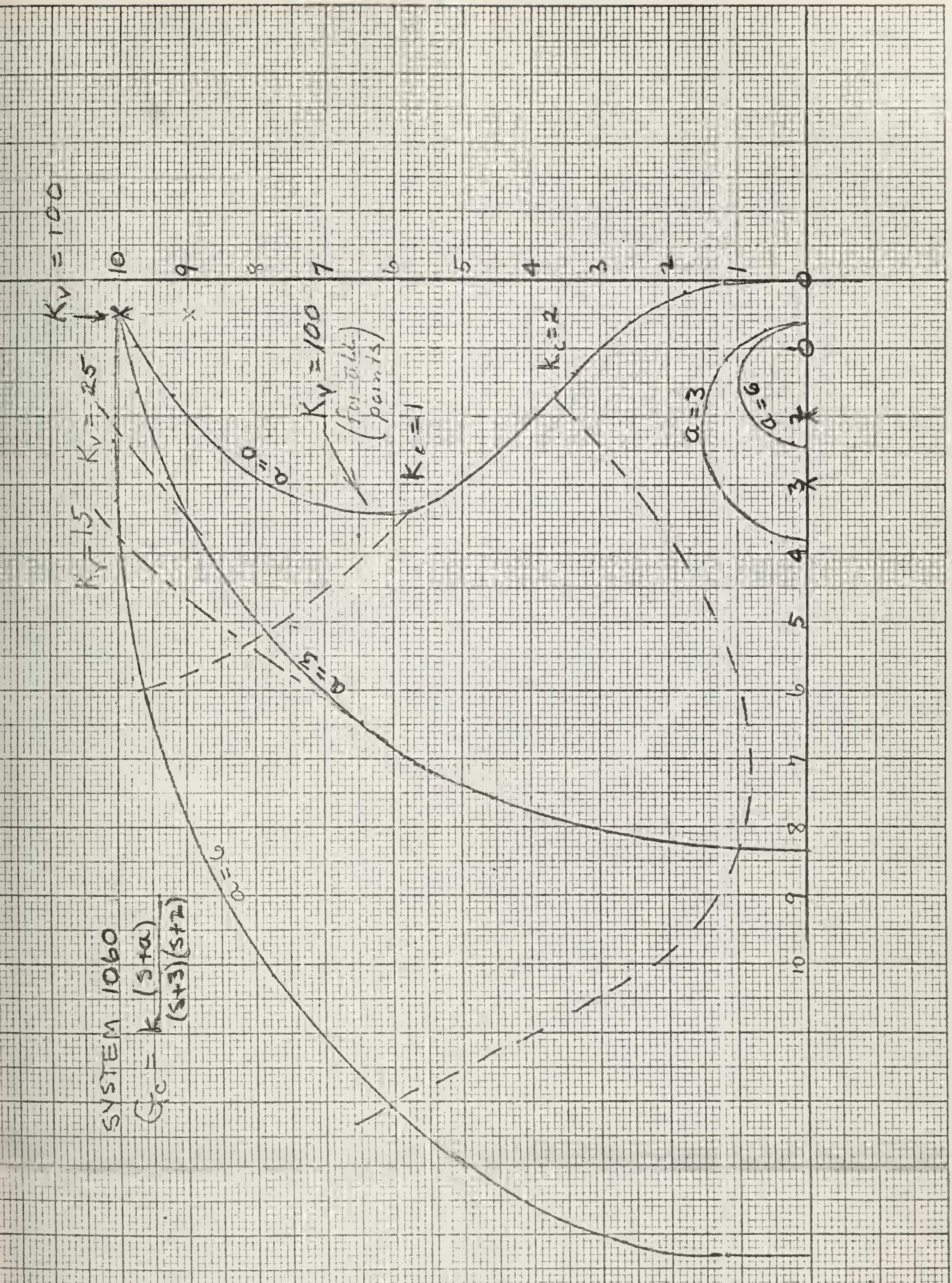

SYSTEM 1060
$G_c = k \frac{(s+a)}{(s+3)(s+2)}$
$K_v = 100$
$K_v = 25$
$K_v = 15$
$K_v = 100$
(for odd points)
$K_c = 1$
$K_c = 2$
$a = 0$
$a = 3$
$a = 6$
$a = 9$

figure 5-10

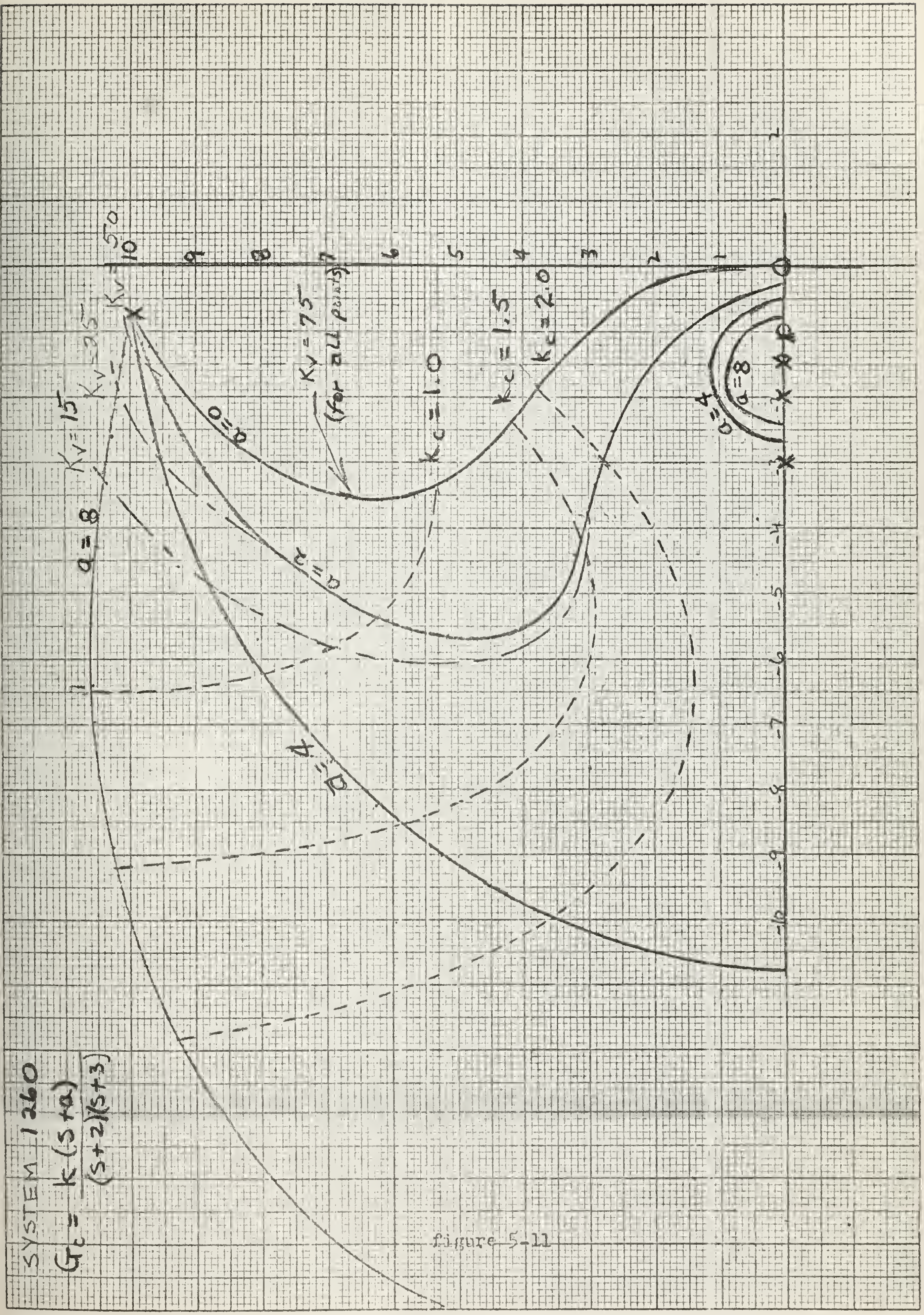
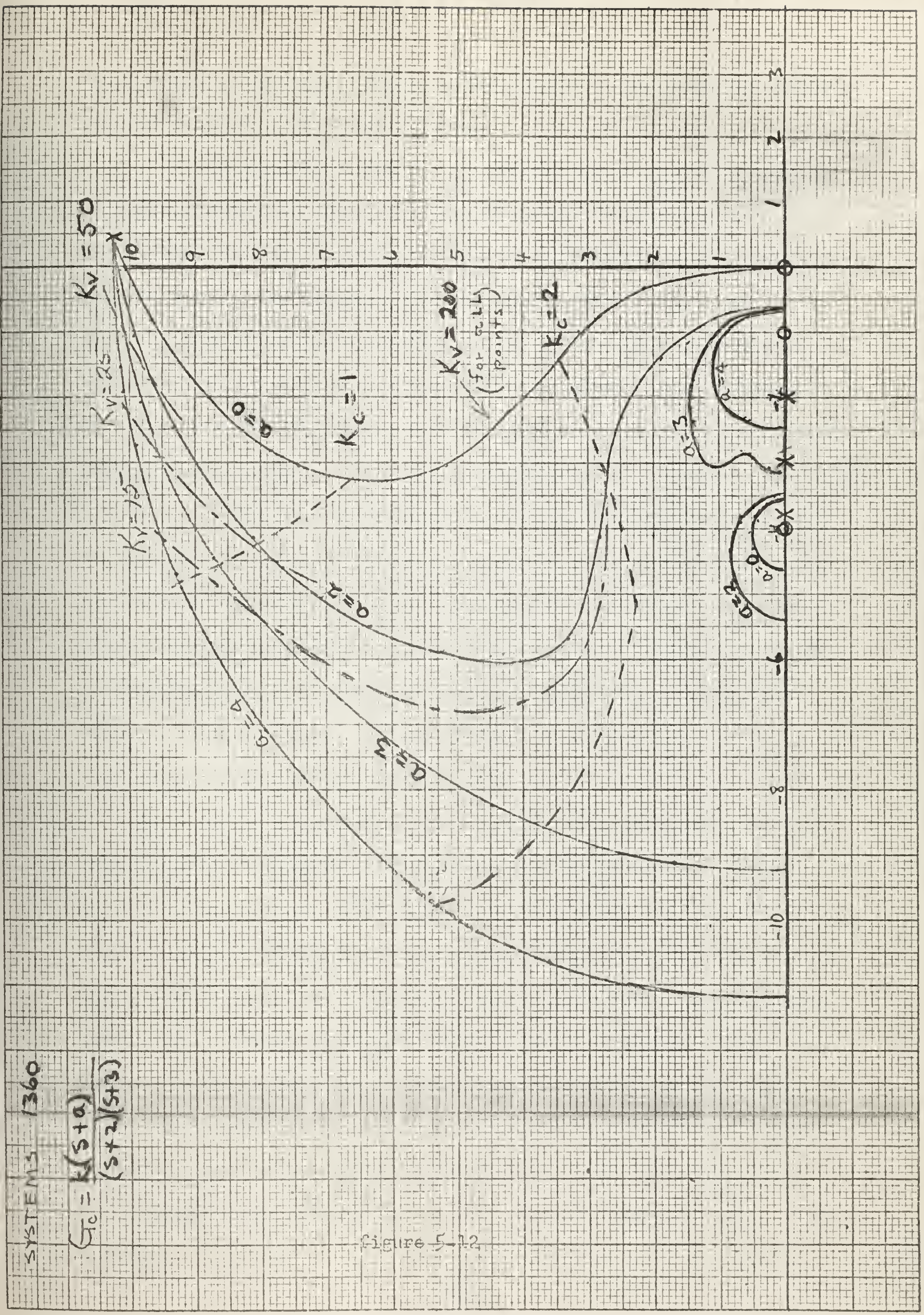

SYSTEM 1260
$G_c = \frac{k(s+a)}{(s+2)(s+3)}$
a = 8
Kv = 15
Kv = 35
Kv = 50
Kv = 75
(for all points)
a = 0
a = 4
a = 8
kc = 1.0
kc = 1.5
kc = 2.0

figure 5-11



SYSTEMS 1360
$G_c = \frac{K(s+a)}{(s+2)(s+3)}$
$K_v = 50$
$K_v = 25$
$K_v = 15$
$K_v = 200$ (for all points)
$K_c = 1$
$K_c = 2$
a = 3

figure 5-12

be obtained by varying a; however, the minimum value of ω_n is limited by that obtainable for a equal to 0.

Although the effect of the compensator on the three systems is basically similar, there are some minor differences which should be noted. One of these is the fact that the value of K_v when a equals 0, differs between the systems. For the 1060 system, K_v is equal everywhere to 100, while for the 1260 and 1360 systems K_v equals 75 and 200 respectively. The only other difference worthy of mentioning is the lack of correlation in both the gain contours and the root loci for the three systems. This is only noticeable for a large or ζ small. Apparently, in view of the fact that a greater lack of correspondence was observed between the stable and unstable systems than between the stable ones, this difference is a function of that between the dominating complex roots of the uncompensated systems. Nevertheless, it is significant to note that excellent correspondence did occur for values of a up to about 3 and of ζ down to about 0.4.

C. Partially satisfactory compensators.

Three of the compensators investigated are considered to be only partially satisfactory in compensating the systems for either or both of two reasons. The first is the fact that the compensator does not produce stability for all values of a, while the second is the fact that when a satisfactory value of a is chosen, stability only occurs for k_c within the finite range of values listed in table 5-1. However, based on the above, compensation of the 1060 system using these three compensators is an exception because neither of the above reasons apply. Nevertheless, this system has been included at this time because it is very similar to the other two systems in all other respects.

A brief discussion of the effect of these three compensators on the three basic systems follows.

(1) Lead network.

Over all, the effectiveness of the lead network is considerably less than that of the two previously discussed in part (B) above. While it does provide compensation, the effectiveness of this compensation is not too favorable and in some cases detrimental to the stability of a basically stable system. In addition the flexibility of this compensator is somewhat restricted, which also reduces its effectiveness.

Primarily, the effectiveness of this compensator depends on two factors: the size of the compensator's pole and the relative size of its zero with respect to this pole. If the pole of the compensator is greater than that of the motor function then this compensator will cause instability to occur for a equal to 0 as shown in figures 5-7 and 5-8. At the same time if a is greater than 0, instability may or may not occur depending on its relative size with respect to the compensator's pole. Specifically, as shown in figures 5-7 and 5-8 stability is maintained for a slightly greater than 2 which is slightly greater than one-half the size of the compensator's pole. It is of interest to also note here that even if instability does occur as a result of this compensation, a further increase in gain is all that is necessary to again render the system stable. Now on the other hand, for the case where the compensator's pole is equal to or less than that of the motor function, the lead network will only cause instability to occur when k_c is equal to infinity as shown in figures 5-4 and 5-5.

To a lesser extent, the effectiveness of the lead network

depends on whether the uncompensated system is stable or unstable. Figures 5-6 and 5-7 illustrate this compensator's effect on a system which is basically unstable. From these root loci one might readily conclude that it is not possible to compensate an unstable system using this compensator with a equal to 0. However, for a sufficient value of gain, compensation does render the system stable when a does not equal 0.

One of the factors which detracts from the effectiveness of the lead network is the limitations placed on the variables a and k_c by the necessity for stability. The range of combinations of ζ and ω_n available in the selection of roots depends primarily on a and k_c. In particular ζ can be decreased by decreasing k_c and maintaining a constant or by increasing a and holding k_c constant provided the mean value of k_c is small (less than 0.1 for figure 5-8). However, if the mean value of k_c is large (about 2.0 for figure 5-8), then the effects of varying a and k_c would be just the opposite.

Other than the fact that the dominating complex roots of the uncompensated systems are not the same, there is only one significant difference between the compensated systems' root loci. This difference is the fact that the K_v for the a equal to 0 root locus, although a constant, is different for each system. Nevertheless, in spite of these differences the correspondence between the root loci of the three compensated systems is excellent for other than small values of ζ. However, the limiting ζ for good correspondence depends on a.

(2) "30" compensator with a equal to 0.

The use of first derivative and proportional feedback signals together does constitute effective compensation, but this compensating effect is due more to the proportional part than the first derivative

effect is due more to the proportional part than the first derivative part of the feedback signal. Figures 5-12 to 5-15 show the effect of using this type of compensation on both stable and unstable basic systems. It is readily apparent from these root loci that as $\underline{a}$ increases the usefulness of the compensator improves also. The reason for this is more apparent if one examines the mathematics involved. From method I of section 1, the expression for the compensated open loop function F_{o_c} in terms of the uncompensated open loop function F_{o_u} (assuming $G_b = \frac{S+a}{S+b}$) is:

$$F_{o_c} = F_{o_u} \frac{S+b}{(S+b) + k_c(S+a)(S+C)}$$

where the compensator $G_c = k_c \frac{(S+C)}{1}$

Therefore, as c goes to infinity

$$F_{o_c} \rightarrow F_{o_u} \frac{(S+b)}{k_c(S+a)(S+C)} = \frac{G_a G_m}{k_c \, G_c}$$

which means that, with a reduction in gain, the root locus of the compensated system approaches an inherently stable system as the proportional component approaches infinity.

Due to the predominate influence of $\underline{a}$, one is led to wonder what is the value of using a first derivative component at all. The answer is none, unless one is interested in relocating the basic roots such that ζ and ω_n are less: then use of this compensator may be useful.

The root loci of the 1030, 1230 and 1330 systems differ significantly in just two respects. One of these is the fact that the dominating complex roots of the uncompensated systems are different: the other is the fact that the K_v for the three root loci for $\underline{a}$ equal to 0 are quite different. But in spite of these two differences the three root loci are nearly similar for other than

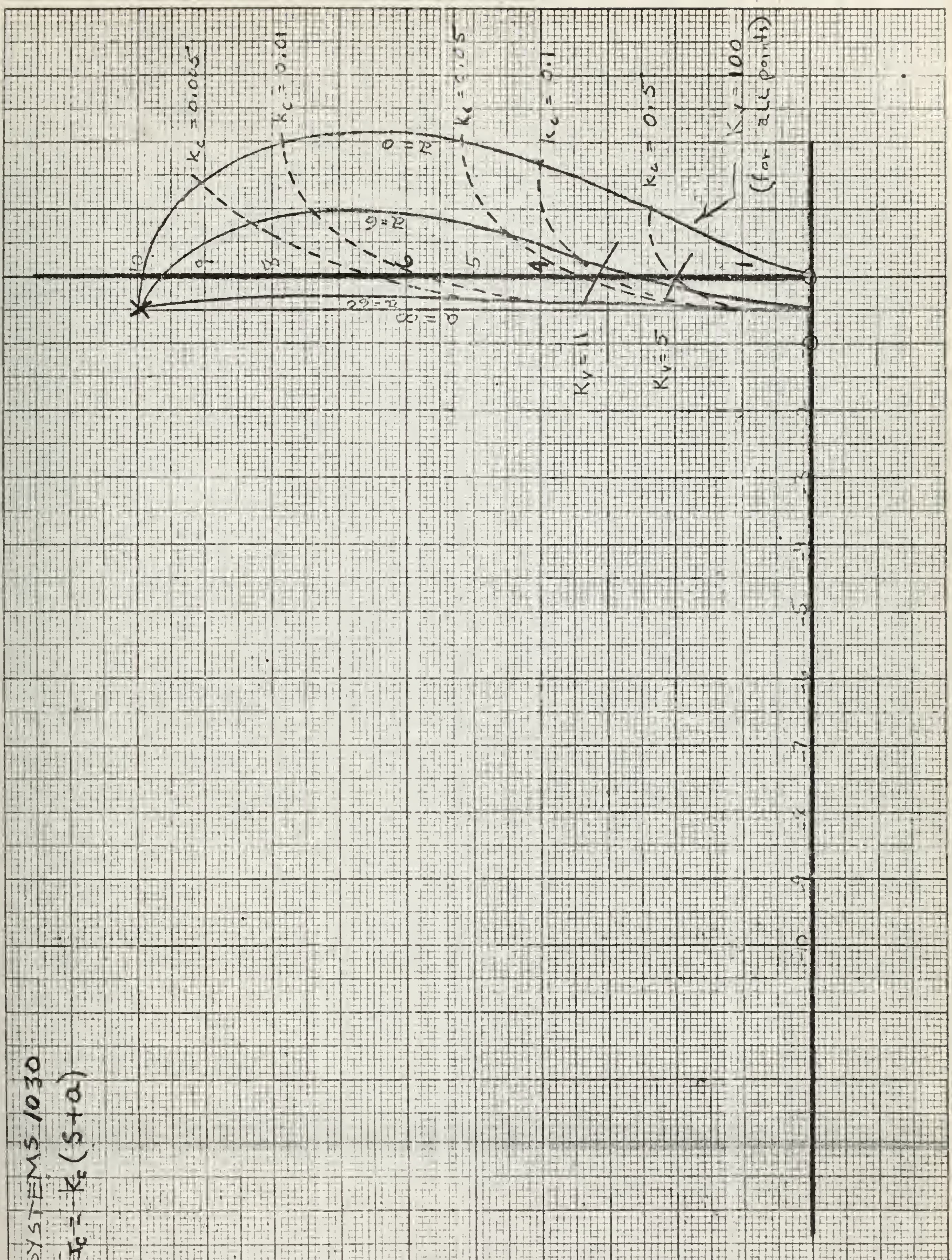
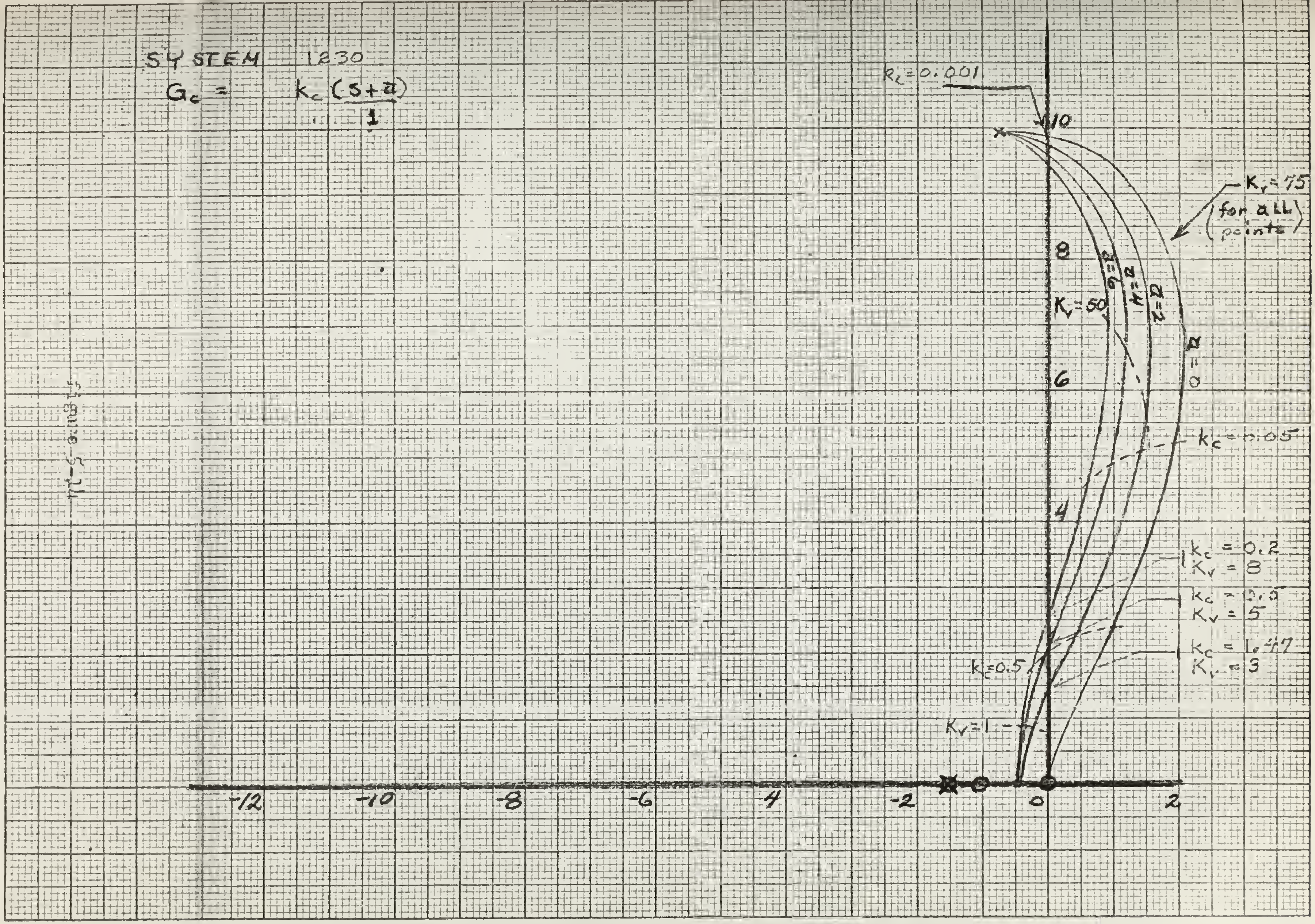

SYSTEMS 1030
Gc = Kc (S+a)
Kc = 0.005
Kc = 0.01
Kc = 0.05
Kc = 0.1
Kc = 0.5
Kv = 100 (for all points)
Kv = 11
Kv = 5

figure 5-13



SYSTEM 1230
$G_c = \dfrac{k_c\,(s+a)}{1}$
$k_c = 0.001$
$K_v = 75$ (for all points)
$K_v = 50$
$a = 6$
$a = 4$
$a = 2$
$a = 0$
$k_c = 0.05$
$k_c = 0.2$ $K_v = 8$
$k_c = 0.3$ $K_v = 5$
$k_c = 1.47$ $K_v = 3$
$k_c = 0.5$
$K_v = 1$
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10

Figure 5-14

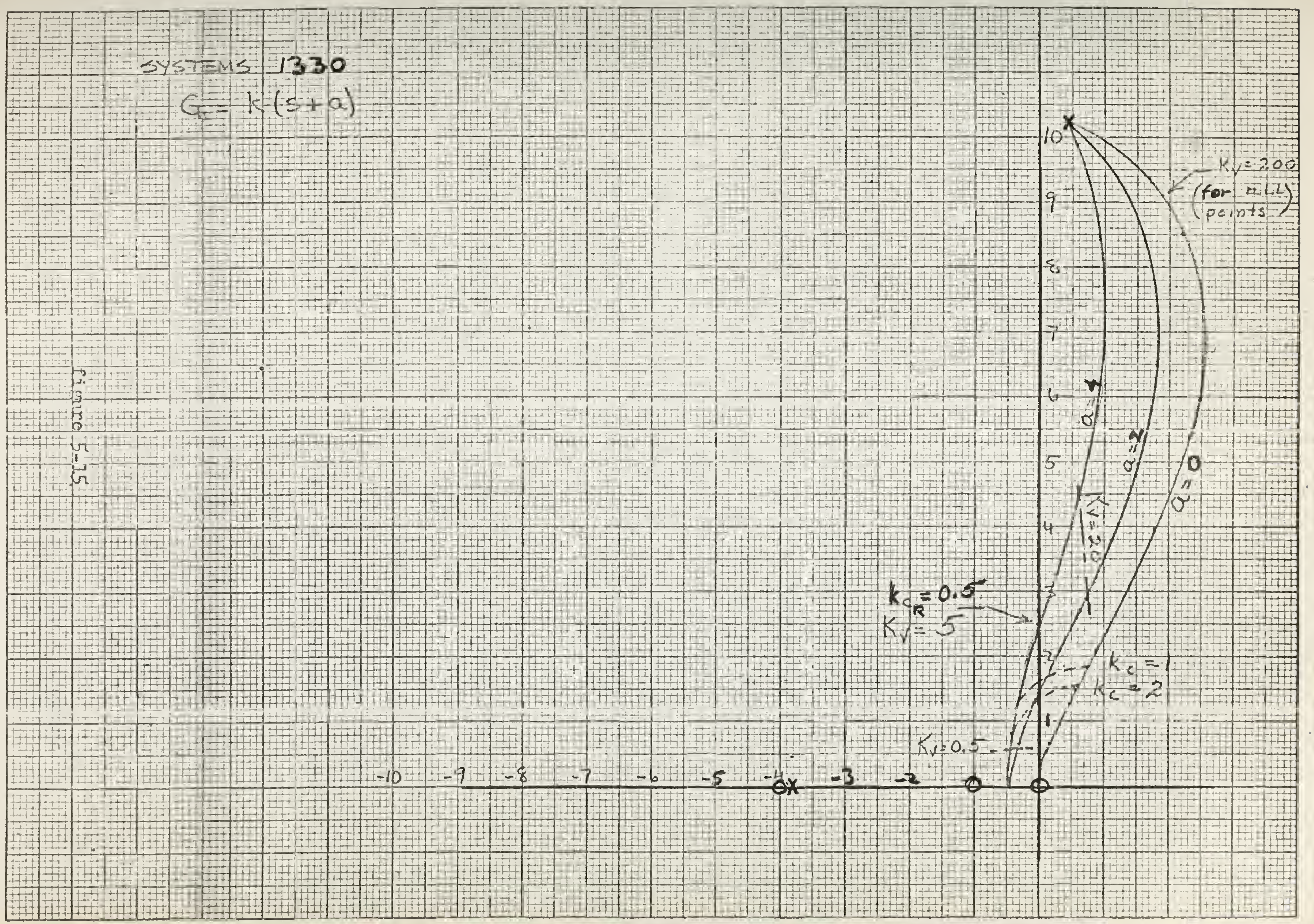

SYSTEMS 1330
G = k(s+a)
$K_v = 200$ (for all points)
a = 4
a = 2
a = 0
$K_v = 20$
$k_{c_R} = 0.5$
$K_v = 5$
$k_c = 1$
$k_c = 2$
$K_v = 0.5$
-10 -9 -8 -7 -6 -5 -4 -3 -2
1 2 3 4 5 6 7 8 9 10

figure 5-15

small values of ζ .

(3) "50" compensator.

Like the compensator just previously discussed, this compensator's effectiveness is also primarily a function of a. As a increases the limiting gain of the stable system, k_{cr} and K_v, both increase. This effect is clearly shown by the gain contours and the limiting values of k_c and K_v in figures 5-16 to 5-18 and table 5-1.

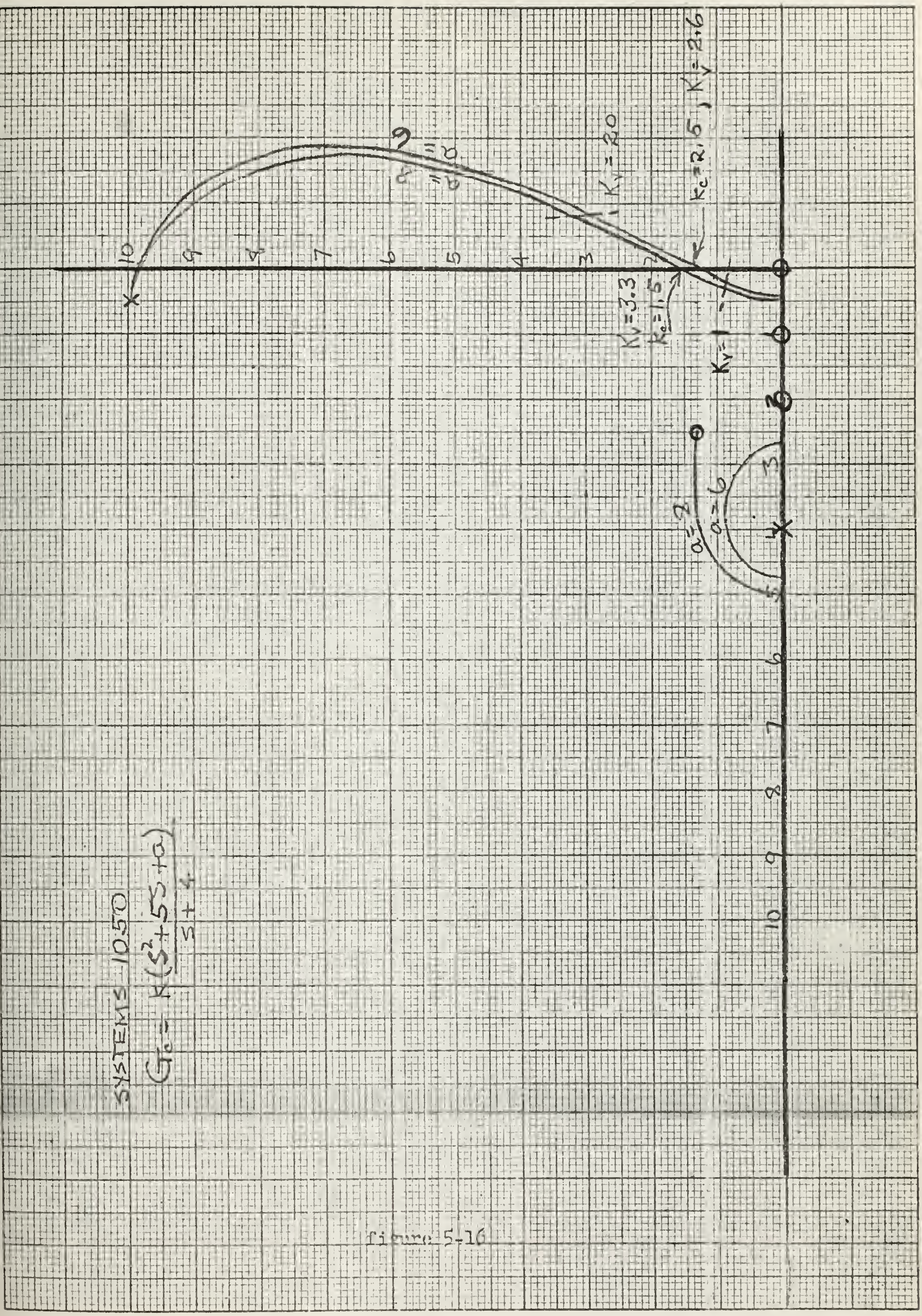
In addition, the limited amount of flexibility which is inherent in this compensator is also similar to that for the "30" compensator (for a not equal to 0). Unless very small values of k_c are used only values of ζ and ω_n smaller than those for the basic system may be obtained. For the case where the uncompensated system is unstable this is true for all values of k_c. But in the situation where the uncompensated system is stable, use of a very small k_c will produce roots of comparable ω_n but smaller ζ than that of the basic system.

For the stable section of the root loci of systems 1050 and 1250 there does not exist any significant differences particularly for k_c large. However, there is considerable difference noted between the root loci of the stable and unstable systems. This difference, of course, is purely a result of the differences in the location of the uncompensated dominating complex roots.

D. Completely unsatisfactory compensators.

Three of the compensators investigated are completely unsatisfactory. This is due to the fact that they fail to compensate the basic system in any way. These compensators are:

1. first derivative feedback - effect shown in figures



SYSTEMS 1050
$G_o = \frac{K(s^2+5s+a)}{s+4}$
a = 9
a = 6
$K_v = 20$
$K_c = 2.5$, $K_v = 2.6$
$K_v = 3.3$
$K_c = 1.5$
$K_v = 1$
a = 2
a = 6

figure 5-16

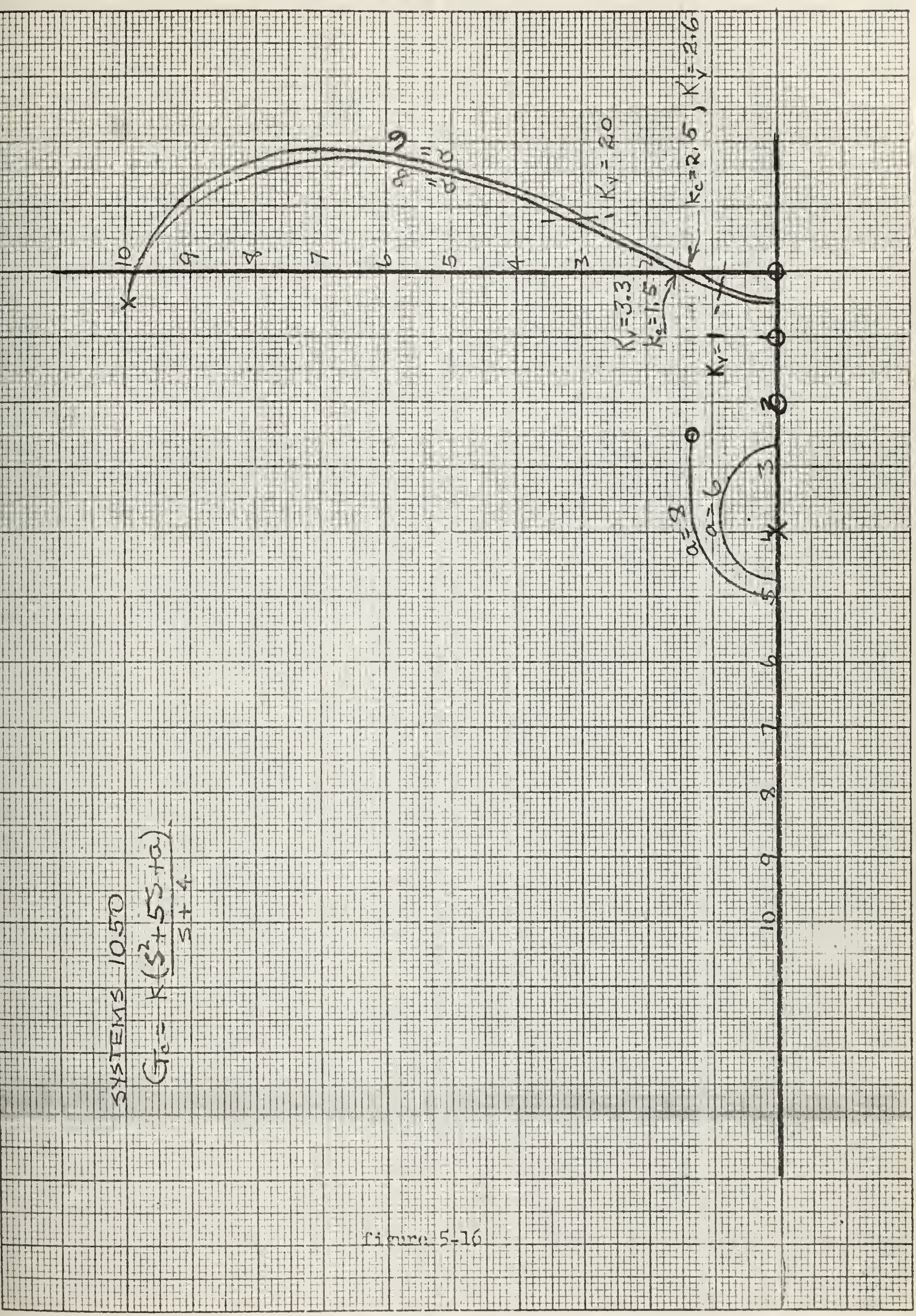


Figure 5-16

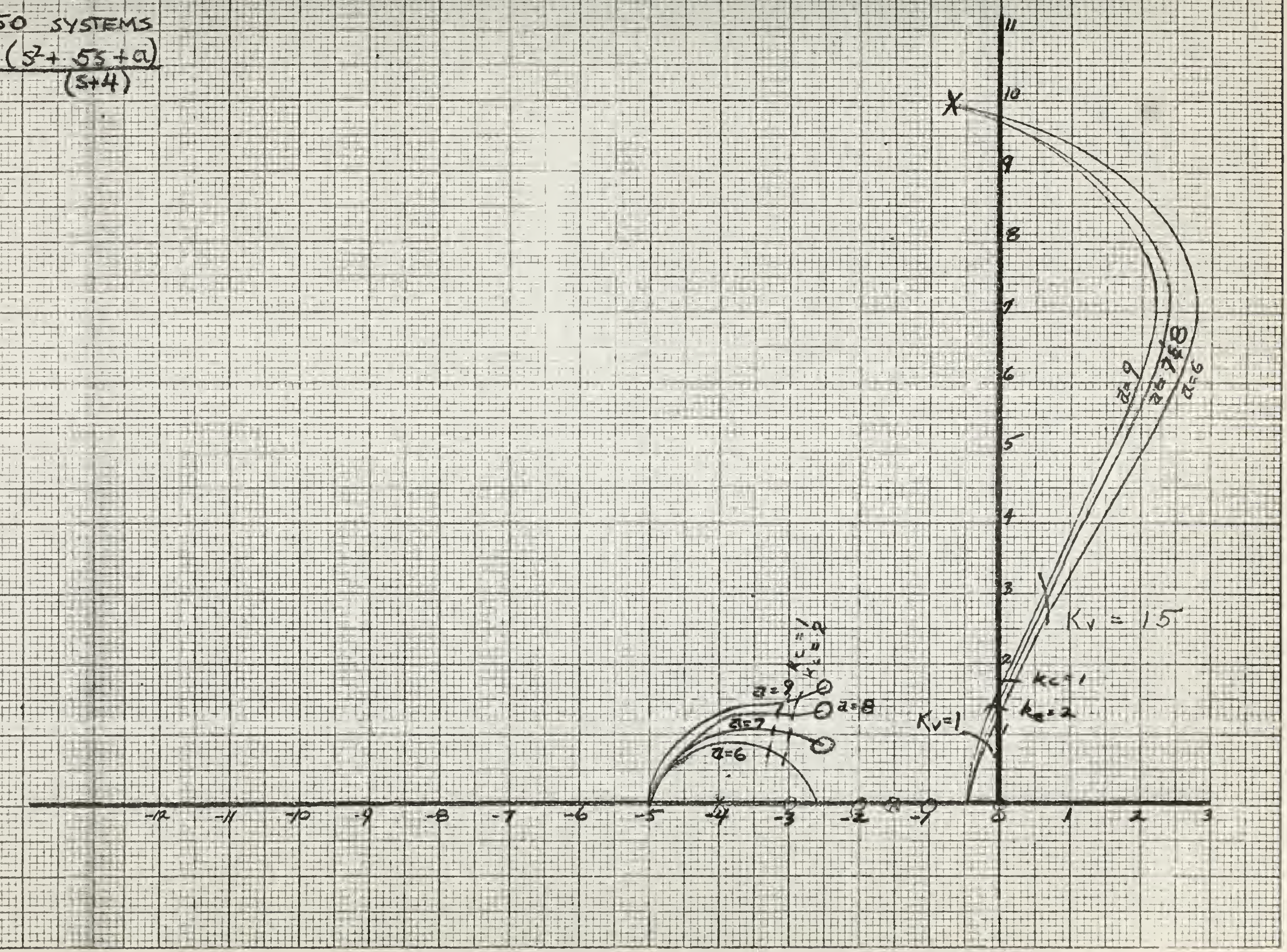

1250 SYSTEMS
$G_c = \frac{k_c(s^2 + 5s + a)}{(s+4)}$
a=9
a=7&8
a=6
a=8
a=7
$K_v = 15$
$k_c = 1$
$k_c = 2$
$K_v = 1$
-12
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11

Figure 5-17.

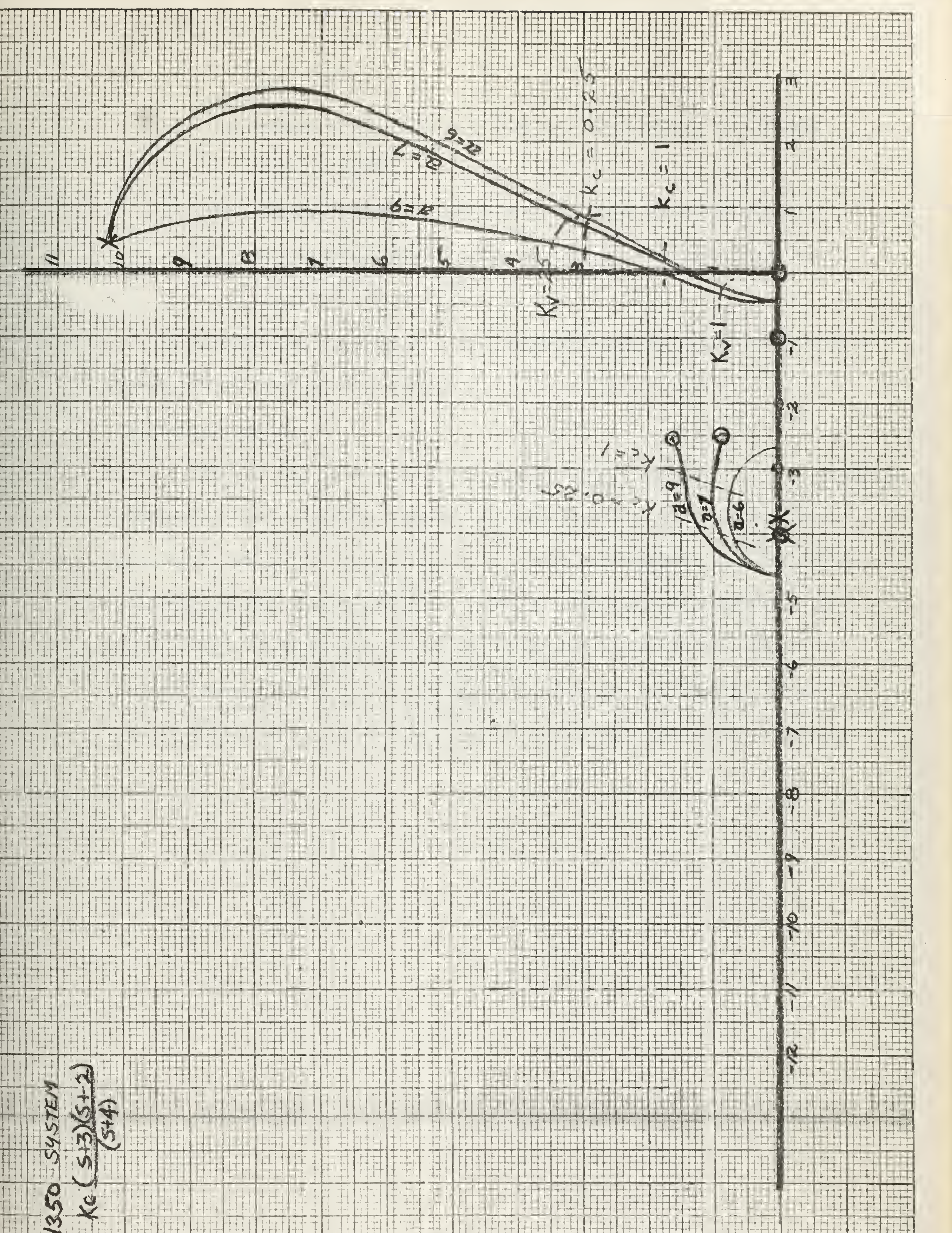


figure 5-18.

5-13 to 5-15.

2. second derivative feedback - effect shown in figures 5-19 to 5-21.

3. second derivative with proportional feedback ("40" compensator with $\underline{a}$ not equal to 0) - effect shown in figures 5-19 to 5-24.

In view of the failure of these compensators to stabilize, further discussion of them is not warranted.

E. Normalization.

In view of the fact that three systems were included in this group, a deeper insight into the problem of normalization of root loci is possible. Actually, normalization has been implemented to some extent by virtue of the fact that these three systems have been grouped together. This fact implies that any other type one, servo system having a second order motor function may also be included in this group provided its G_b function has an equal number of poles and zeros.

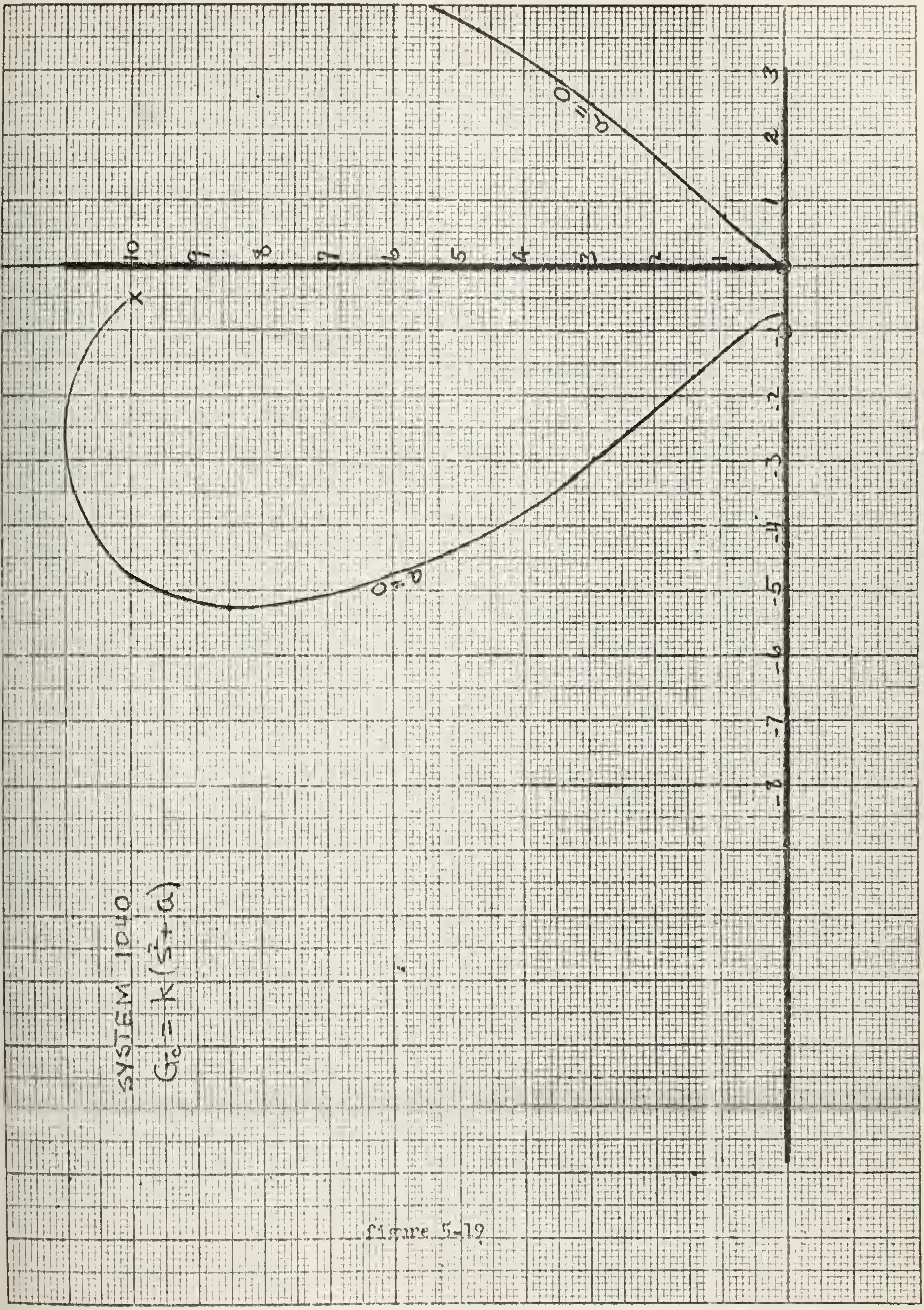


figure 5-19

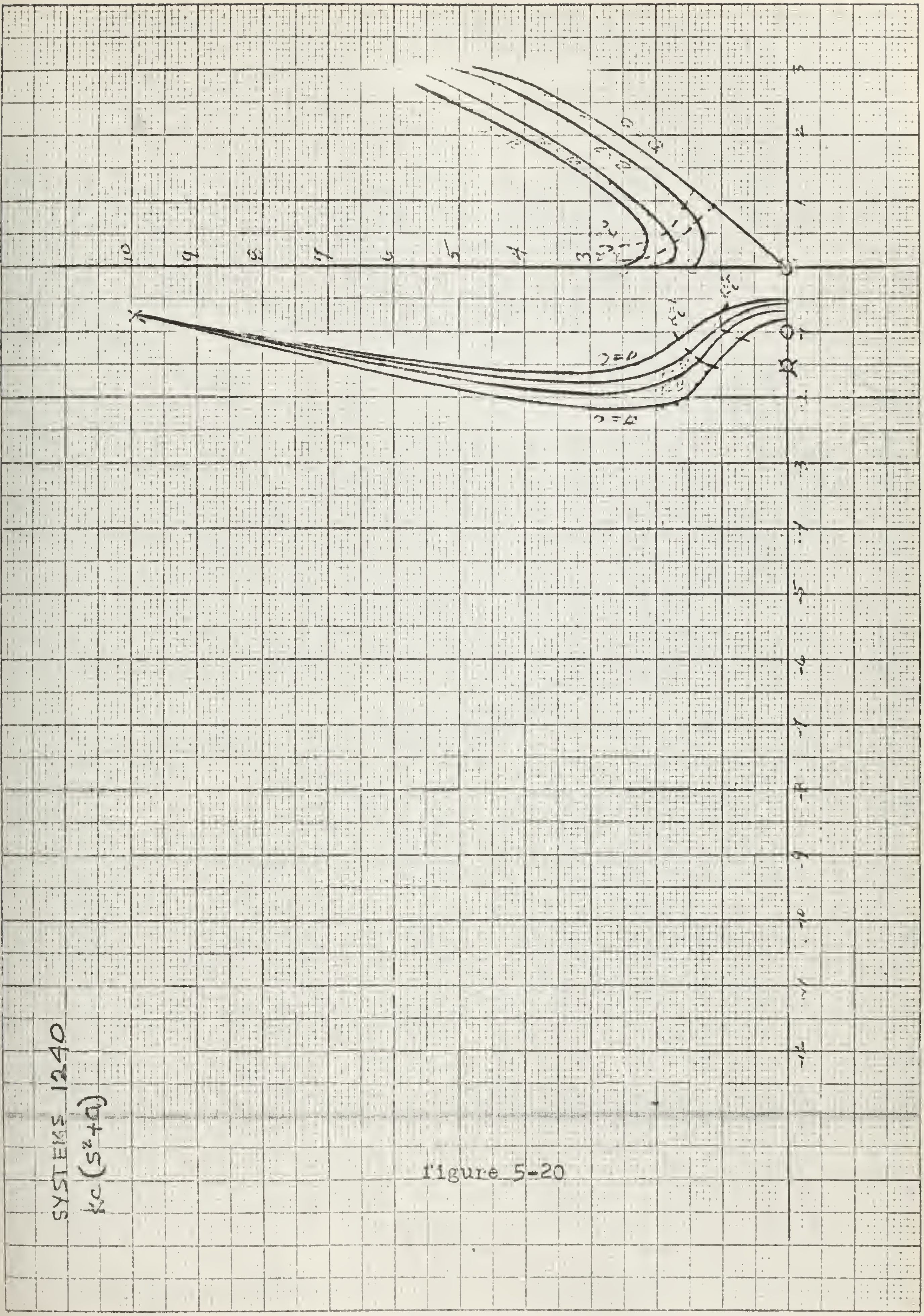


figure 5-20

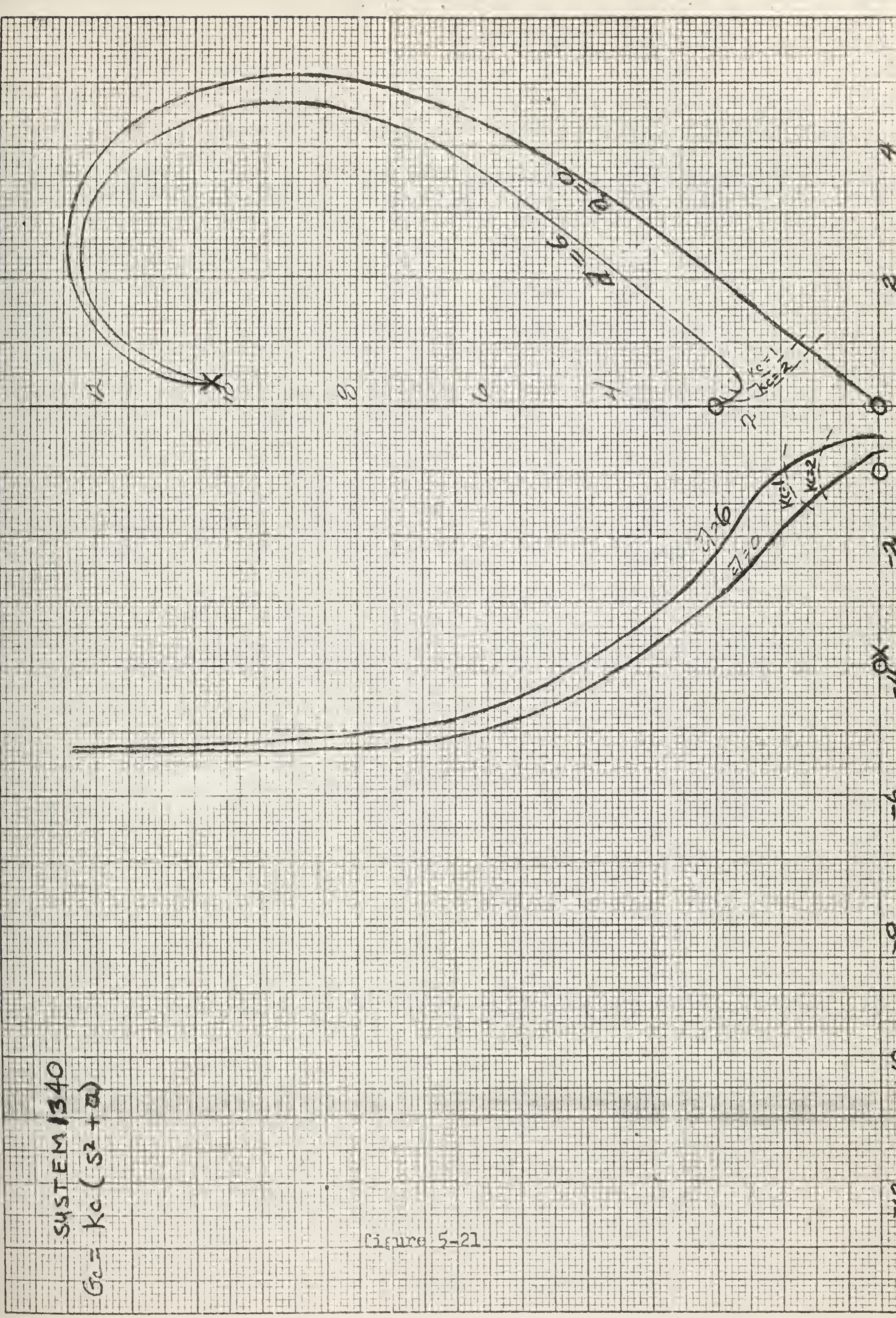


figure 5-21

[illegible]

[illegible]

Compensated system	k	Lower limit K_{cr}	ω_v	Upper limit K_{cr}	ω_v
1810	2	0.02[illegible]	7[illegible].9[illegible]		
	4	0.022	7[illegible].075		
	[illegible]	0.013	7[illegible].7[illegible]		
	8	0.011	7[illegible].7[illegible]		
[illegible]	0	0.0[illegible]	100.0[illegible]	∞	100.0[illegible]
	1	0.0[illegible]	[illegible]7.[illegible]7[illegible]	[illegible].1[illegible]	[illegible].10
	[illegible]	0.165	5[illegible].[illegible]	[illegible].02[illegible]	1[illegible].[illegible]
1220	[illegible]	0.055	75	∞	75.0[illegible]
	1	0.100	[illegible].15[illegible]	1[illegible].[illegible]	[illegible].57[illegible]
	2	[illegible]	[illegible]	[illegible]	12.[illegible]
1320	[illegible]	1.7[illegible]	10.731		
	[illegible]	0.100	66.6[illegible]7		
	6	0.0[illegible]2	102.12[illegible]		
1360	0	0.095	200.000		
	2	[illegible].005	1[illegible]		
	[illegible]	0.07[illegible]	[illegible]7.[illegible]9		
	6	[illegible].075	77.[illegible]		
	[illegible]	0.07[illegible]	[illegible].10[illegible]		
1930	0	0.201	100.000		
	2	[illegible]	[illegible].039	1.[illegible]	[illegible].757
	4	[illegible]	[illegible].1	[illegible]	[illegible]
	6	0.[illegible]	[illegible]	[illegible].027	[illegible]
	8	[illegible]	92.[illegible]93	[illegible].114	[illegible]

TABLE 5-1 (contd)

Condensate system	n	Lower limit K_{cr}	Lower limit K_v	Upper limit K_{cr}	Upper limit K_v
1030	0	0.001	75.00		
	2	0.001	73.414	1.764	2.732
	4	1.745	2.825	0.492	4.757
	6	0.505	4.827	0.198	7.575
1330	2	1.764	2.795		
	4	0.492	4.953		
	6	0.237	6.783		
1050	6	0.001	98.232	2.540	2.558
	7	0.001	98.280	2.116	2.629
	8	0.001	98.039	1.470	3.290
	9	0.001	97.800	1.225	3.502
1250	6	0.001	73.804	2.510	2.300
	7	0.001	73.609	2.116	2.606
	8	0.001	73.414	1.470	2.254
	9	0.001	73.221	1.192	2.552
1350	6	2.2555	2.576		
	7	1.919	2.933		
	8	1.586	3.103		
	9	1.192	3.661		

6. Group V - type one system with second order motor function and one excess pole in G_b.

A. General.

Two of the systems investigated fall into this group. They are the 1100 and 1500 systems. The component functions for these systems were selected so as to represent physical components which would be found in a practical servo system. Figures 6-1 and 6-2 illustrate the block diagram for these systems respectively.

Also shown in figures 6-1 and 6-2 are the locations of the roots of the basic systems. The gain of each system has been adjusted to make it unstable in order that a more thorough evaluation of each compensator's competency may be made. Thus, for these systems the primary objective of compensation is stabilization: whereas, the secondary objective is the provision of increased flexibility in the choice of root location.

An examination of figures 6-3 to 6-13 indicates that much similarity exists between corresponding root loci of the two compensated systems. This fact is especially true for the predominating sections of the root loci. Therefore, because of this similarity and in the interest of simplicity, the analysis of the effects of a particular compensator will be made by considering both systems simultaneously and reserving further comment for any significant difference which may exist between them until later.

B. Completely satisfactory compensators.

Three of the compensators investigated provided completely satisfactory compensation. This does not necessarily suggest that they are to be considered as the best compensators - its sole meaning is that only one requirement must be met in order for stability to occur in

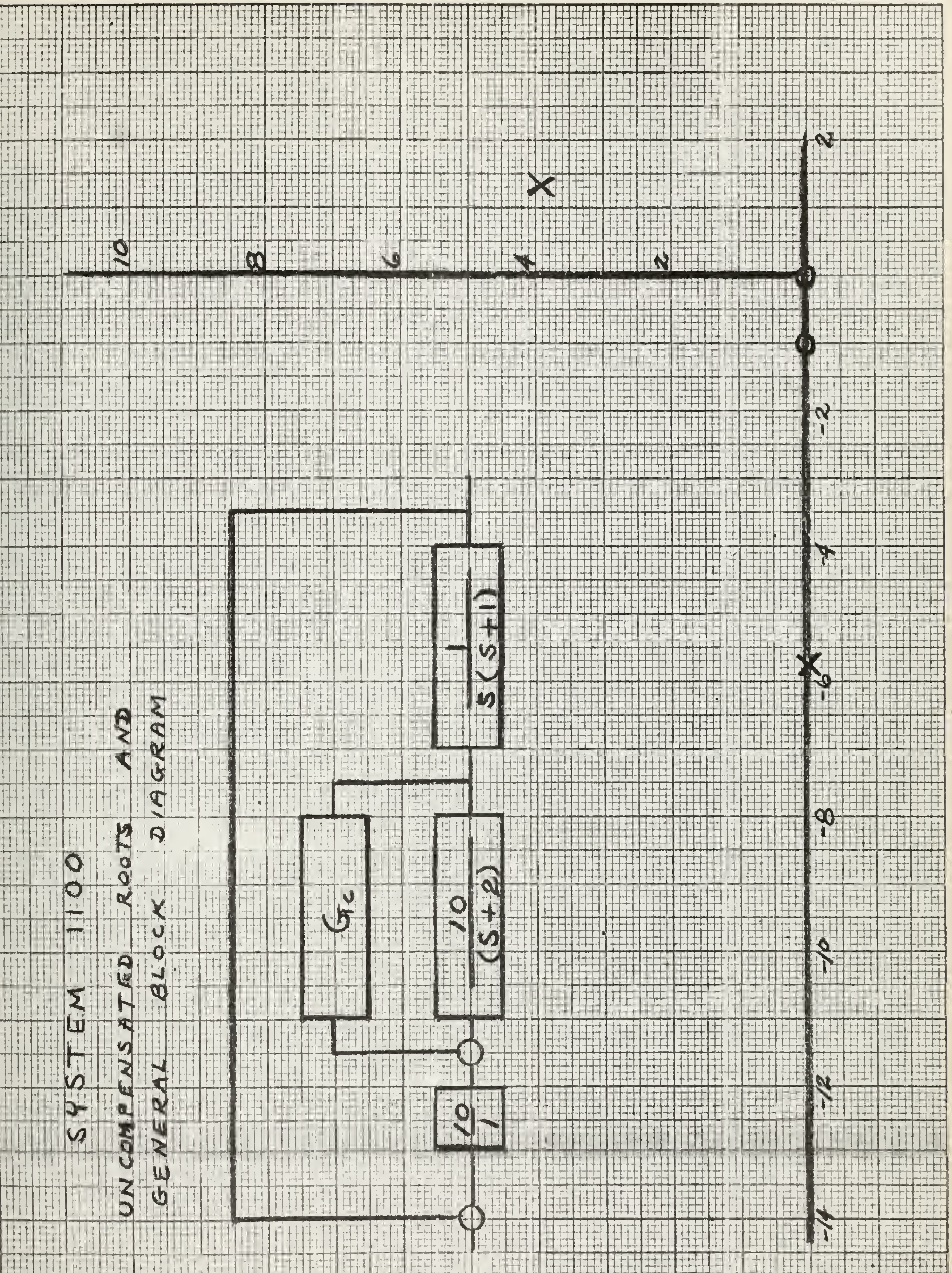

SYSTEM 1100
UNCOMPENSATED ROOTS AND
GENERAL BLOCK DIAGRAM
Gc
10
(S+2)
1
S(S+1)
10
1
10
8
6
4
2
2
-2
-4
-6
-8
-10
-12
-14

figure 6-1

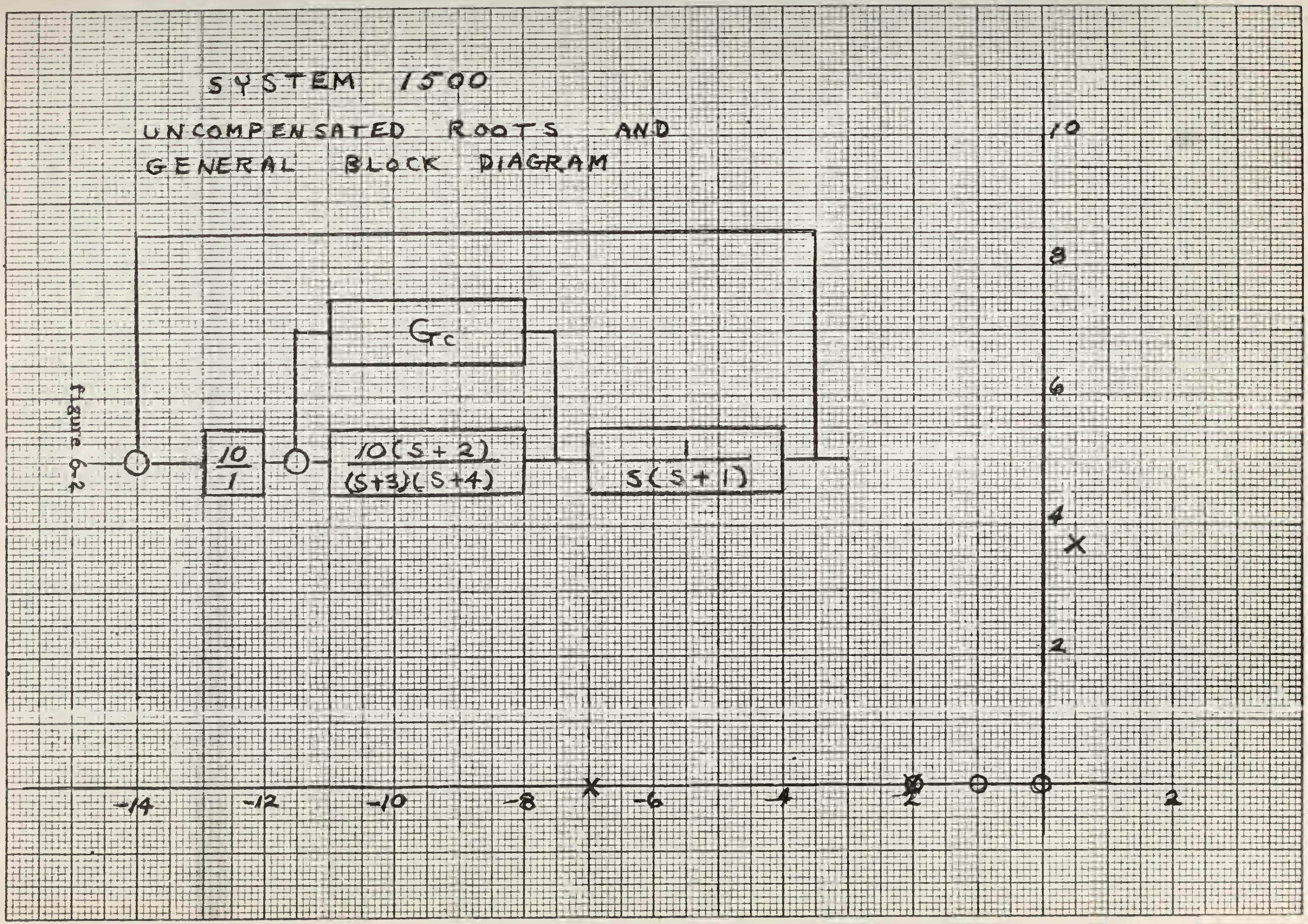

SYSTEM 1500
UNCOMPENSATED ROOTS AND
GENERAL BLOCK DIAGRAM
10/1
Gc
10(S+2)/(S+3)(S+4)
1/S(S+1)
10
8
6
4
2
-14
-12
-10
-8
-6
-4
-2
2

figure 6-2

the compensated system. This requirement is that the compensator's gain, k_c, be greater than a minimum gain, k_{cr}. These values of k_{cr}, which vary with the compensator's zero, a, are listed in table 6-1.

A brief analysis of the effects produced by these compensators follows.

(1) Lag network.

This compensator is quite effective in compensating an unstable system. It not only has the ability to stabilize a system but also it provides the designer with considerable flexibility in meeting specifications.

The ability of this compensator to stabilize can readily be confirmed by referring to the root loci shown in figures 6-3 and 6-4 for a greater than 1.0. Also figures 6-5 and 6-6 for a greater than 1 show the effect of this compensator for a different compensator pole.

The same root loci also give a good indication of the flexibility which is available through use of this compensator. By varying k_c and a, various values of ζ and ω_n may be obtained. In particular, some methods by which ζ may be increased consist of: (1) increasing k_c while maintaining a constant, or (2) increasing a while maintaining k_c constant. The former method will cause ζ to vary from 0 to 1.0, whereas, the latter gives a more limited variation. Likewise, the most obvious method of increasing ω_n is to increase a while maintaining k_c constant. Thus by using any of the above methods or combinations thereof, desirable values of ζ and ω_n may be obtained.

Nevertheless, there are limits which must be considered when using this compensator. Because of the physical size of

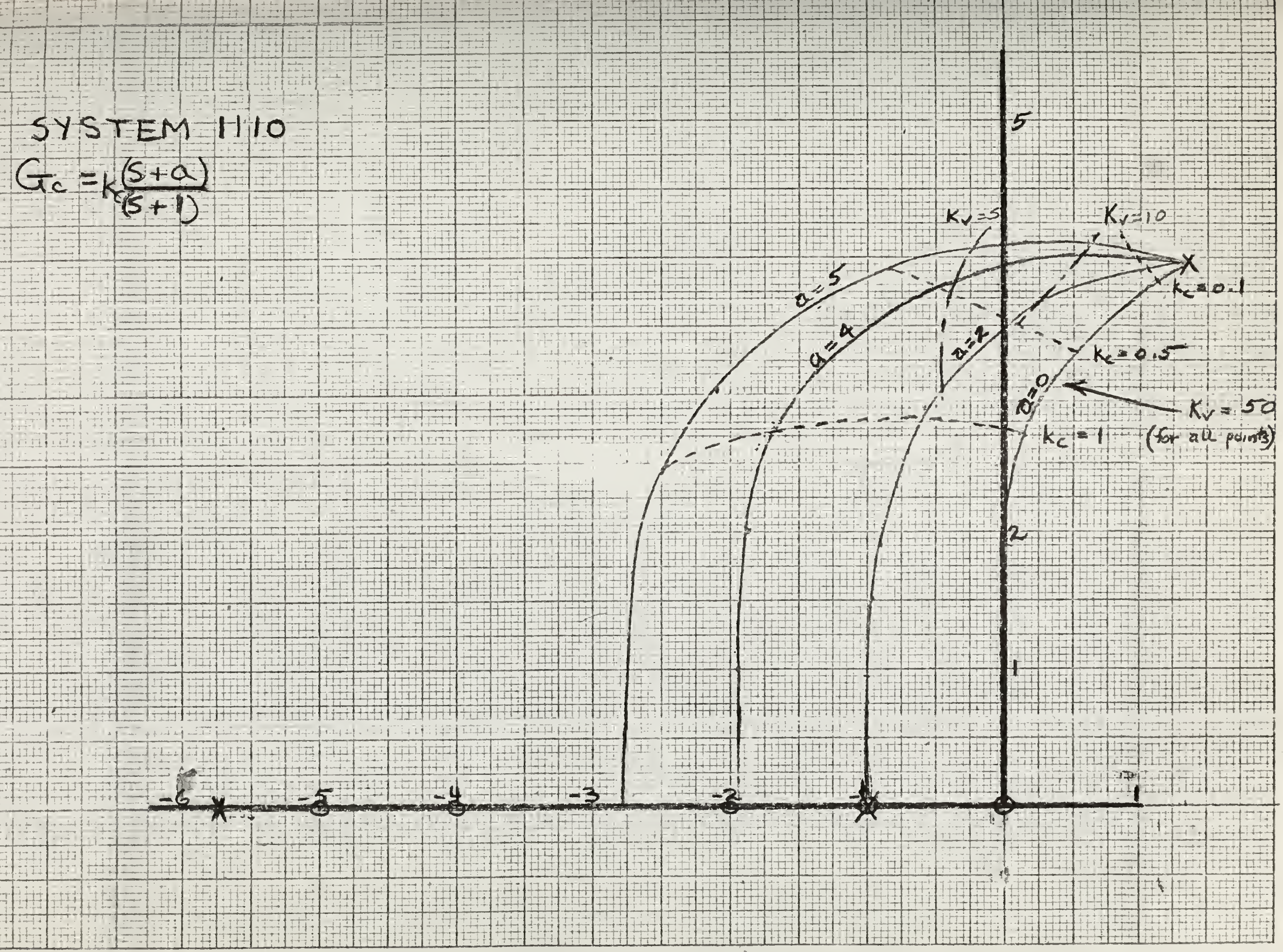
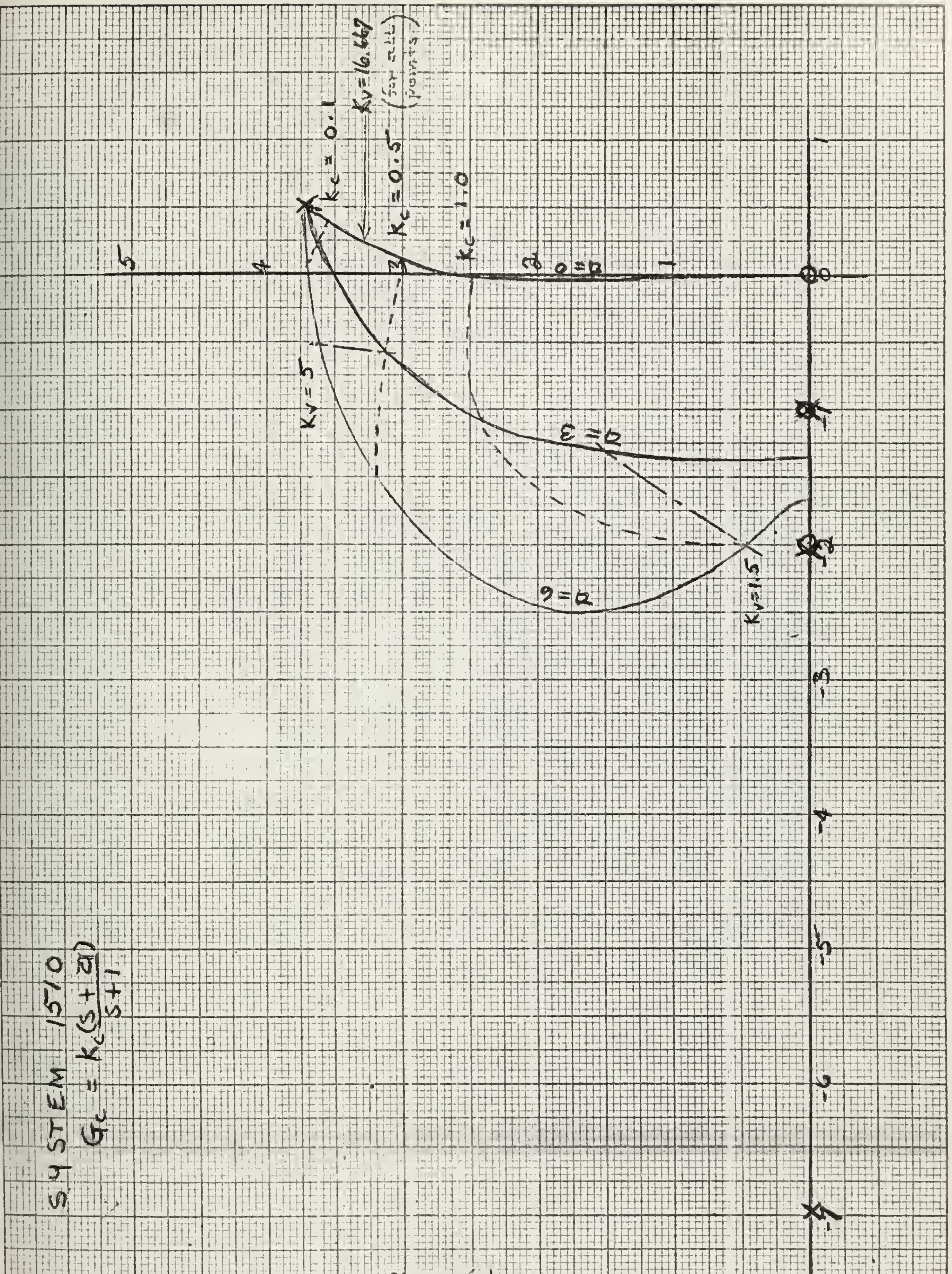

SYSTEM 1110
$G_c = k_c \frac{(s+a)}{(s+1)}$
$K_v = 5$
$K_v = 10$
$k_c = 0.1$
$k_c = 0.5$
$k_c = 1$
$K_v = 50$
(for all points)
$a = 5$
$a = 4$
$a = 2$
$a = 0$
5
2
1
-6
-5
-4
-3
-2
-1
1

Figure 6-3



SYSTEM 1510
$G_c = K_c \frac{(S+Z)}{S+1}$
$K_c = 0.1$
$K_v = 16.667$ (for all points)
$K_c = 0.5$
$K_c = 1.0$
$K_v = 5$
$K_v = 1.5$
$Z = 0$
$Z = 3$
$Z = 6$

figure 6-4

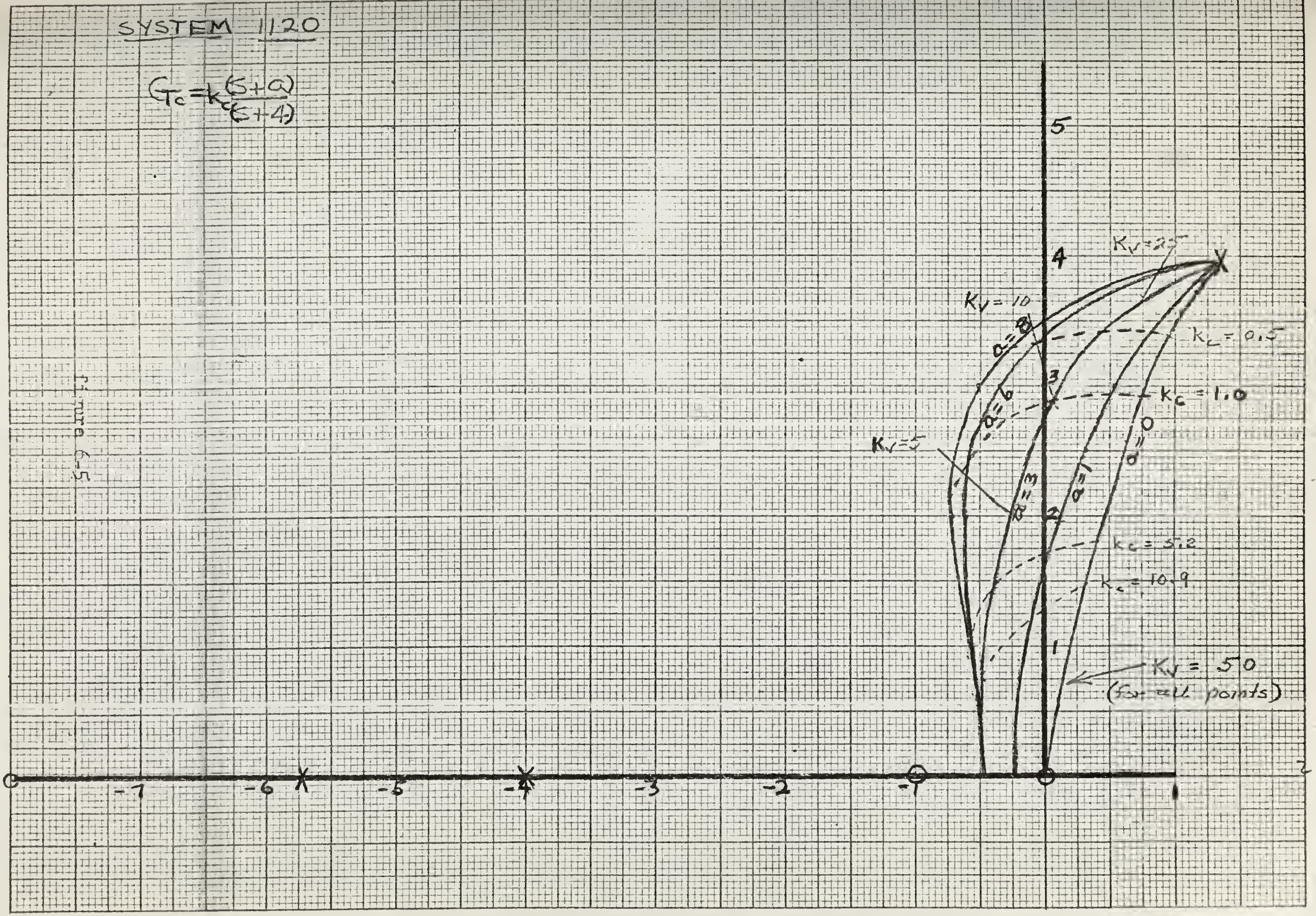

SYSTEM 1120
$G_c = k_c\frac{(s+a)}{(s+4)}$
$K_v = 25$
$K_v = 10$
$K_v = 5$
$a = 8$
$a = 6$
$a = 3$
$a = 1$
$a = 0$
$k_c = 0.5$
$k_c = 1.0$
$k_c = 5.2$
$k_c = 10.9$
$K_v = 50$
(for all points)
5
4
3
2
1
-7
-6
-5
-4
-3
-2
-1

figure 6-5

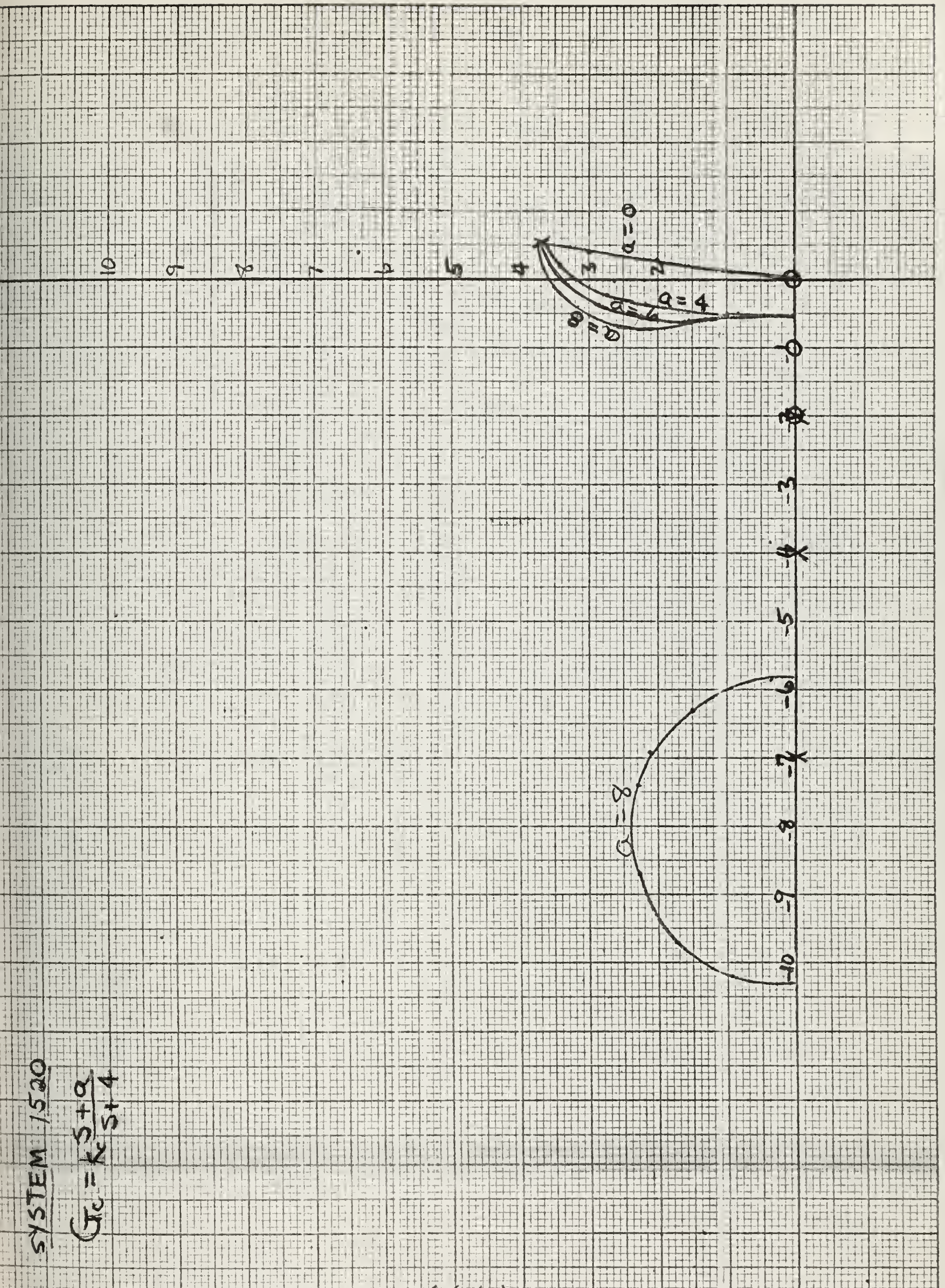


figure 6-6 (a)

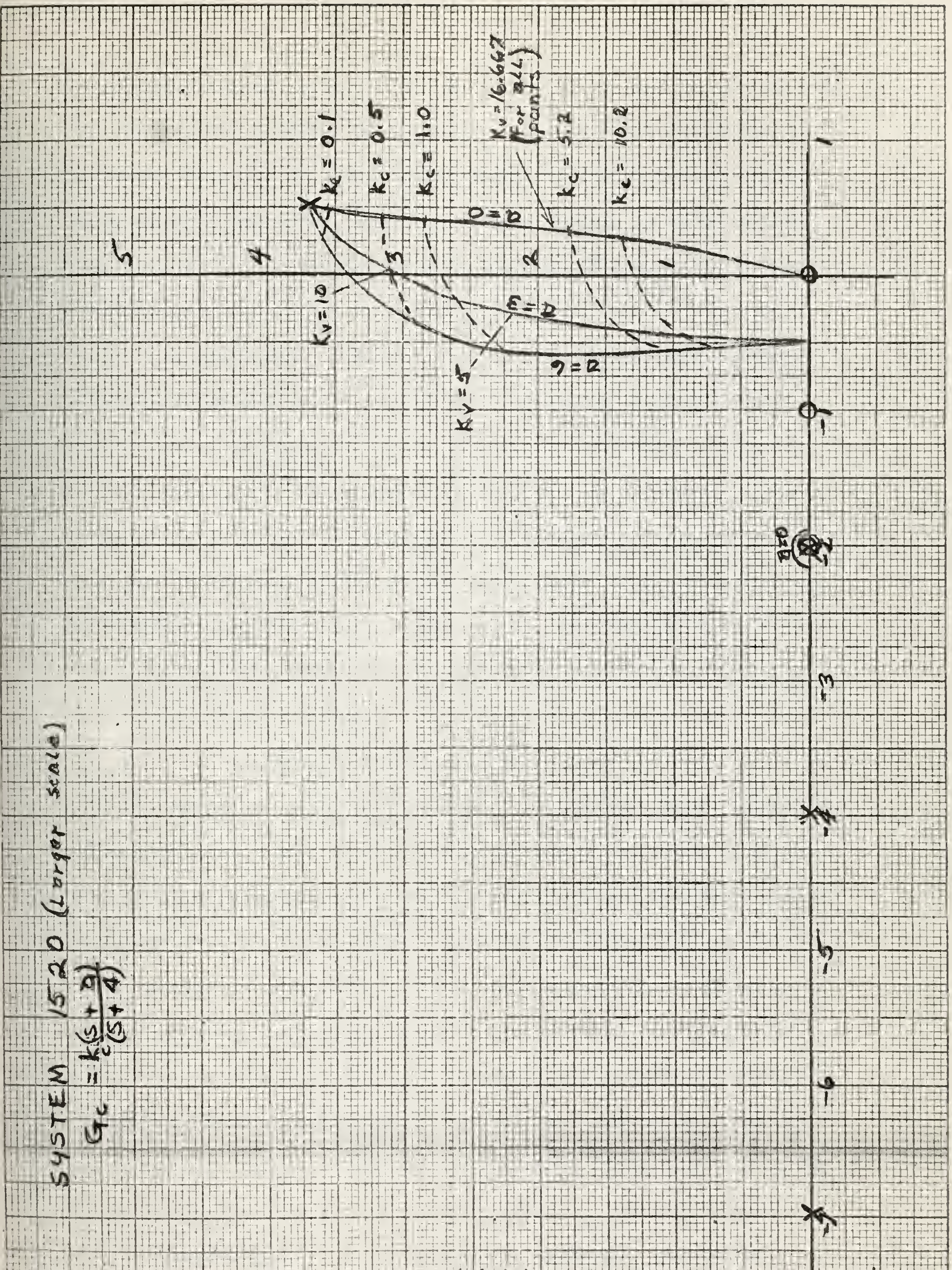

SYSTEM 1520 (Larger scale)
$G_c = \frac{k_c(s+2)}{(s+4)}$
$k_c = 0.1$
$k_c = 0.5$
$k_c = 1.0$
$k_c = 5.2$
$k_c = 10.2$
$K_v = 16.667$ (for all points)
$K_v = 10$
$K_v = 5$
$\zeta = 0$
1
2
3
4
5
-1
-2
-3
-4
-5
-6
-7

figure 6-6 (b)

components required, a is limited in magnitude by that of the lag network compensator's pole. On the other hand, k_c might be limited for different reasons. If design requirements specify a small, steady state velocity lag error, then K_v would be limited to large values. Thus this limitation could reduce the maximum ζ available or in extreme cases nullify the stabilizing effect of this compensator.

A comparison of figures 6-3 and 6-4 reveals the existence of some differences in the root loci. The primary source of these differences is the fact that the roots of the two basic systems are not the same. These differences are exhibited in two ways: a marked change in the shape of the predominating section of the complex root loci, and differences in the extent ω_n changes as a varies. These differences increase with a. Nevertheless, the remarks previously made concerning the trends of ζ and ω_n when varying k_c and a still apply, to each system.

(2) "30" compensator with a not equal to 0.

Compensation by feeding back a combination first derivative and proportional signal ("30" compensator with a not equal to 0) does effectively compensate - though not as well as the lag network - the systems in two ways: it induces stability and increases the flexibility available for root relocation. The flexibility provided by this compensator, as readily shown by the root loci of figures 6-7 and 6-8 for systems 1130 and 1530 respectively, permits the designer a relatively wide selection of ζ and ω_n that can be acquired by variation in k_c and a. Using this compensator, desired values of ζ may be obtained either by increasing k_c above k_{cr}, with a constant or by increasing a while k_c is maintained constant. The former method causes ζ to range from 0 to 1 as k_c increases while the

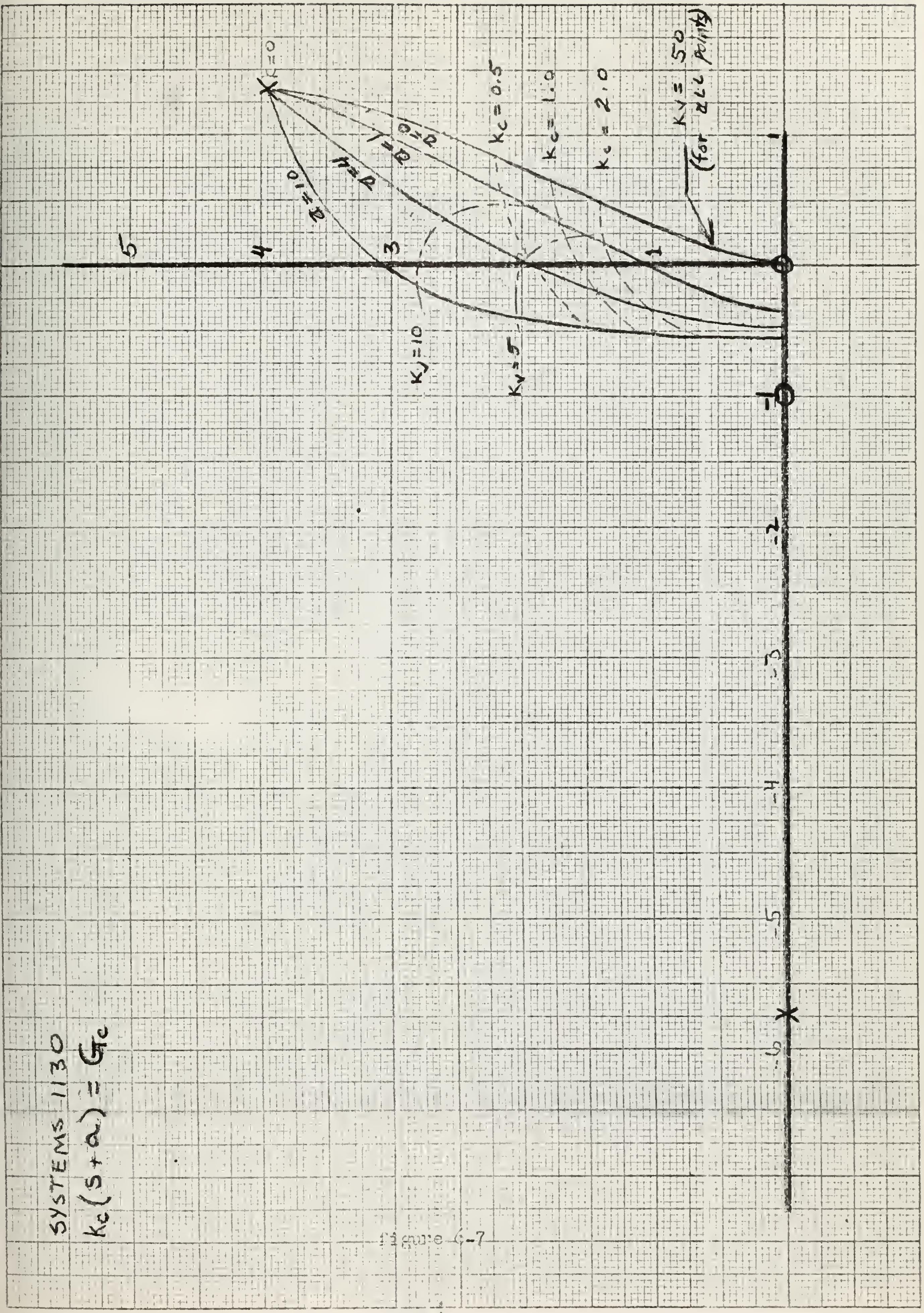

SYSTEMS 1130
Kc(s+a) = Gc
Kc=0
Kc = 0.5
Kc = 1.0
Kc = 2.0
KV = 50
(for all points)
KV=10
KV=5
5
4
3
1
-1
-2
-3
-4
-5
-6

figure 6-7

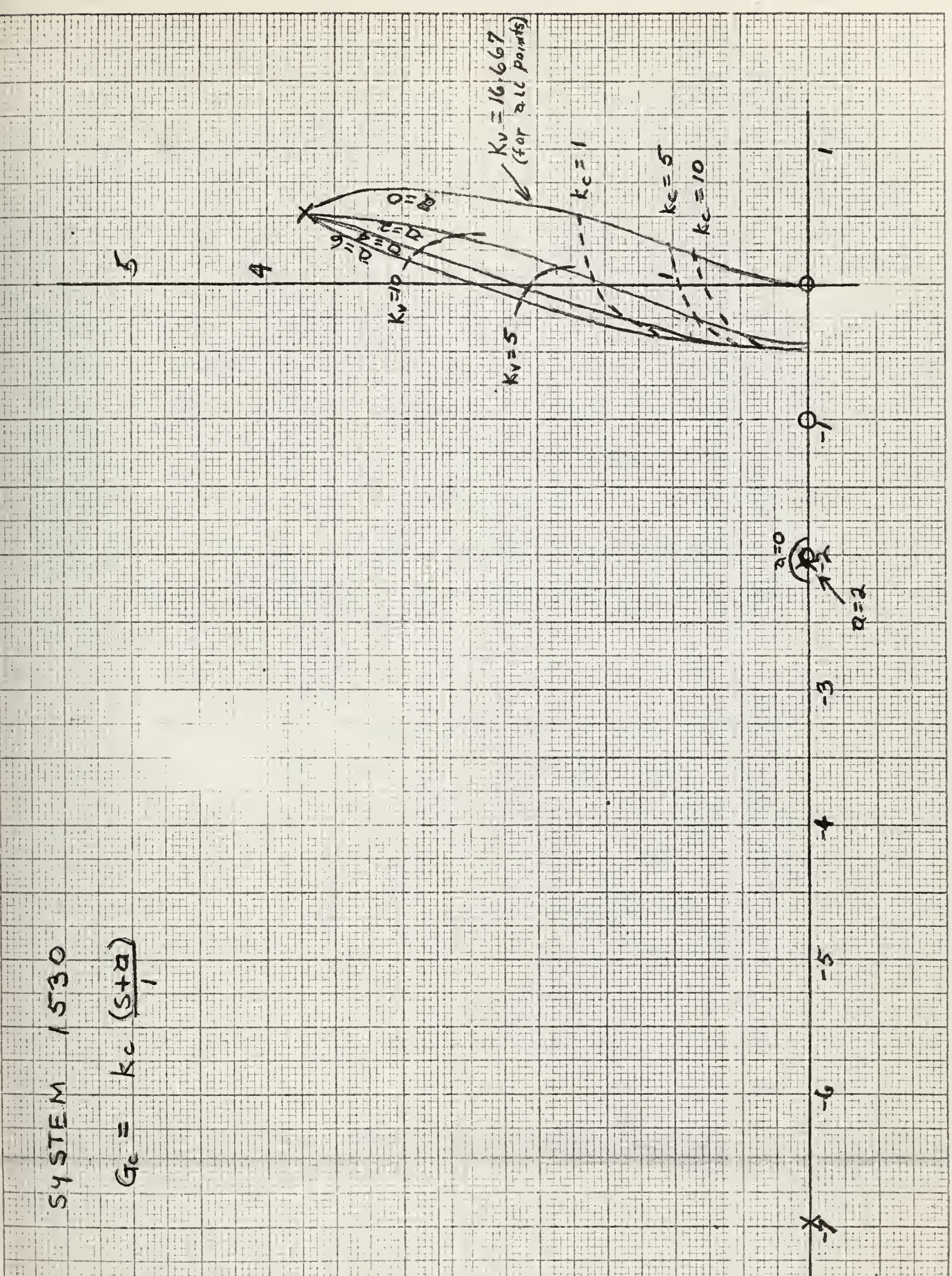

SYSTEM 1530
$G_c = K_c \frac{(S+a)}{1}$
$K_v = 16.667$ (for all points)
$K_c = 1$
$K_c = 5$
$K_c = 10$
$K_v = 10$
$K_v = 5$
$a = 0$
$a = 2$

figure 6-8

latter method causes less variation. Likewise, a desirable increase in ω_n can be obtained in several ways of which the most obvious are: increasing $\underline{a}$ while maintaining either ζ and k_c constant or decreasing k_c with $\underline{a}$ constant.

Only two differences have been noted between the root loci of the 1130 and 1530 systems, and these are relatively small. The most significant of these differences is the fact that the similarity between the predominating sections of each system's root loci disappears for values of ζ less than 0.2; whereas, the similarity is very good for values of ζ greater than 0.2. The other difference was the fact that the root loci of the 1530 system contains two complex sections. Nevertheless, the additional effect of this second section is negligible.

(3) "50" compensator.

Although this compensator does provide satisfactory compensation of the systems, it is not as effective as the two just previously discussed. While it does provide the same degree of stability the flexibility provided is considerably reduced. In particular, the possible values of ζ available and the methods of obtaining its variation through use of the "50" compensator are quite limited. As shown in figures 6-9 and 6-10, variation in ω_n by changing $\underline{a}$ alone is very small: however, ω_n may be increased significantly by increasing k_c while maintaining $\underline{a}$ constant. But increasing k_c from k_{cr} while maintaining $\underline{a}$ constant will also cause ζ to vary from 0 to 1 depending on the magnitude of k_c. Thus, because of the restrictions usually placed on the selection of ζ, the flexibility is also restricted.

The root loci for the two compensated systems show close

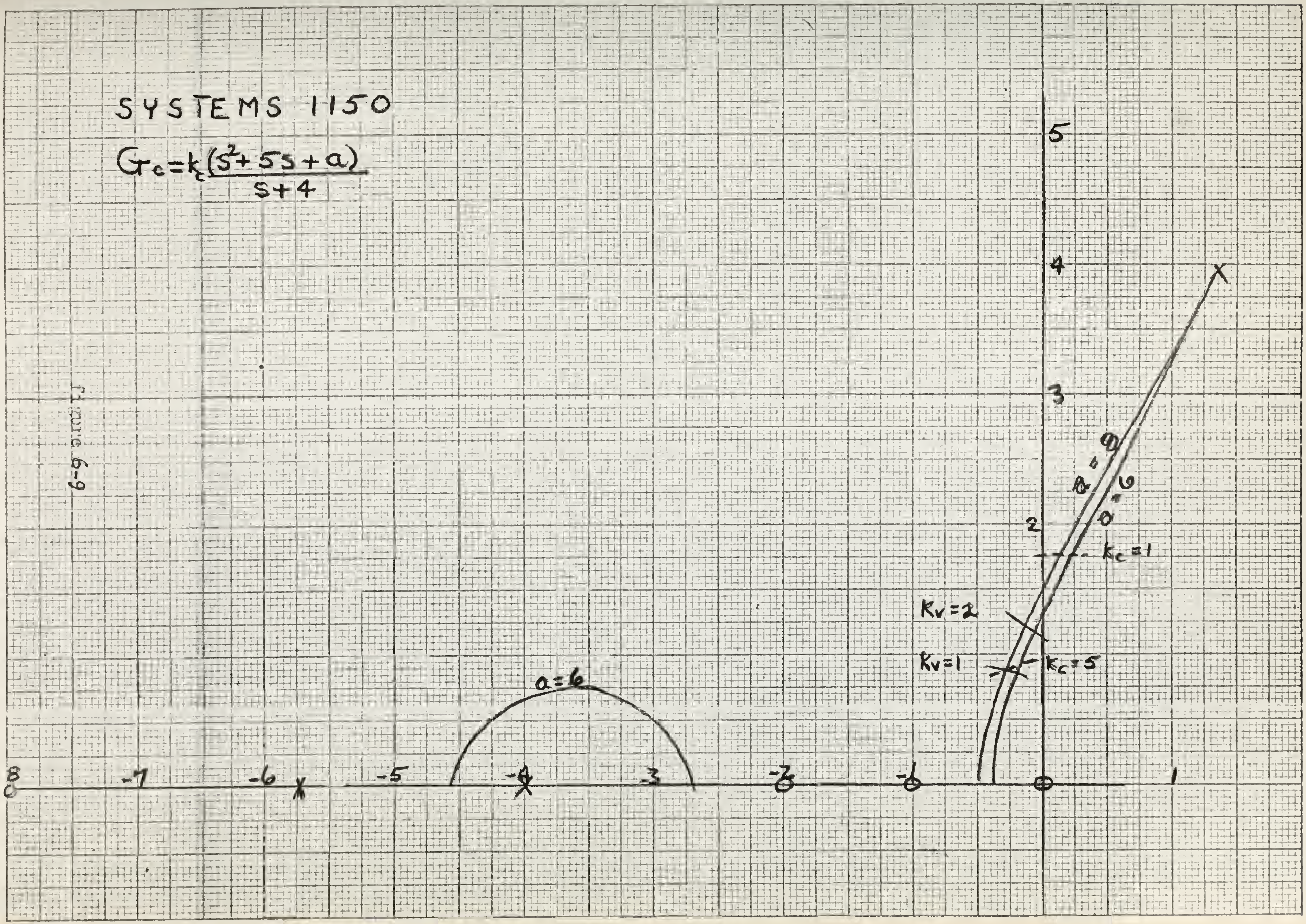

SYSTEMS 1150
$G_c = k_c \dfrac{(s^2+5s+a)}{s+4}$
5
4
3
2
1
$k_c = 1$
$k_c = 5$
$K_v = 2$
$K_v = 1$
$a = 6$
-8
-7
-6
-5
-4
-3
-2
-1

figure 6-9

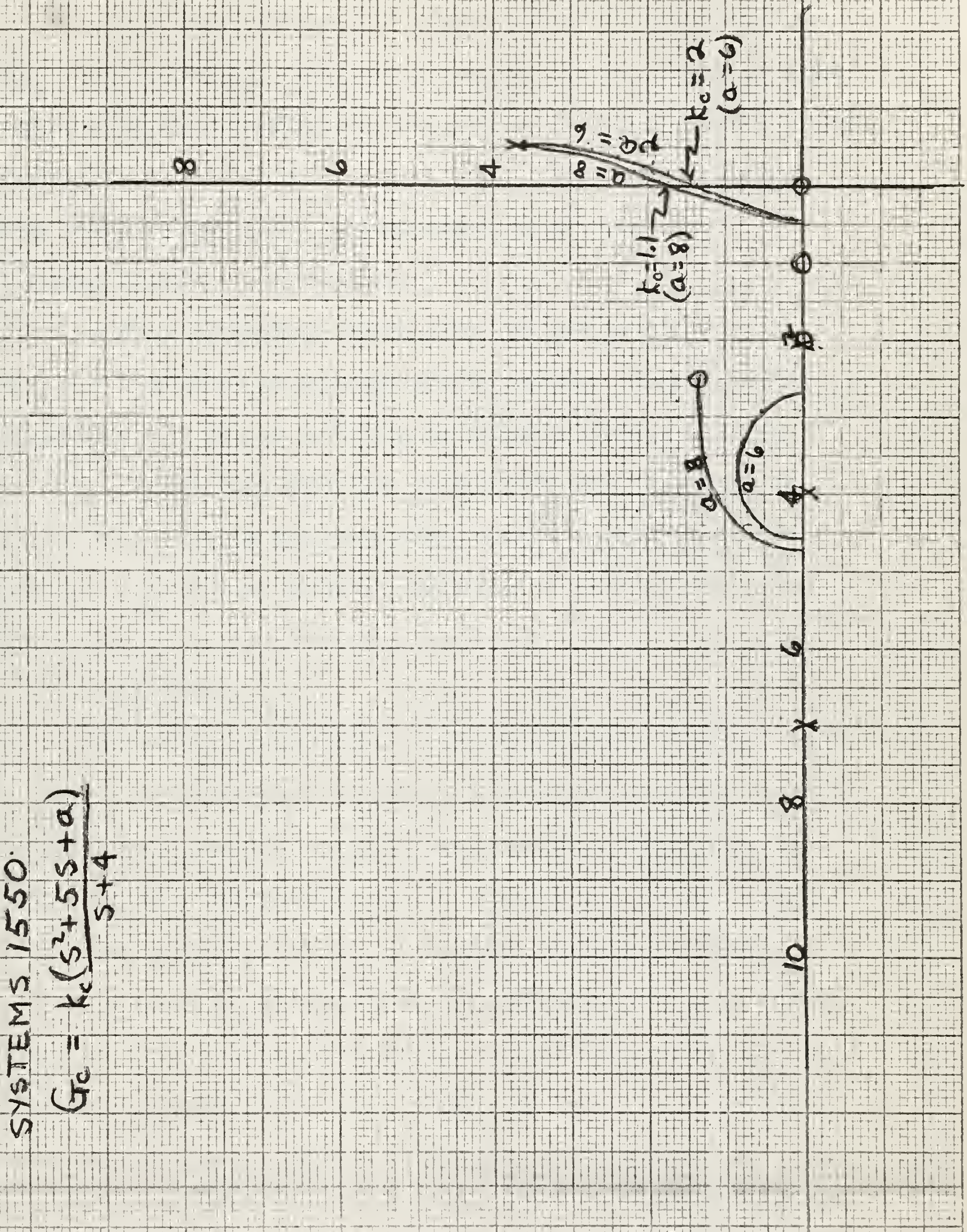


figure 6-10

similarity. The only significant difference lies in the fact that the roots of the two uncompensated systems differ in number and location. This causes the root loci having the largest number of poles and zeros to be more complicated, but not too much with respect to the predominating section of complex root loci. Therefore, these particular sections show close correspondence in all respects.

C. Partially stable compensators.

Two of the compensators investigated are considered to be only partially satisfactory in compensating the system. This is due to the fact that they do not stabilize an unstable system for every value of a. In one case stability is realized only for values of a greater than zero; in the other case, stability occurs only for low values of a. A more detailed discussion of these compensators follows.

(1) Lead network.

The effectiveness of the lead network in compensating this servo system is considerably limited. It will not stabilize for all values of a. In addition, when it does succeed in stabilizing the basic system, the flexibility which it provides the designer, while somewhat similar to that of the lag network, is more restricted.

As shown in figures 6-3 to 6-6, the criterion which seems to determine this compensator's ability to stabilize is the size of a. For all the cases investigated, except the one shown in figure 6-4, the compensator does not stabilize the system when a equals 0. In the case where stability does occur for a equal to 0 (shown in figure 6-4) it is only marginal, and therefore, it is not a good compensator. Moreover, even though the value of a is appropriately selected, stability only occurs when k_c is greater than the values of k_{cr} listed

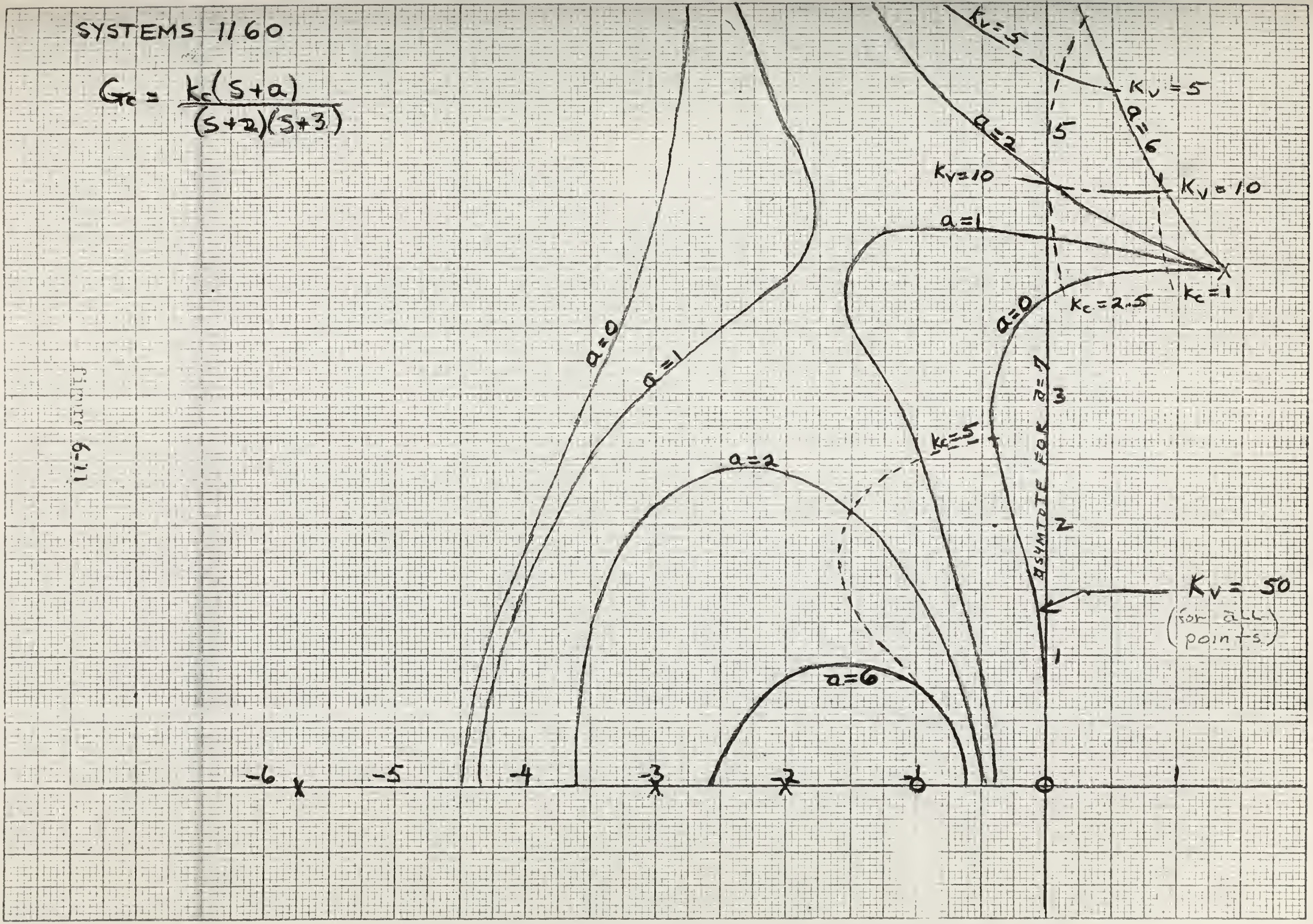
in table 6-1.

The flexibility provided by this compensator, while somewhat similar to that of the lag network, actually seems to supplement it. This can be observed by comparing the effect of both compensators on the root loci of any one of figures 6-3 to 6-6. Not only is there a lack of a radical transition between the root loci of the two differently compensated systems, but also the values of ω_n available using one compensator supplements those available using the other; and yet, the method of varying ω_n is similar for both. Also similar are the methods and extent of variation of ζ. This can be varied from 0 to 1 by increasing k_c above k_{cr} while maintaining a constant, or it can be varied to a lesser extent by increasing a while maintaining k_c constant.

The only difference between the effect of the lead network on the two basic systems can be attributed to the small difference in their uncompensated root location. For ζ greater than approximately 0.3 there is no significant difference, but for a ζ less than this the ω_n available in system 1120 is slightly less than that available in system 1520.

(2) "60" compensator.

The effectiveness of the "60" compensator, depending on the value of a used, can be either fair or poor. This is due to the fact that limitations placed on both stability and flexibility depend on a. As shown in figures 6-11 and 6-12, for a not greater than 1, the general characteristics are: stability occurs for a equal to 1 or less, all values of ζ from 0 to 1 are available, and a reasonable variation in ω_n is possible. However, for a greater than 1 the general characteristics are: stability depends on a, all possible



SYSTEMS 1160
$G_c = \frac{k_c(s+a)}{(s+2)(s+3)}$
$K_v = 5$
$K_v = 5$
$K_v = 10$
$K_v = 10$
$a = 6$
$a = 2$
$a = 1$
$a = 0$
$a = 0$
$a = 1$
$a = 2$
$a = 6$
$k_c = 1$
$k_c = 2.5$
$k_c = 5$
asymtote for $a = 7$
$K_v = 50$
(for all points)
-6
-5
-4
-3
-2
-1
1
2
3
5

figure 6-11

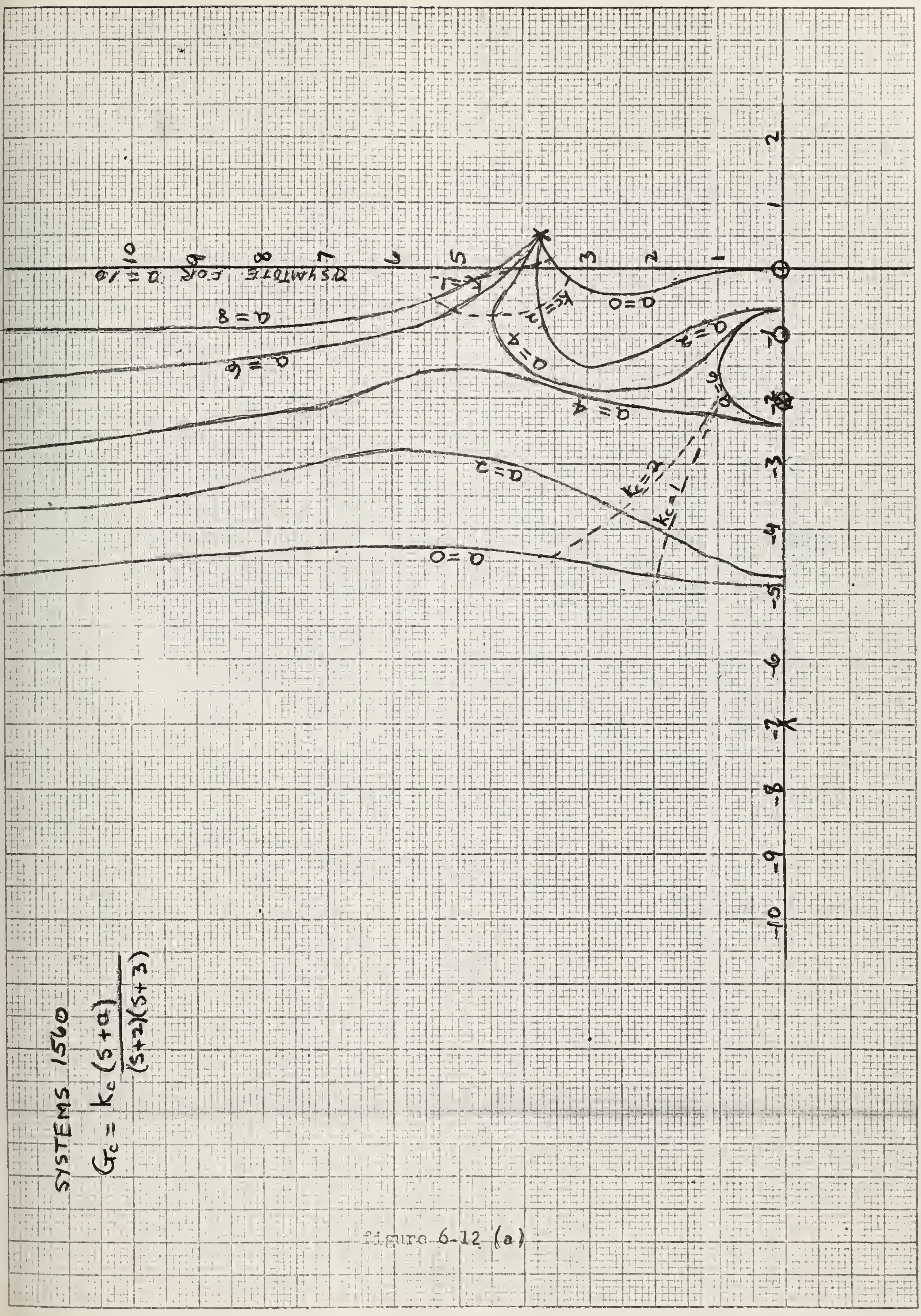

SYSTEMS 1560
$G_c = k_c \frac{(s+a)}{(s+2)(s+3)}$
asymptote for a = 10
a = 8
a = 6
a = 4
a = 2
a = 0
$k_c = 1$
$k_c = 2$

figure 6-12 (a)

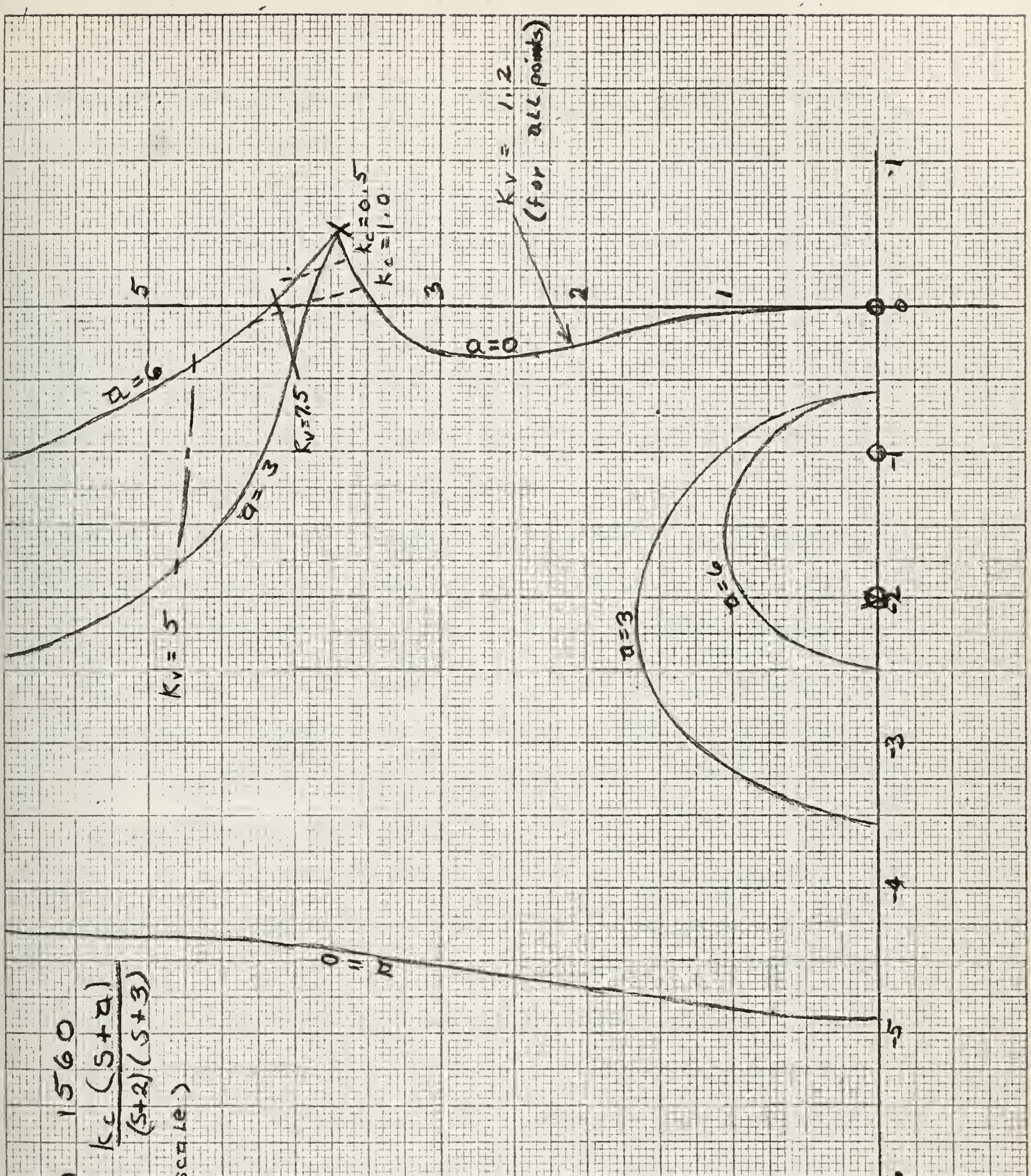

SYSTEM 1560
Gc = Kc (S+a) / (S+2)(S+3)
(Larger scale)
Kv = 1.2
(for all points)
Kc = 0.5
Kc = 1.0
a=6
a = 3
a=0
Kv=7.5
Kv = 5
a=3
a=6
a = 0
5
3
2
1
0
-1
-2
-3
-4
-5
-6
-7

figure 6-12 (b)

values of ζ are not available, and variation of ω_n is restricted. Further discussion of this compensator can be simplified by dividing it into two parts - a greater than 1 and a not greater than 1.

If a is not greater than 1 this compensator is not only capable of stabilizing but also has a reasonable degree of flexibility. Provided k_c of the compensator is greater than the appropriate value of k_{cr} listed in table 6-1, the "60" compensator will stabilize the Group V system. However, for k_c large (approximately 10 for the cases investigated) the stability becomes only marginal when a is set to 0. The flexibility available is favorable in that all values of ζ from 0 to 1 are possible for a particular value of a when k_c is varied upward from k_{cr}; while, a smaller range of increase in ζ is possible by increasing a as k_c is maintained constant. Also a variation in a, particularly with ζ in the 0.4 to 0.7 range, gives a reasonable selection of values of ω_n. Increases in ω_n can be obtained by increasing a.

On the other hand, when a takes on values greater than 1 the situation is quite different. Stability will not occur for all values of a; as a matter of fact, the system becomes unstable when a takes on the value which causes the root loci to become asymtotic with the imaginary axis (this is 7 and 11 for the 1160 and 1560 systems respectively). At the same time the flexibility is reduced considerably, particularly with respect to the variation of ζ. As a increases above 1, there develops a finite range for which the values of ζ are unobtainable. Hence, as a approaches its upper limit only very large and very small values of ζ are still available. In addition, although relatively large values of ω_n may be selected, the variation possible in ω_n becomes less as a approaches its limit. As

shown in figure 6-12 for a equal to 2, the selection of ζ is limited approximately to values less than 0.4 or greater than 0.7 which is certainly unfavorable for most servo applications.

The root loci for the two compensated systems show a considerable degree of similarity: however, a few differences do exist due to a difference in the system's uncompensated roots. If one considers only the situation where a does not exceed 1, then these differences are nearly insignificant. The primary one of these is the fact that good correspondence between the predominate sections of the complex root loci no longer occurs for values of ζ less than about 0.2.

(3) "40" compensator with a equal to 0.

For nominal values of a the combination of second derivative and proportional feedbacks does not provide satisfactory compensation. As shown in figures 6-13 and 6-14 satisfactory compensation occurs only after a is increased to the extent that the proportional component of the feedback signal dominates. Then this compensator can more appropriately be considered to consist of proportional feedback only. Because this type of compensation is no more than a gain reduction in the open loop function it will not be discussed any further.

D. Completely unsatisfactory compensators.

Two of the compensators investigated were considered to be completely unsatisfactory. These are:

(1) first derivative feedback only - shown in figures 6-3 to 6-6

(2) second derivative feedback - shown in figures 6-13 and 6-14.

These compensators were completely incapable of stabilizing the

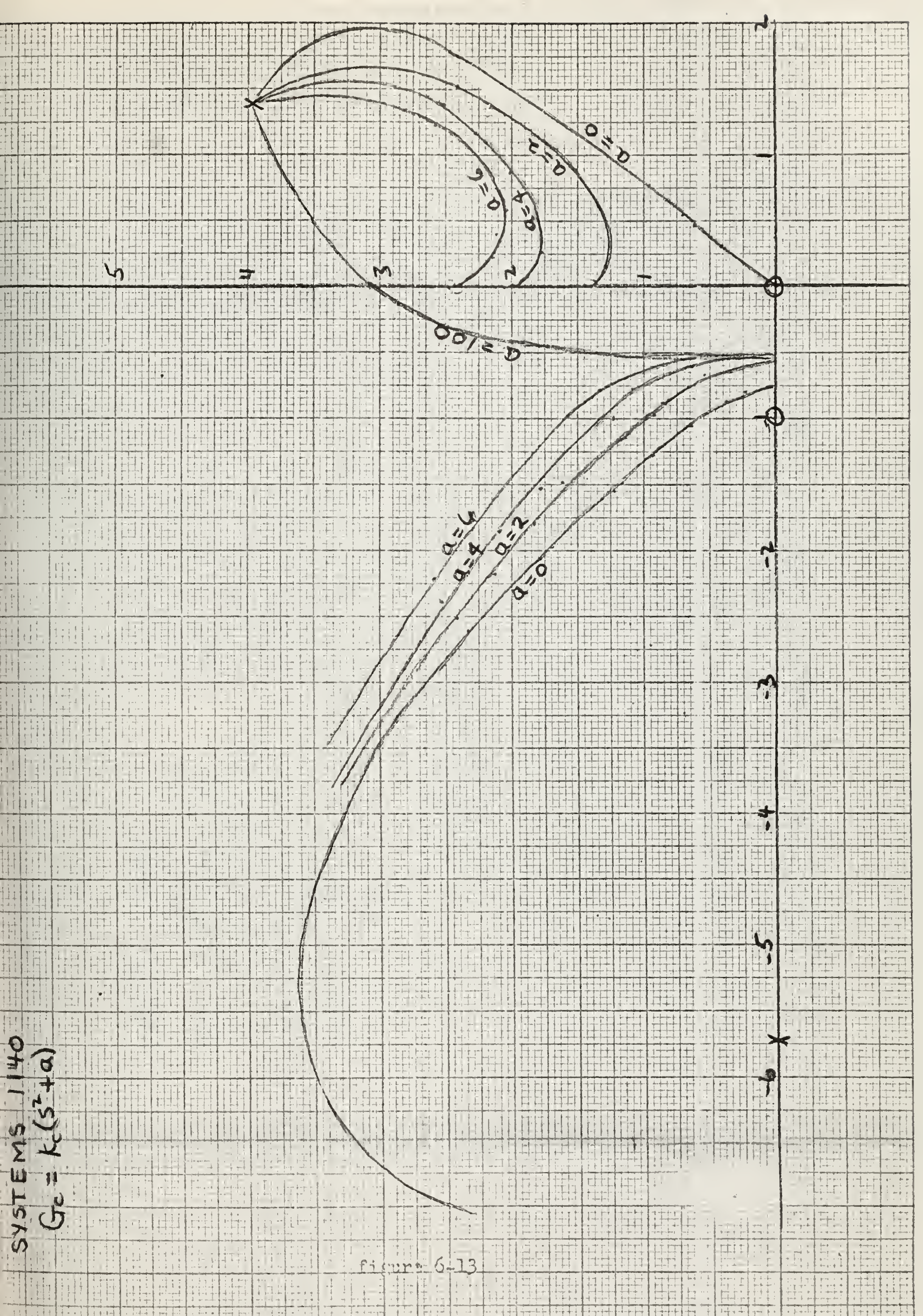


figure 6-13

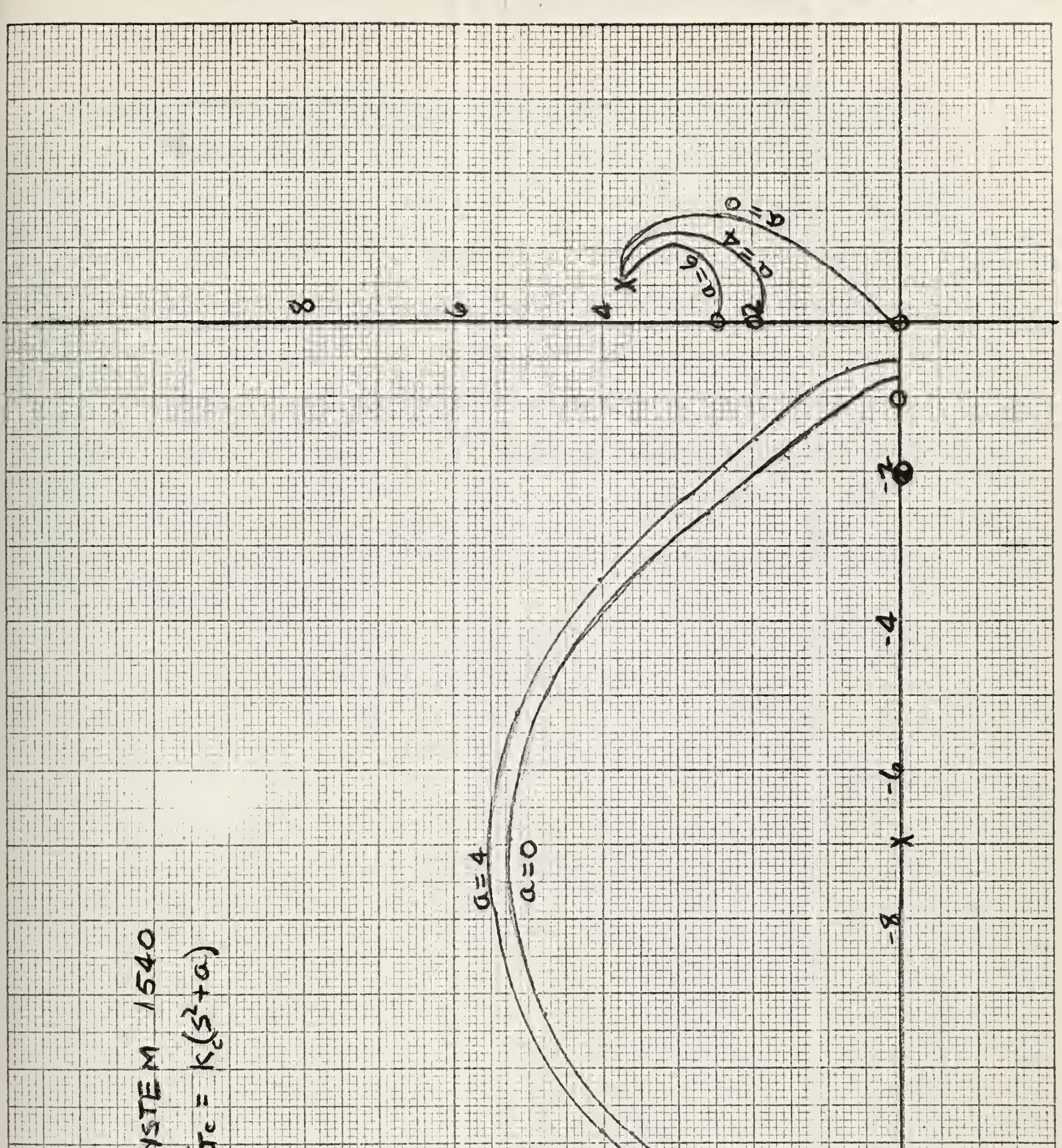


Figure 6-14

two basic systems; therefore, they will not be discussed any further.

F. Normalization.

In view of the fact that two systems have been included in Group V, it is possible to note the degree to which normalization has been implemented with respect to this group's root loci. A comparison of these root loci reveals the fact that all the remarks made in section 5 with respect to the normalization of these root loci also apply in this section. Therefore one is referred to sections 5 and 10 for a detailed discussion on the methods of extending these plotted root loci to systems which are nearly similar but whose parameters are of a different magnitude.

TABLE 6-1

APPROXIMATE LIMITS OF STABILITY

Compensated system	$\underline{a}$	k_{cv}	K_v
1110	2	0.51	8.0
	3	0.410	6.990
	5	0.30	5.85
	6	0.25	5.70
1510	0	1.021	16.667
	1	0.380	10.207
	3	0.198	8.779
	6	0.120	7.60
1120	1	10.901	3.413
	2	1.225	8.940
	4	0.851	9.519
	6	0.450	11.4
	7	0.400	11.1
1520	2	1.083	8.759
	3	0.591	9.588
	4	0.380	10.207
	6	0.236	10.486
	8	0.161	10.845
1130	1	4.979	1.931
	2	1.586	2.965
	3	0.814	3.785
	4	0.505	4.501
	5	0.345	5.192
	6	0.259	5.694

TABLE 6-1 (contd)

Compensated system	$\underline{a}$	k_{cv}	K_v
1530	2	1.225	3.279
	4	0.342	5.083
	6	0.137	7.021
1150	6	2.540	2.494
	7	2.116	2.562
	8	1.470	3.185
1550	6	2.116	2.649
	7	1.47	3.152
	8	1.225	3.279
	9	1.021	3.452
1160	0	2.54	50
	2	2.555	9.509
	3	2.60	6.5
	4	3.091	4.823
	6	5.26	1.829

7. Group VI - type one, fourth order system.

A. General.

This group consists of system 1800. It is a type one, fourth order system. The block diagram, along with the uncompensated roots are shown in figure 7-1. In the uncompensated condition it is unstable, having two complex roots in the right plane.

B. Completely satisfactory compensators.

(1) Lead Network.

The loci for this compensator are shown in figures 7-2 and 7-3 for a less than 1 and a less than 4 respectively. These networks are capable of stabilizing the system except for the limiting value of a equal to 0 for which the system is unstable. A complete range of ζ, from 0 to 1 may be obtained; however, the bandwidths are extremely small for optimum values of ζ from .4 to .7. In general, the higher values of ω_n are obtainable as the value of a approaches the value of the compensator pole. This is, in effect, approaching pure proportional feedback. However, it might also be noticed that stability would require greater than unity feedback. In general, the value of ζ increases and ω_n decreases as k_c is increased. There is also a minimum value of compensator gain, k_{cr}, which must be met for stability. This value is in general fairly high. A list of k_{cr} for values of a is given in table 7-1.

(2) First derivative plus proportional feedback.

These loci are shown in figure 7-4 for a greater than 0. Pure first derivative feedback (a equal to 0) is unstable and it is apparent that the proportional component has an important effect on the compensation. A complete range of ζ's are also available with this compensator, but its bandwidth for high ζ's is limited.

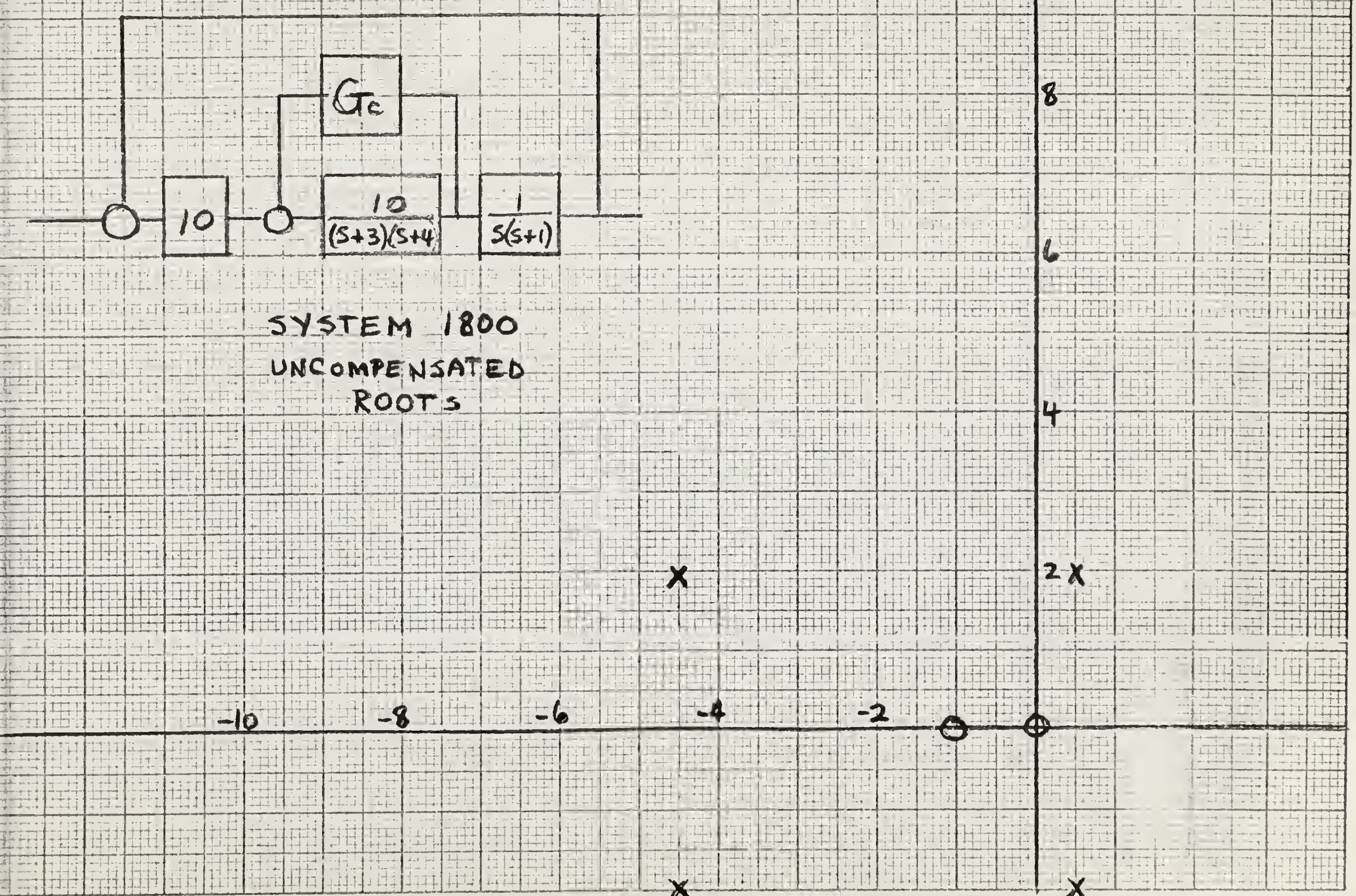

10
G_c
$\frac{10}{(S+3)(S+4)}$
$\frac{1}{S(S+1)}$
SYSTEM 1800
UNCOMPENSATED
ROOTS
8
6
4
2
-10
-8
-6
-4
-2

Figure 7-1

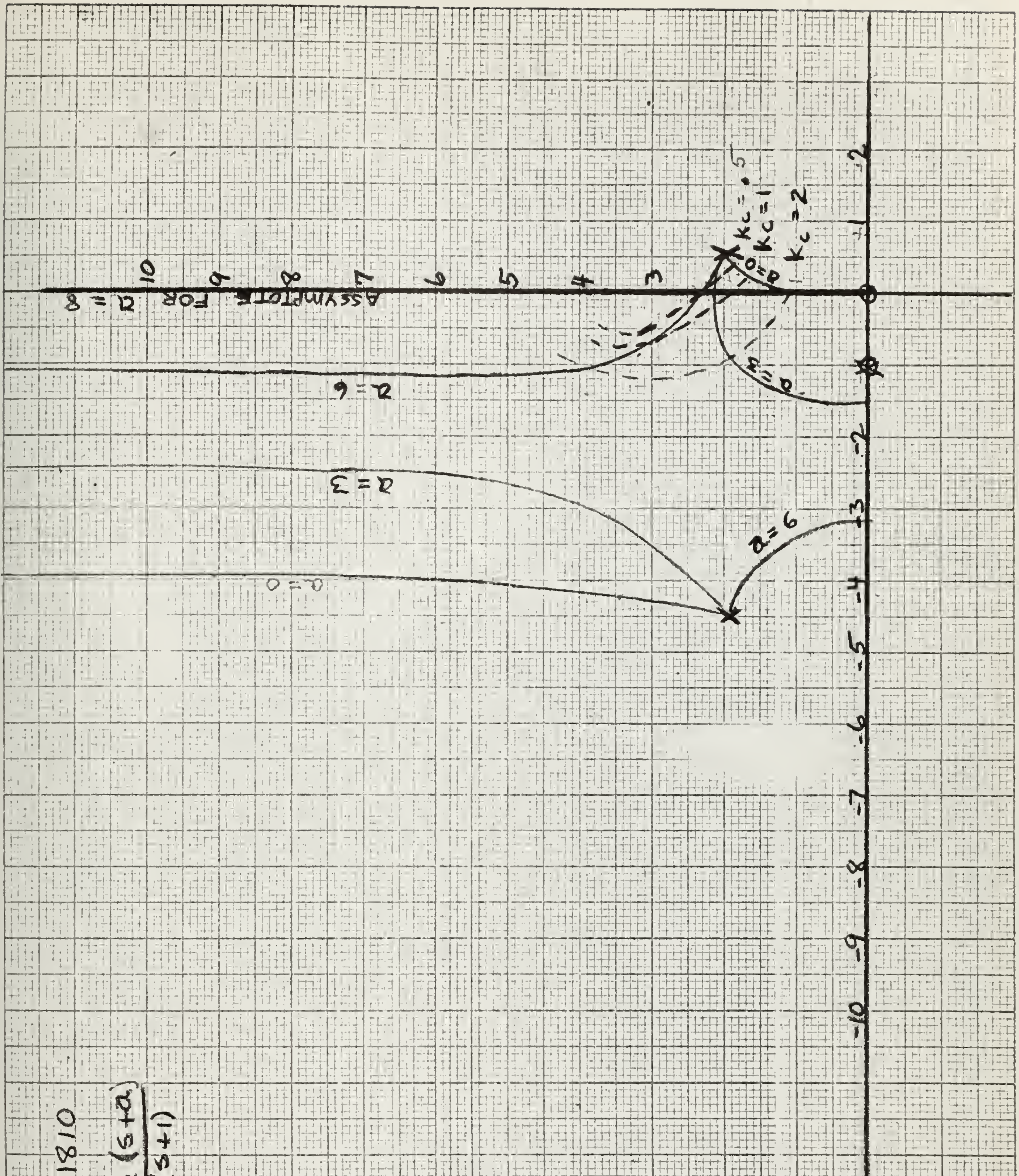
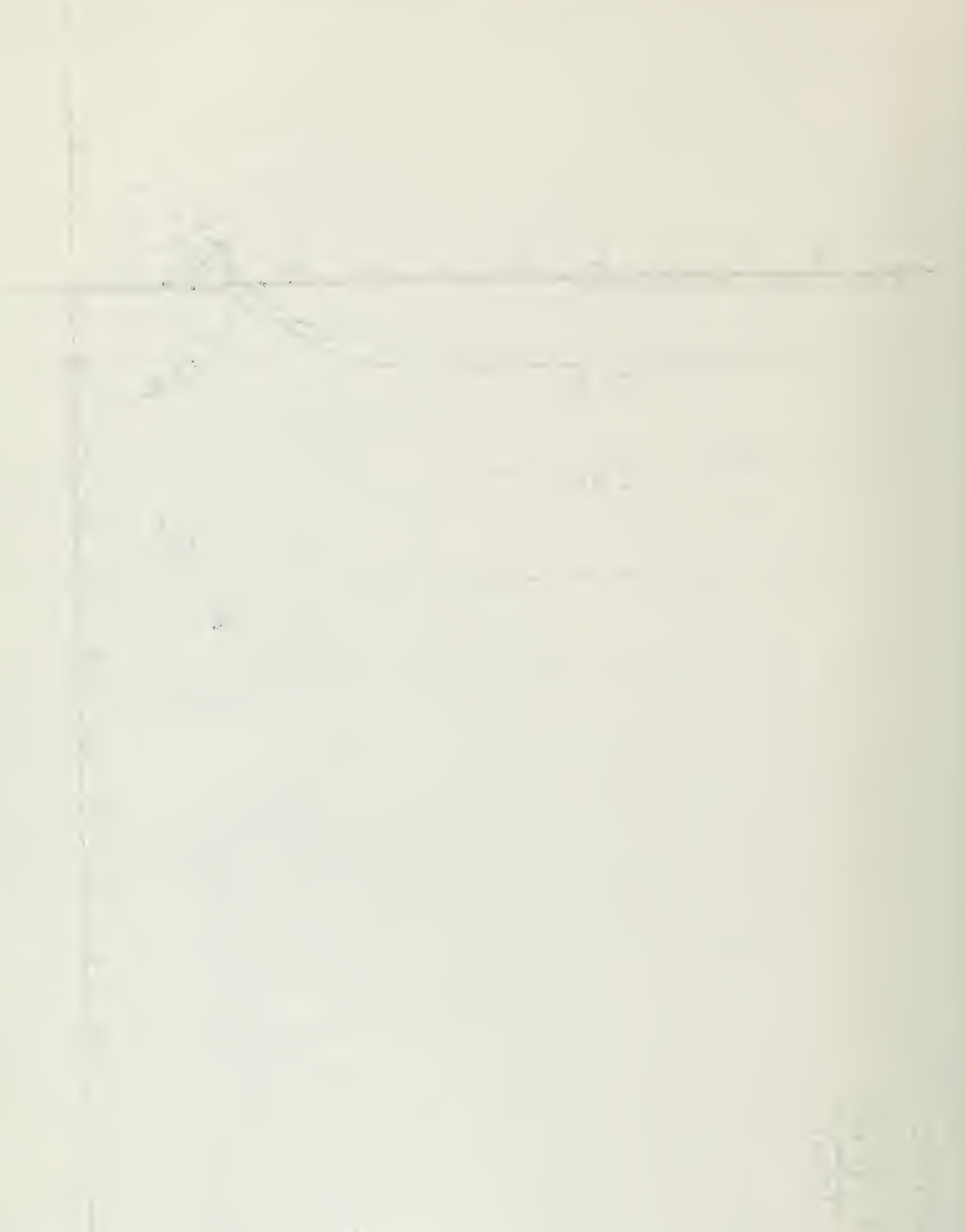

SYSTEM 1810
$G_c = \frac{K_c(s+a)}{(s+1)}$
ASSYMPTOTE FOR a = 8
a = 6
a = 3
a = 0
$K_c = .5$
$K_c = 1$
$K_c = 2$

Figure 7-2(a)



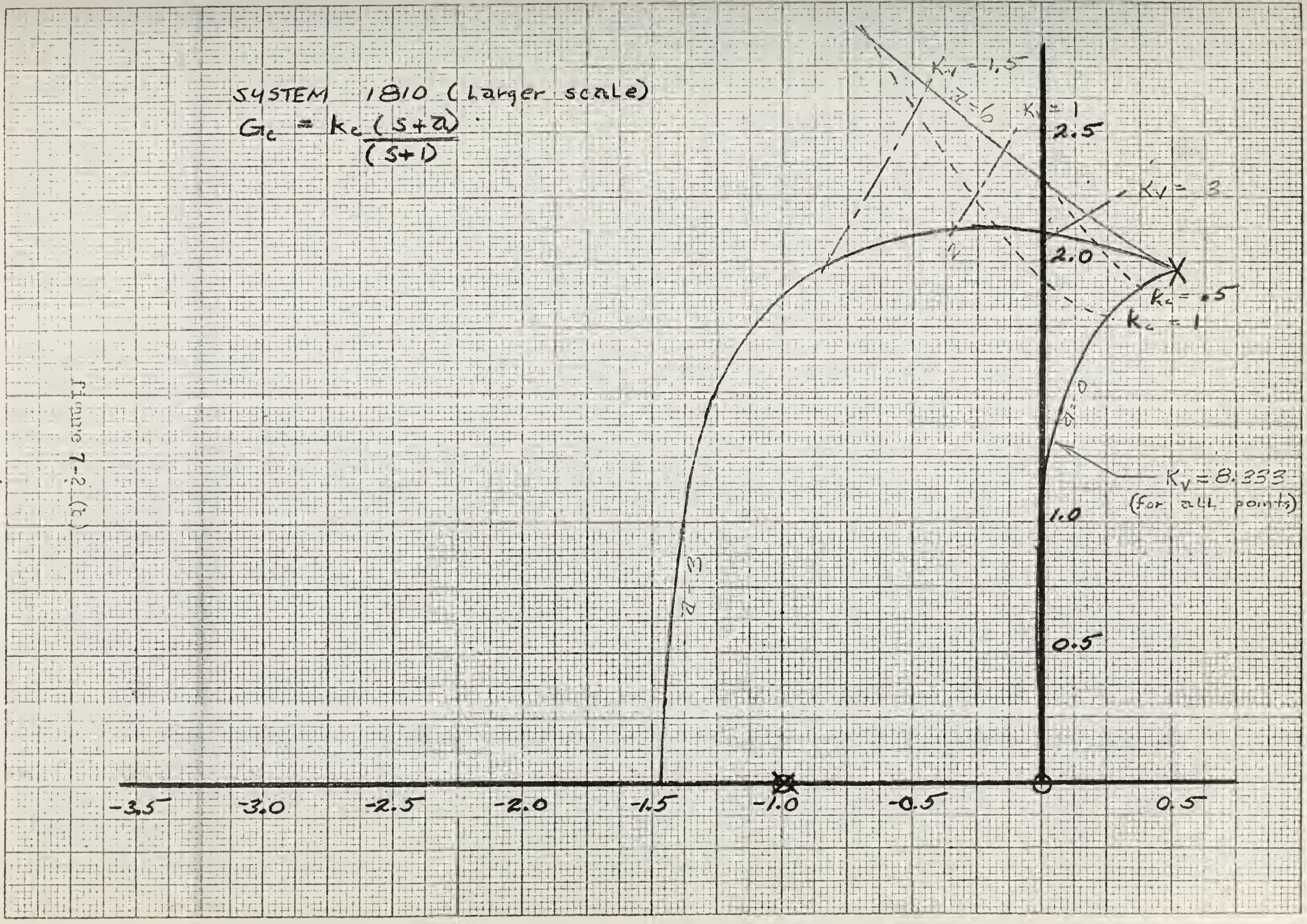

SYSTEM 1810 (Larger scale)
$G_c = k_c \frac{(s+a)}{(s+1)}$
$K_v = 1.5$
$a = 6$
$K_v = 1$
$K_v = 3$
$k_c = .5$
$k_c = 1$
$a = 0$
$a = 3$
$K_v = 8.333$
(for all points)
2.5
2.0
1.0
0.5
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.5

Figure 7-2 (b)

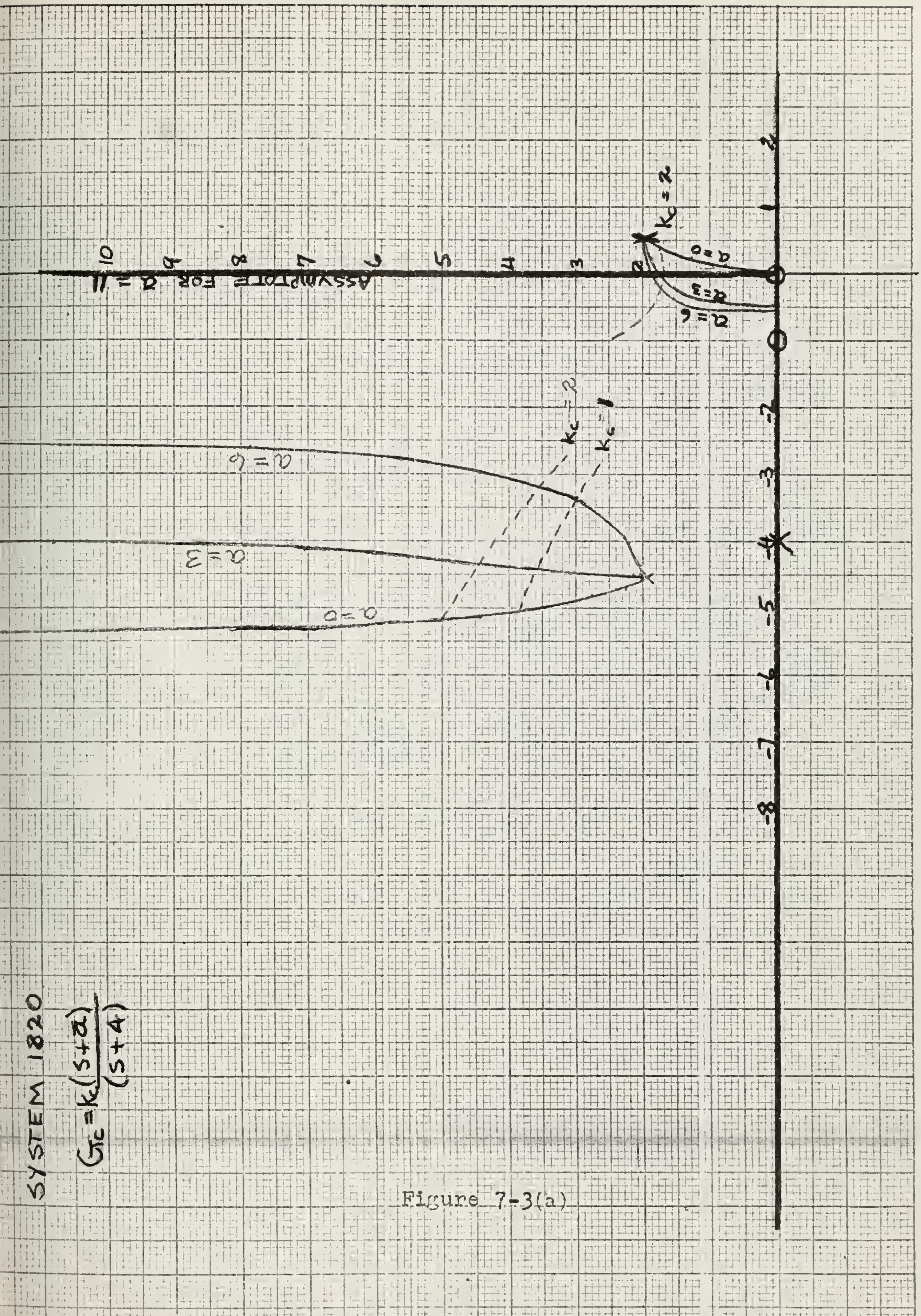

SYSTEM 1820
$G_{Tc} = \frac{K_c(s+a)}{(s+4)}$
a=0
a=3
a=6
K_c=1
K_c=2
K_c=2
ASSYMPTOTE FOR a = 11

Figure 7-3(a)

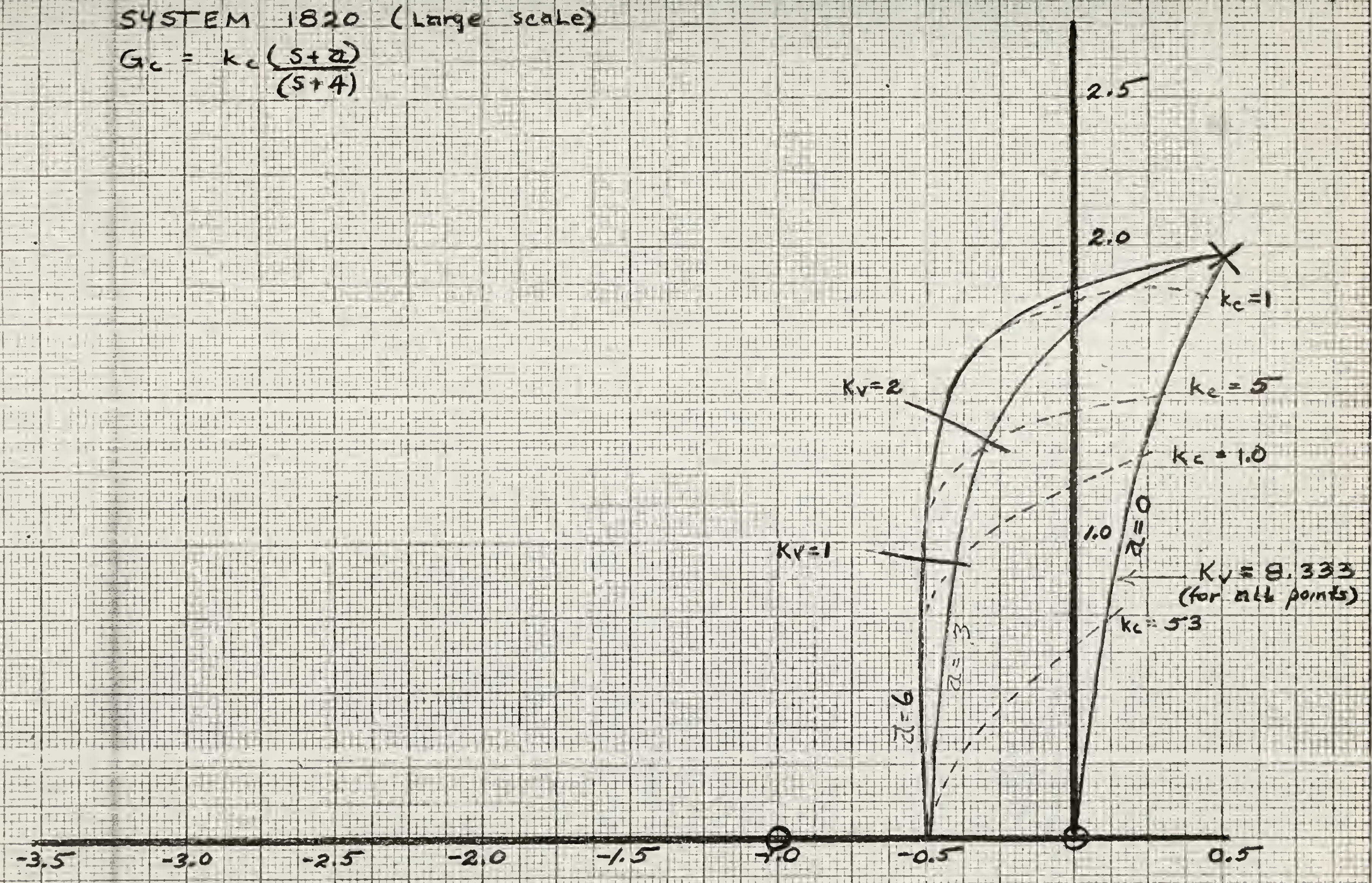

SYSTEM 1820 (Large scale)
$G_c = k_c \frac{(s+z)}{(s+4)}$
2.5
2.0
1.0
$k_c = 1$
$k_c = 5$
$k_c = 1.0$
$k_c = 53$
$K_v = 2$
$K_v = 1$
$K_v = 8.333$ (for all points)
$z = 0$
$z = 2$
$g = 2$
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.5

figure 7-3 (b)

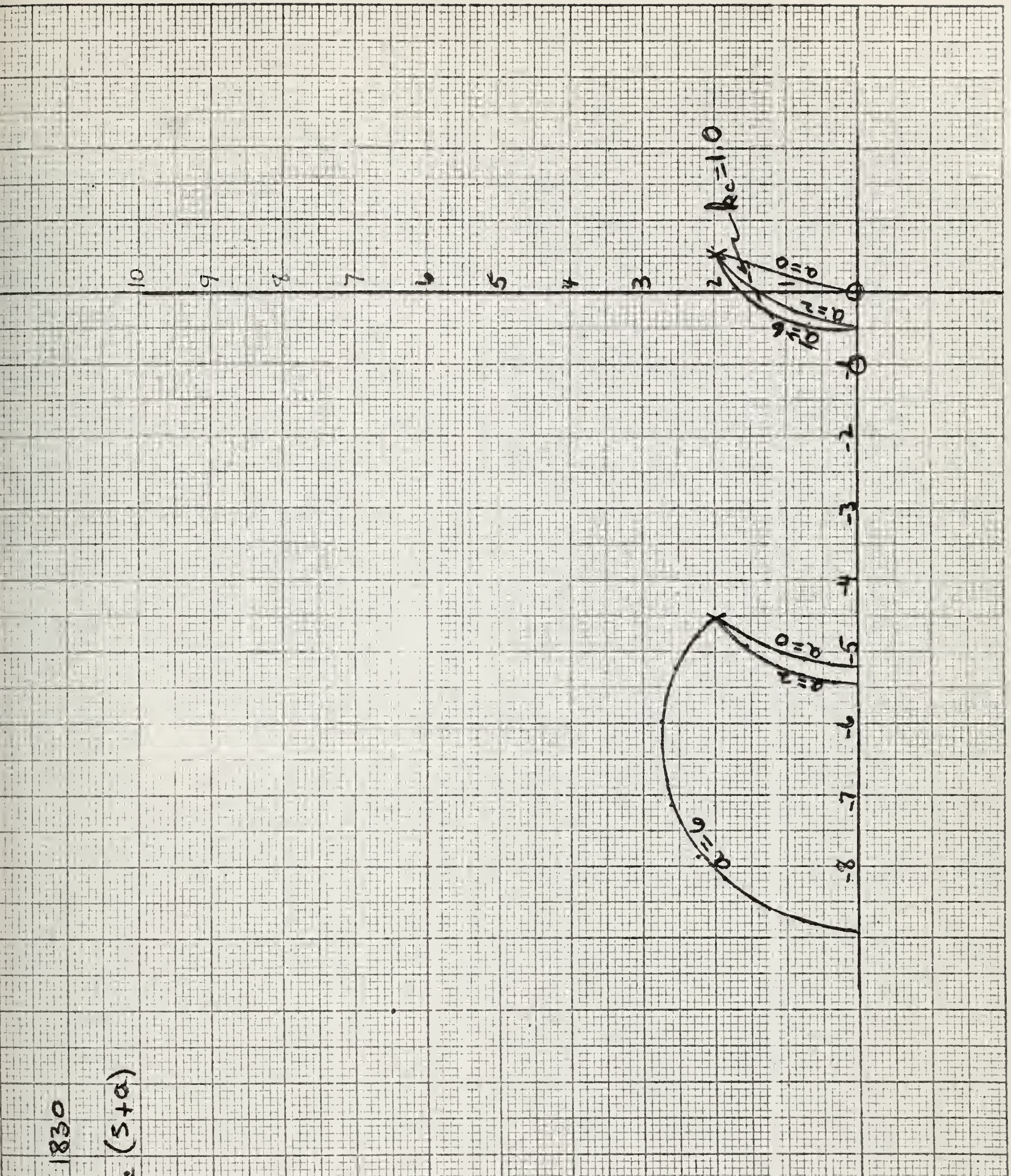


Figure 7-4(a)

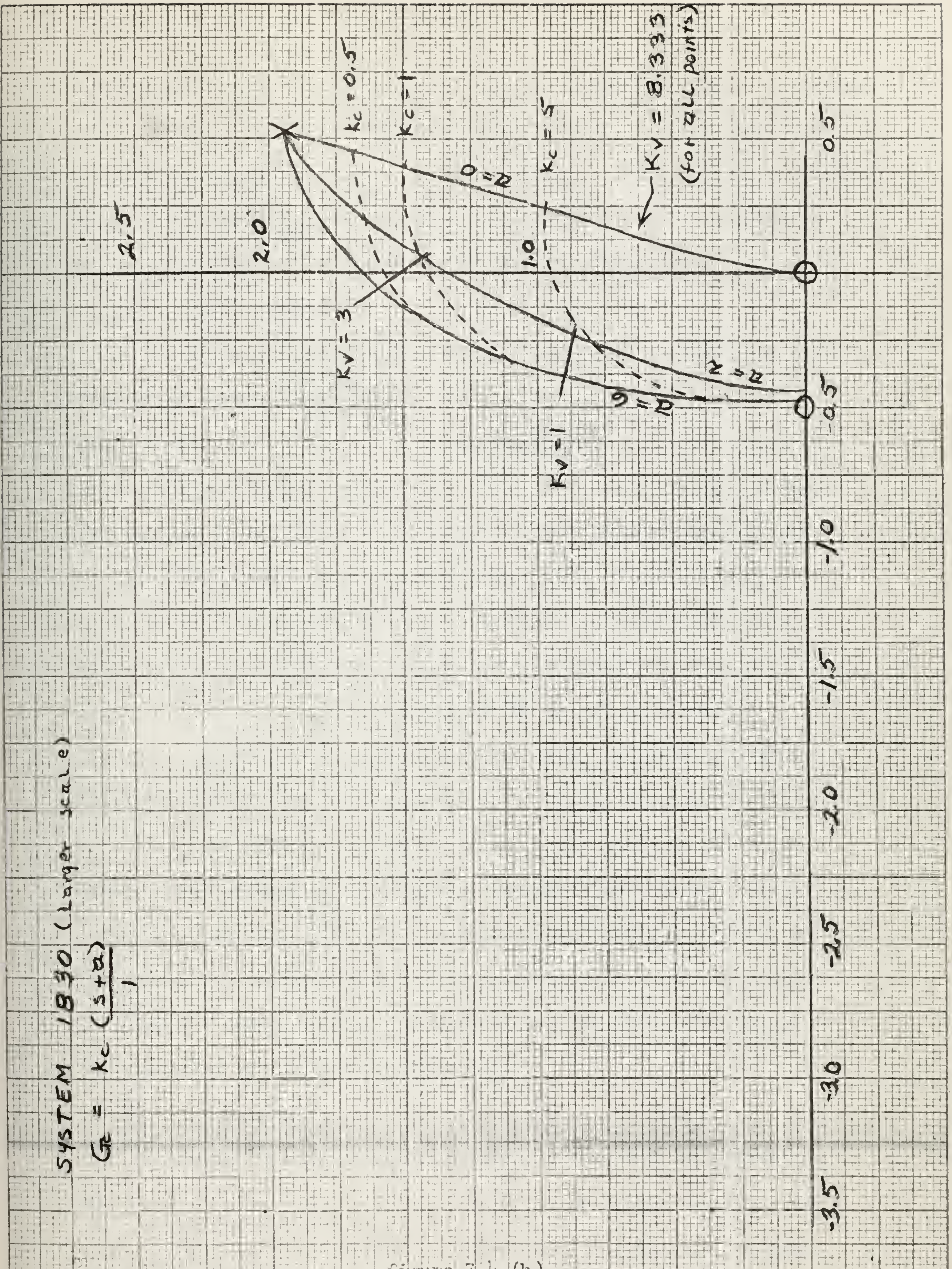

SYSTEM 1830 (Larger scale)
$G_c = K_c \frac{(s+a)}{1}$
Kv = 8.333 (for all points)
Kc = 0.5
Kc = 1
Kc = 5
Kv = 3
Kv = 1
2.5
2.0
1.0
0.5
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5

figure 7-4 (b)

Increasing the value of a increases ζ and decreases ω_n slightly, but its effect is not too noticeable about a equal to 8. Increasing k_c also increases ζ, but decreases ω_n. There is also a minimum value of gain, k_{cr}, to be met for stability. It is important to note that an increase of a reduces k_{cr} quite considerably.

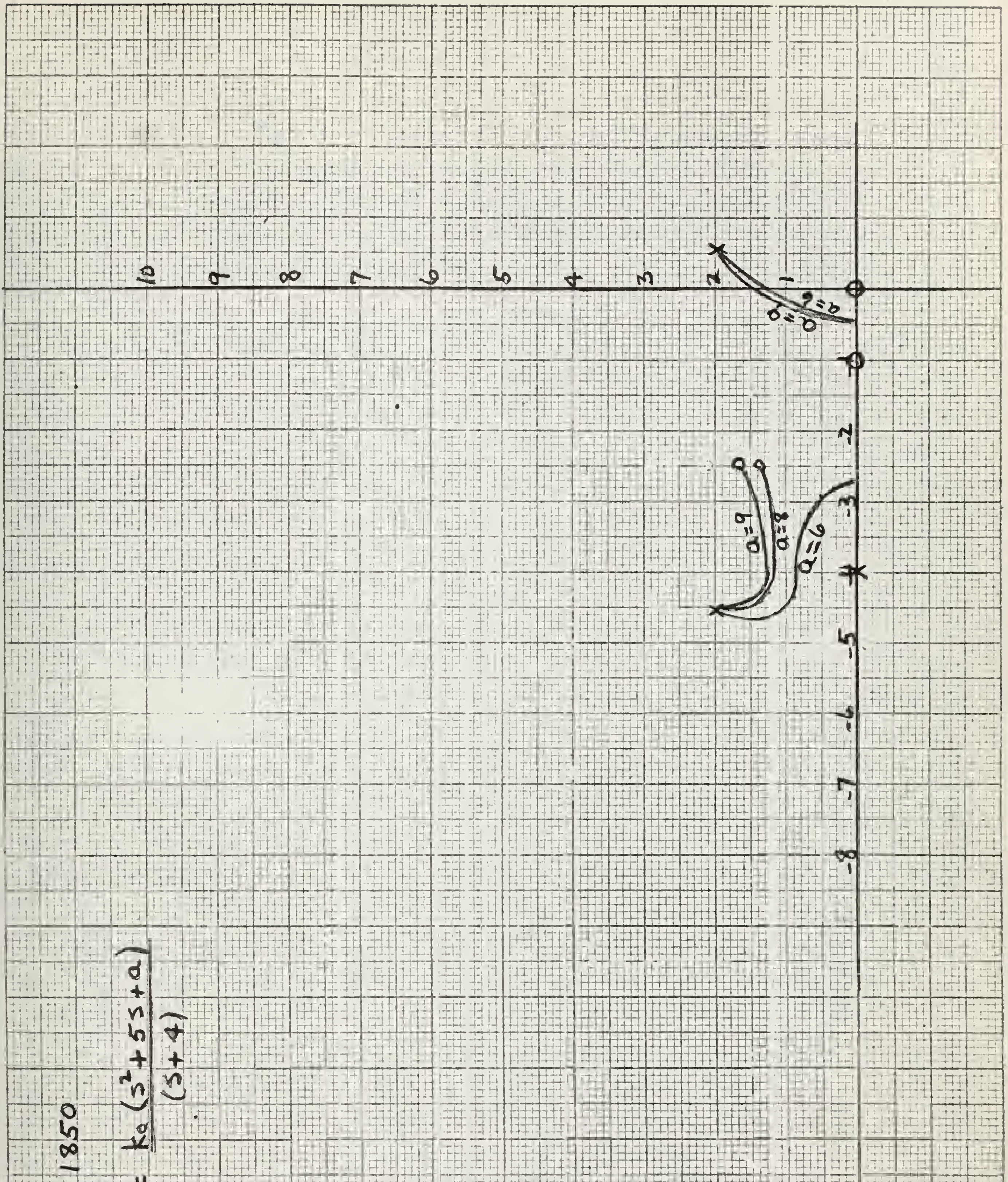
(3) "50" compensator.

These loci are shown in figure 7-5 for the normal "50" compensator of $G_c = k_c \frac{(S^2 + 5S + a)}{(S+4)}$. Figure 7-6 also shows loci for a modified "50" compensator of $G_c = k_c \frac{(S^2 + bS + 8)}{(S+4)}$, with a family of curves as b was varied from 1 to 6. The normal compensator has loci very similar to the first derivative plus proportional feedback compensator. In general, there is not much change as a is raised above the value of 6. However, it should be noted that the "far" roots begin to increase their $j\omega_c$ value and this might have some noticeable effects. A complete range of ζ's are available, but again the bandwidth is somewhat limited. As k_c is increased, ζ is increased and ω_n is decreased slightly. Again there is a minimum value of k_{cr} for stability, values of which are given in table 7-1. Although this appears to be a fairly favorable compensator, it is shown in figure 7-6, that if a is held constant and b is varied, it is possible for the system to become unstable. This has the effect of moving the system's complex zeros towards the right hand plane.

C. Partially satisfactory compensators.

(1) Lag network.

These loci are given in figures 7-2 and 7-3 for a greater than 1 and a greater than 4 respectively. Even though this compensator is listed as only partially satisfactory it is also the most flexible, and probably the best, compensator investigated. It is



SYSTEMS 1850
$G_c = \frac{k_a(s^2+5s+a)}{(s+4)}$
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
a=9
a=8
a=6

Figure 7-5(a)

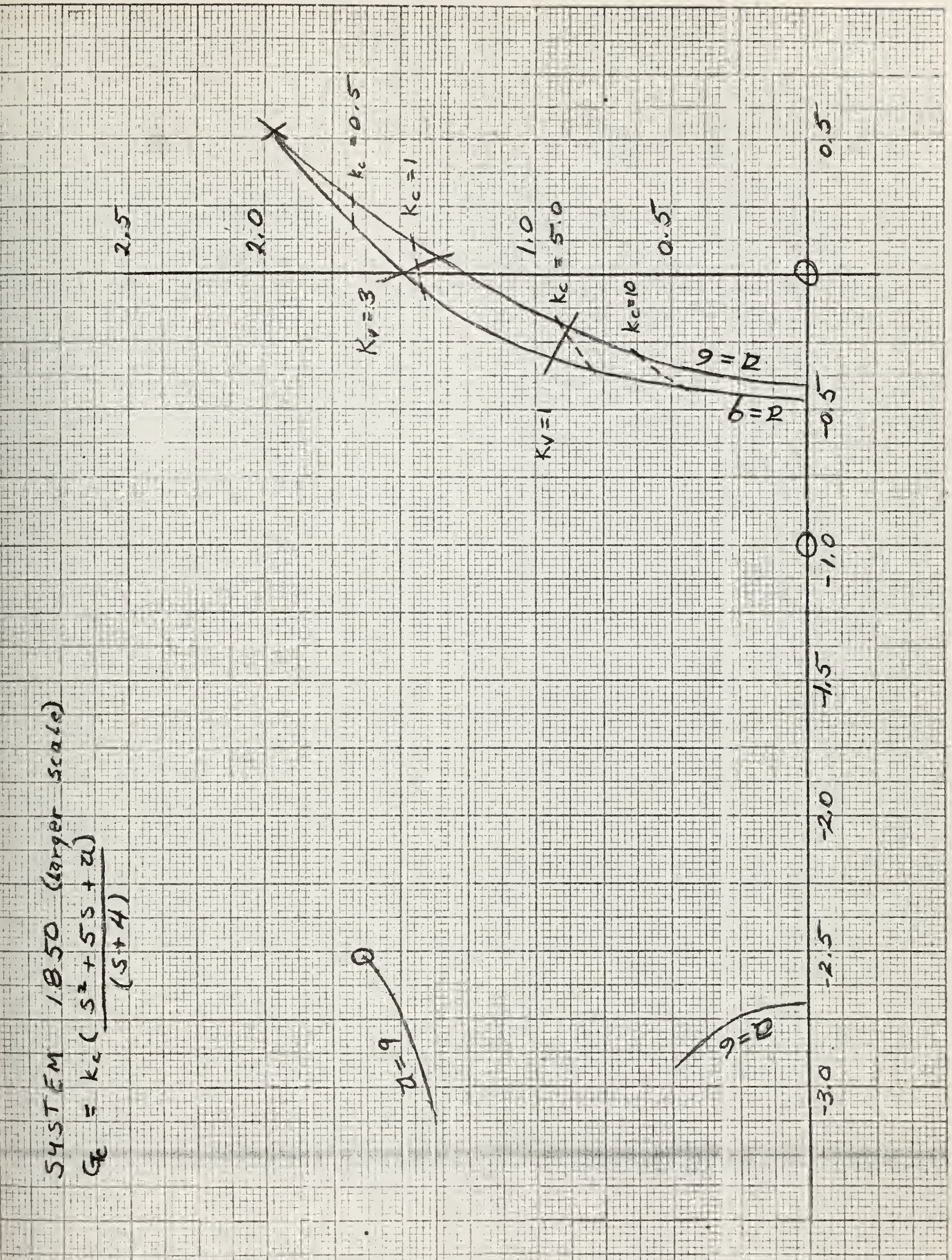

SYSTEM 1850 (larger scale)
kc = 0.5
kc = 1
kc = 5.0
kc = 10
Kv = 3
Kv = 1
2.5
2.0
1.0
0.5
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0

figure 7-5 (b)

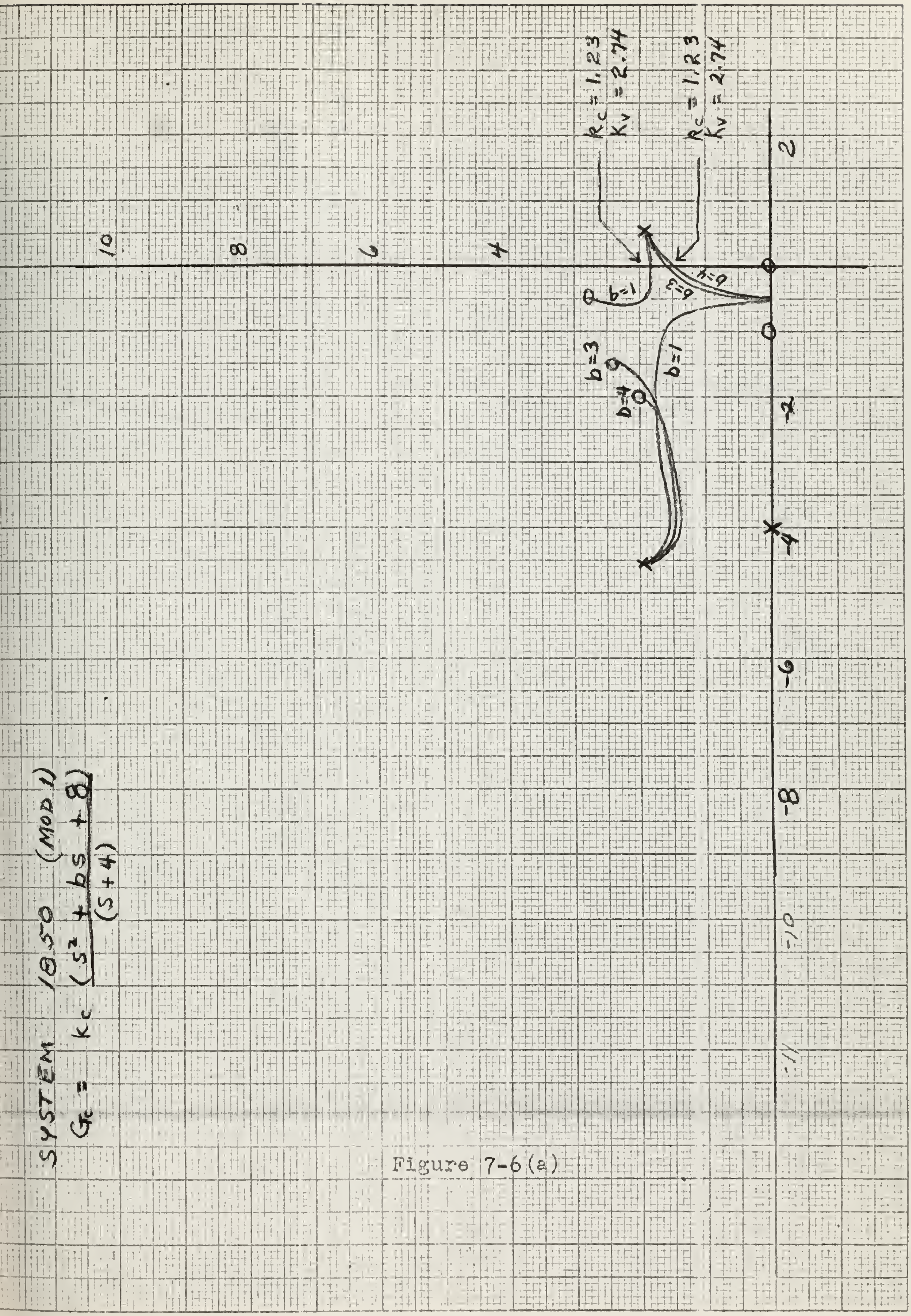

SYSTEM 1850 (MOD 1)
$G_c = K_c \frac{(s^2 + bs + 8)}{(s+4)}$
$K_c = 1.23$
$K_v = 2.74$
$K_c = 1.23$
$K_v = 2.74$
b=1
b=3
b=4
b=3
b=4
b=1
10
8
6
4
2
-2
-4
-6
-8
-10
-11

Figure 7-6(a)

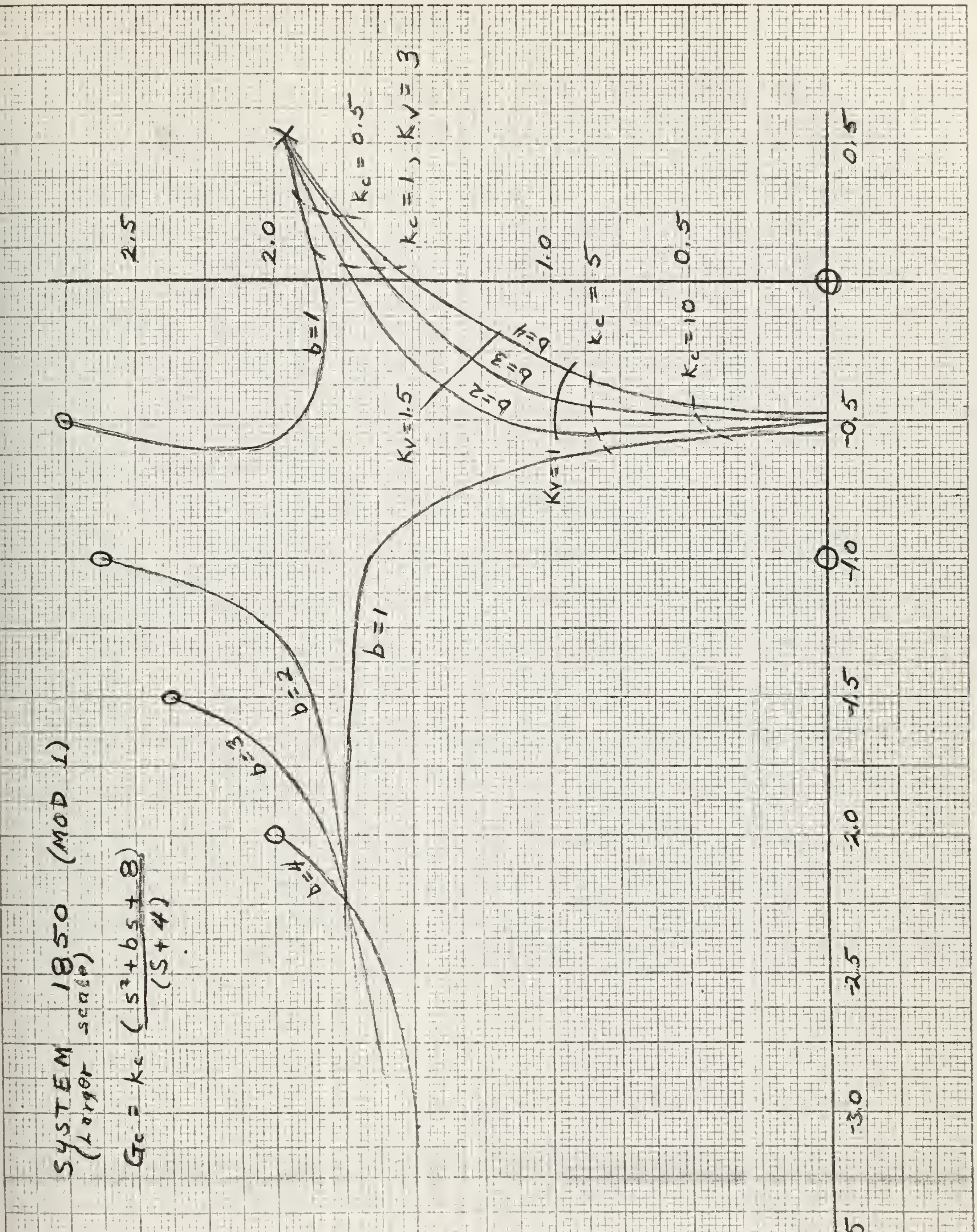

SYSTEM 1850 (MOD 1)
(Larger scale)
$G_c = K_c \frac{(s^2+bs+8)}{(s+4)}$
b=1
b=2
b=3
b=4
$K_c = 0.5$
$K_c = 1$, $K_v = 3$
$K_c = 5$
$K_c = 10$
$K_v = 1.5$
$K_v = 1$
2.5
2.0
1.0
0.5
0.5
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5

figure 7-6 (b)

only partially satisfactory since the systems become unstable for higher values of a. However, for the proper range of values of a, a wide range of ζ 's and ω_n 's are available. For the lower values of a, ζ will increase and ω_n will decrease slightly as compensator gain, k_c, is increased. By holding k_c constant and increasing a, ω_n will increase. There is the limiting case for instability, however, as previously mentioned. These values are shown in table 7-1. It should also be noted that the lower compensator pole (10) is more satisfactory than the larger (20).

(2) Second derivative plus proportional feedback.

It was found that pure second derivative feedback was completely unsatisfactory, but that the system could be stabilized by a high amount of proportional component, a. These loci are given in figure 7-7 for a greater than 0. Here the complex zeros due to the compensator are located on the $j\omega_c$ axis and increase as the square root of a. The system is unstable for low values of a but becomes stable as a is increased. After stability is attained a wide range of ζ 's and ω_n's are obtainable. Note that the minimum computer gain for stability, k_{cr}, is very low in these cases. Otherwise the loci are very similar to the first derivative plus proportional compensator.

(3) "60" Compensator.

This compensator is capable of stabilizing the system; however, it has a very limited range for values of a to be effective. As seen from the loci in figure 7-8, increasing a beyond a value of 5 will make the system completely unstable. For values of a somewhat lower there is both a minimum and maximum value of computer gain, k_{cr}, for stability. For even lower values of a, the system then

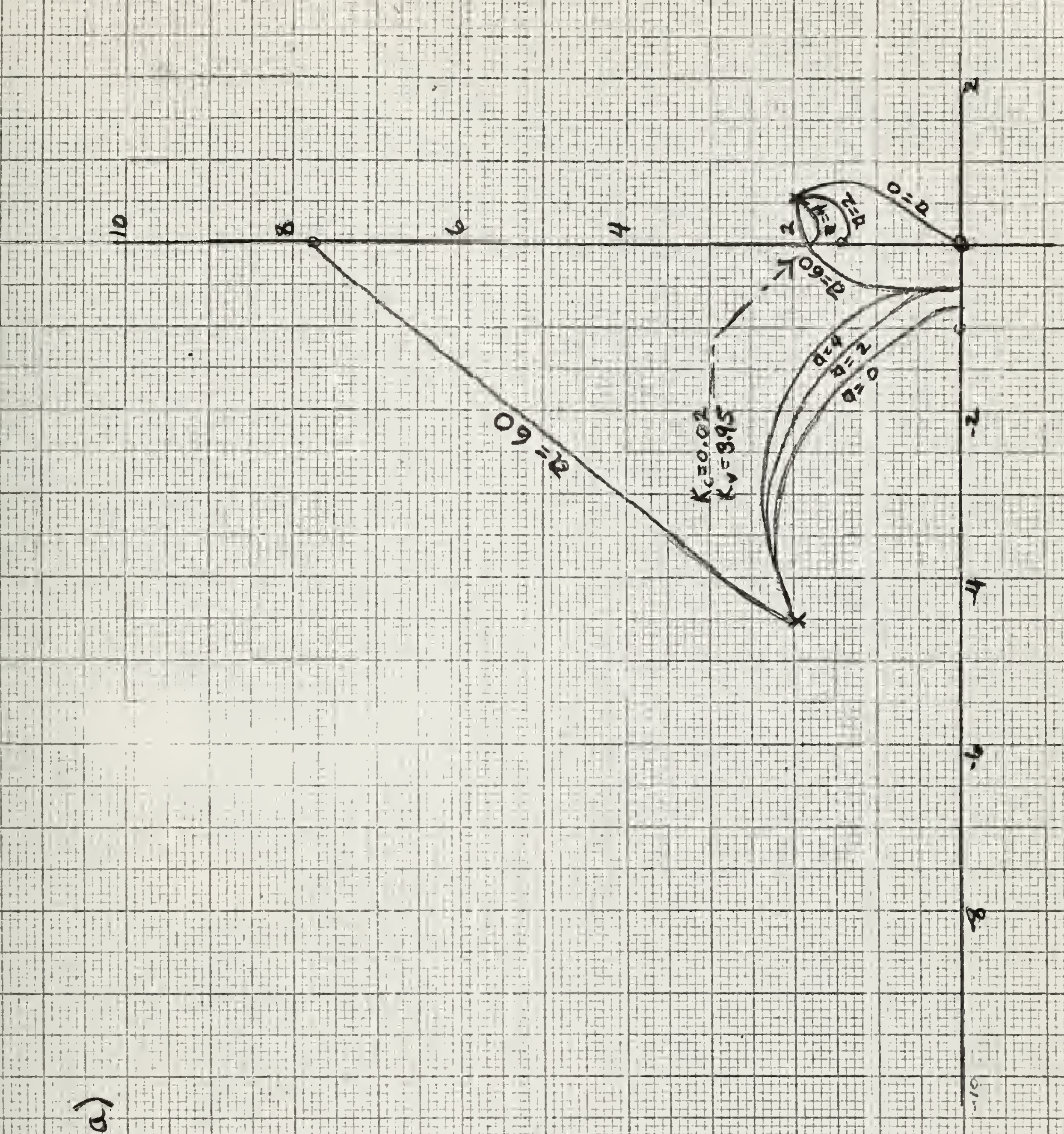


Figure 7-7

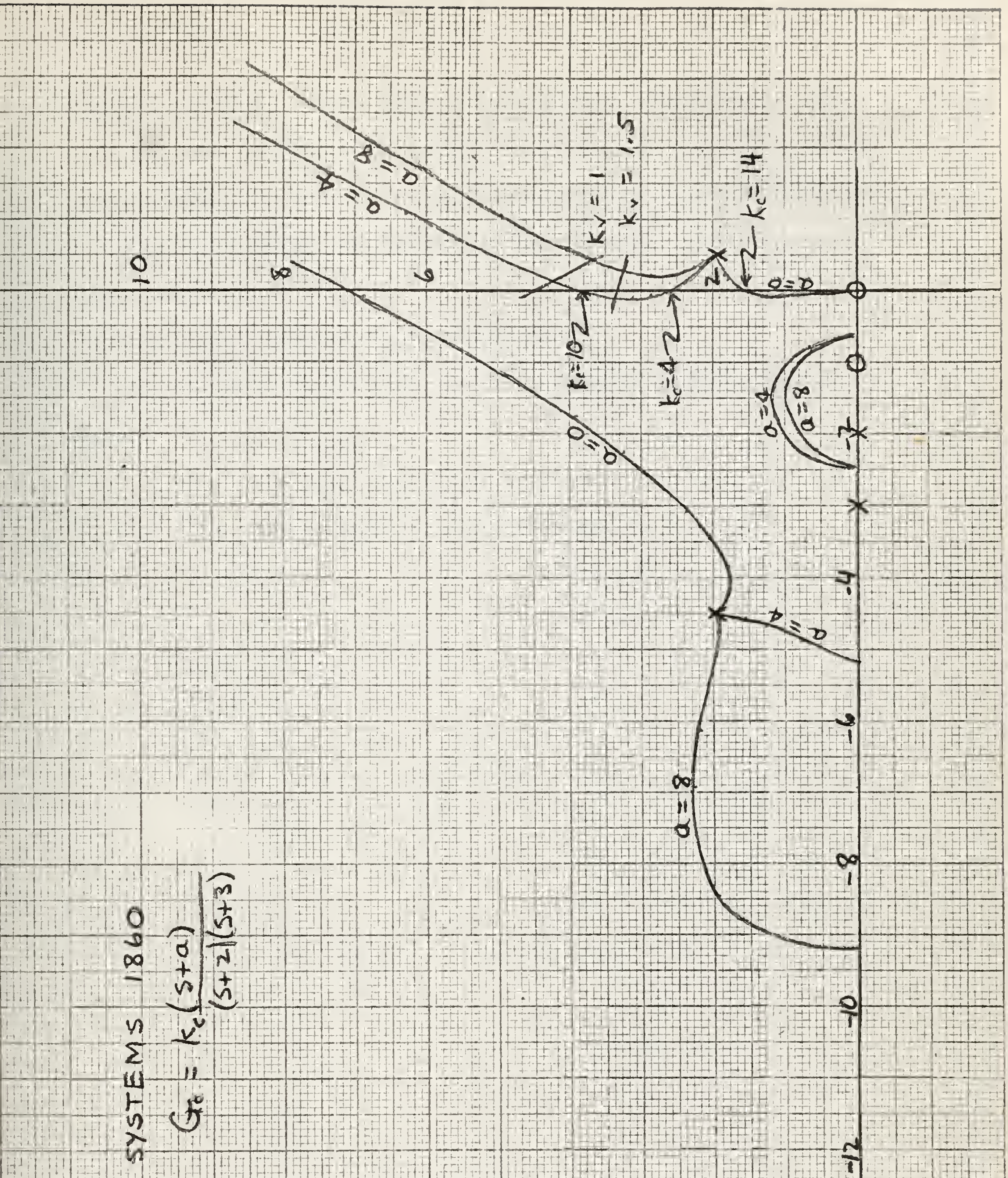

SYSTEMS 1860
$G_o = \frac{K_c(s+a)}{(s+2)(s+3)}$
a=8
a=4
a=0
$K_v = 1$
$K_v = 1.5$
$K_c = 14$
$K_c = 10$
$K_c = 4$
a=8
a=4
a=0
a=4
a=8
10
8
6
2
-2
-4
-6
-8
-10
-12

Figure 7-8

becomes completely stable above a minimum k_{cr}. These latter seem to be the more desirable locations. In general, however, only small values of ω_n are obtainable and the compensator does not appear to be as desirable as the single lead or lag network. Table 7-1 gives values for critical gain.

D. Completely unsatisfactory compensators.

The following compensators were incapable of stabilizing the system:

(1) Pure first derivative feedback

(2) Pure second derivative feedback.

TABLE 7-1

APPROXIMATE LIMITS OF STABILITY

Compensator	a	Lower limit k_c	Lower limit K_v	Upper limit k_c	Upper limit K_v
10	3	0.8	2.8		
10	6	0.5	2.3		
20	3	2.0	3.6		
20	6	1.5	2.9		
30	2	1.3	2.6		
30	6	0.3	3.4		
50	6	2.0	2.4		
50	9	1.0	2.8		
50 mod 1 (b=1)	8	1.2	2.7		
50 mod 1 (b=4)	8	1.2	2.7		
40	60.	0.025	3.6		
60	0	25.	8.3		
60	3	3.5	3.5	17.0	1.0
60	4	4.0	2.6	10.0	1.3
60	6	unstable			

8. Group VII - type one system with second order motor function and three excess poles in G_b.

A. General.

Only one of the systems investigated falls into this group. The block diagram of this system, the 1900 system, is illustrated in figure 8-1.

Also shown in figure 8-1 are the roots of the uncompensated system. Because these roots indicate instability, the primary objective of compensation is to stabilize this system. However, because design specifications extend further than just demanding stability there are other requirements for compensation. Thus a secondary point to be considered in observing the effect of the compensators is the flexibility provided by use thereof.

B. Completely satisfactory compensator.

Of those investigated only one compensator was considered to be completely satisfactory: that is, limited in its capacity to stabilize only by the requirement that k_c be greater than a minimum gain k_{cr}. The root loci of this compensator, the so called "50" compensator, are shown in figure 8-2; and the values of k_{cr}, which depend on the value of a, are listed in table 8-1.

Although it is a fact that the "50" compensator is the only one investigated which stabilizes regardless of the value of a, it is not meant to be implied that this is the most effective compensator. Actually the effectiveness of this compensator, while no worse, is also no better than that of the ones considered to be only partially satisfactory. Thus, in spite of its ability to stabilize, the flexibility of this compensator is not too significant.

Nevertheless, because it is applicable to all compensated systems

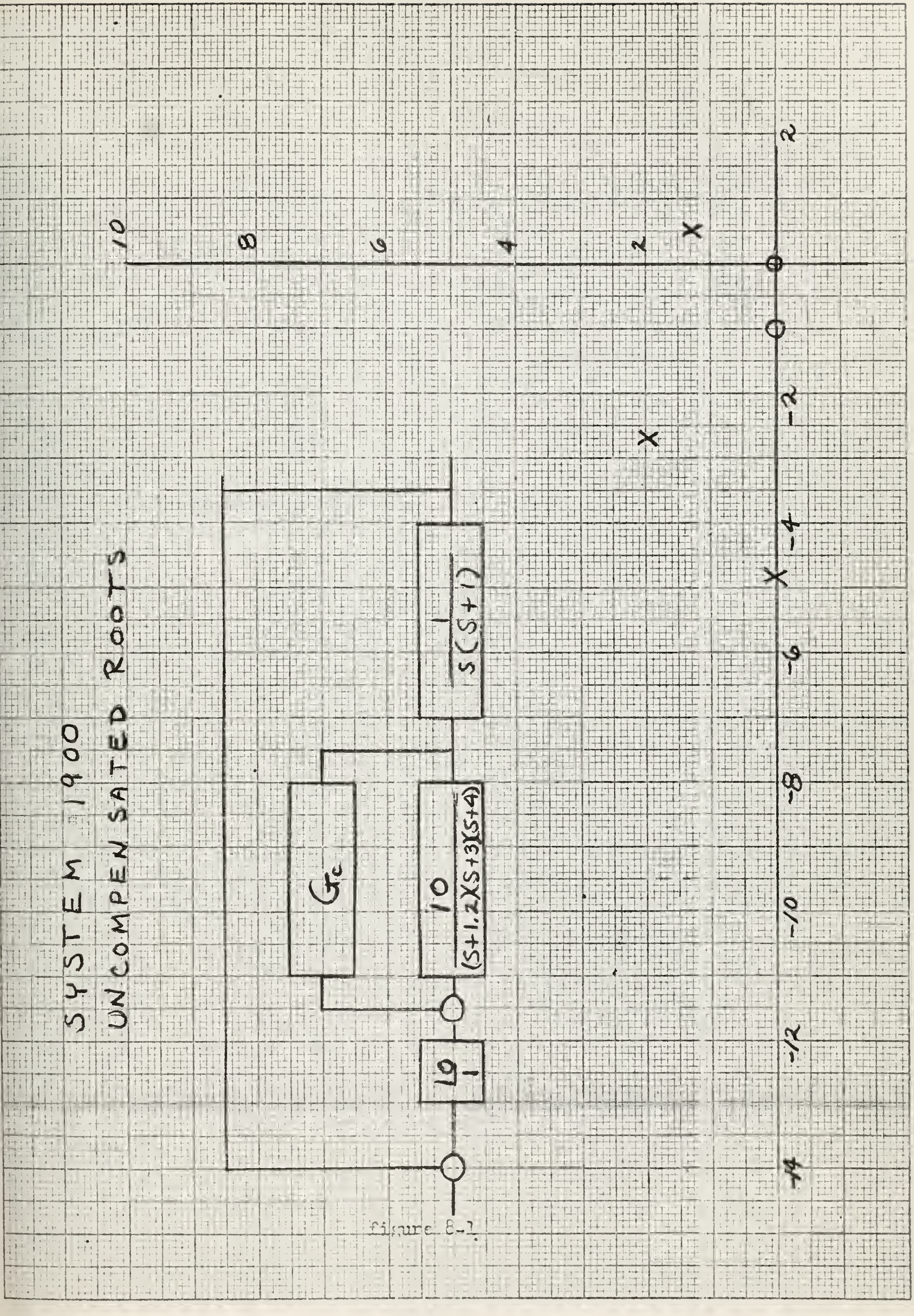

SYSTEM 1900
UNCOMPENSATED ROOTS
G_c
$\frac{10}{(S+1.2)(S+3)(S+4)}$
$\frac{1}{S(S+1)}$
$\frac{10}{1}$
10
8
6
4
2
2
-2
-4
-6
-8
-10
-12
-14

figure 8-1

SYSTEM 1950

$$G_c = k_c \frac{(s^2+5s+a)}{(s+4)}$$

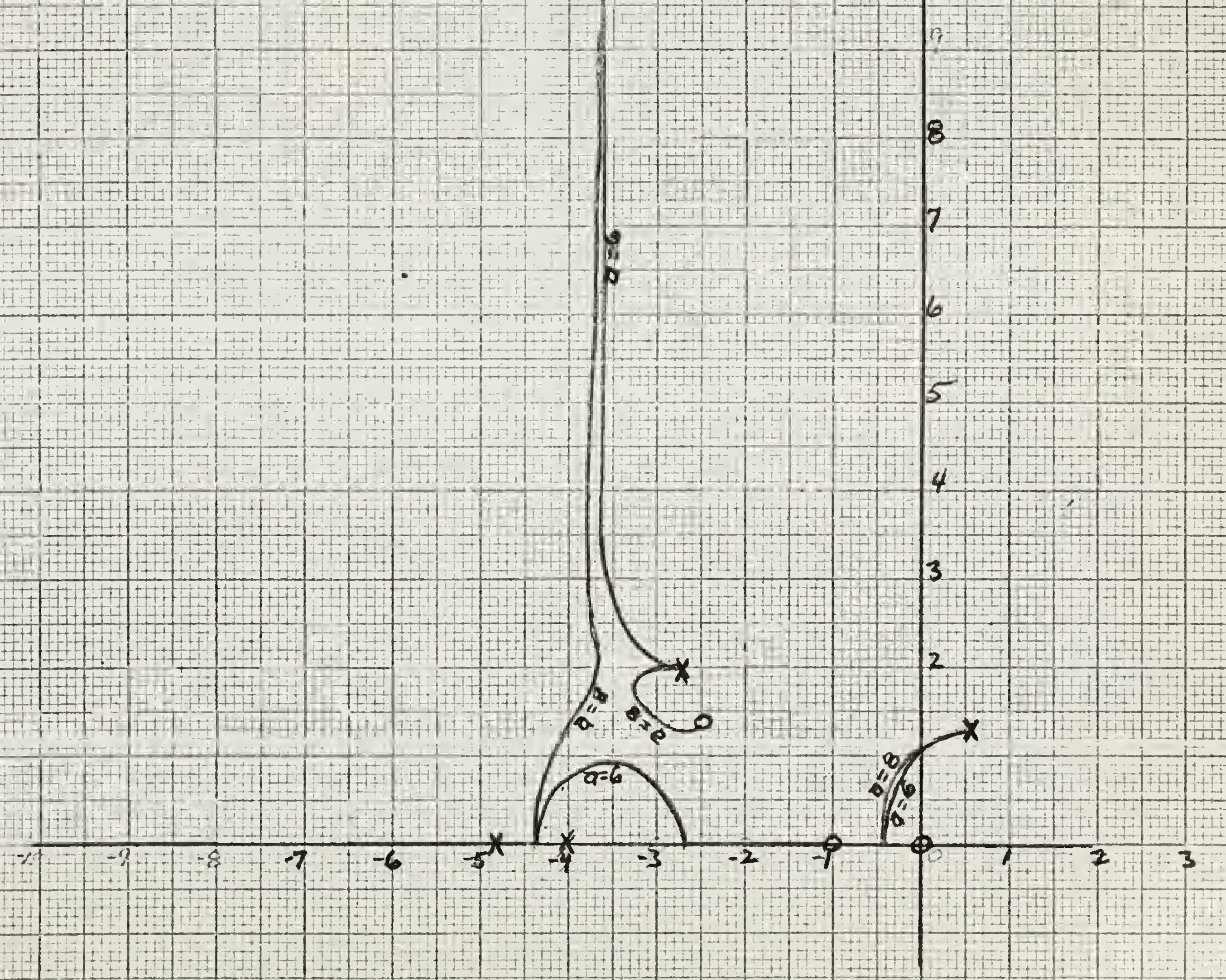


figure 8-2 (a)

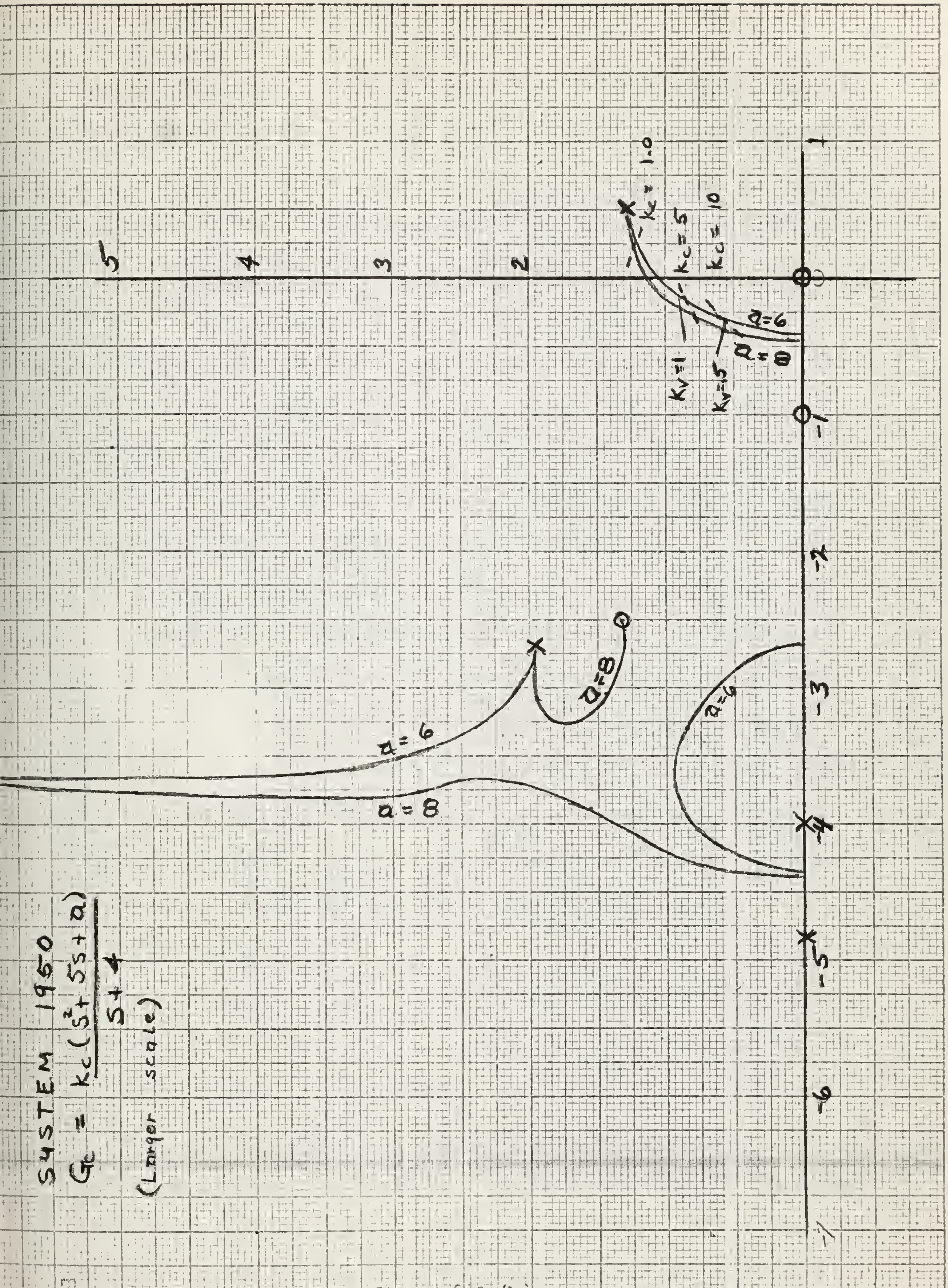

SYSTEM 1950
$G_c = \frac{K_c(s^2+5s+a)}{s+4}$
(Larger scale)
a = 6
a = 8
a=8
a=6
a=6
a=8
$K_c = 1.0$
$K_c = 5$
$K_c = 10$
$K_v=1$
$K_v=1.5$
1
2
3
4
5
-1
-2
-3
-4
-5
-6
-7

figure 8-2 (b)

in this group except system 1940, it is of value to mention briefly the variation in ζ and ω_n which is possible using this compensator. Essentially there are two methods of varying ζ. One way is to vary k_c while maintaining a constant: the other is to vary a while holding k_c constant. The former method allows ζ to vary from 0 to 1 as k_c increases above k_{cr}. The latter method causes the variation in ζ to be more restricted. In the case of ω_n its variation may be caused by varying a. However, its range of variation is not only smaller but also less consistent than that of ζ. Thus where flexibility with respect to variation in ω_n is desirable this compensator is definitely not too favorable.

C. Partially satisfactory compensators.

Five of the compensators investigated are only partially satisfactory as a compensator of the basic system. The reason for evaluating these compensators as such is two fold. First, the compensator does not provide stabilization for all values of a, and second, the compensator only induces stability when the magnitude of k_c is within a small finite range. Either one or both of these reasons may apply to each of the compensators discussed briefly below.

(1) Lag network.

Depending on the ratio of a to the magnitude of the compensator's pole, the root loci of figures 8-3 and 8-4 show that the lag network compensator has two degrees of effectiveness - fair or poor. When this ratio is considerably larger than unity, stringent limitations on a and k_c are necessary in order to insure stability. These limitations, coupled with the fact that roots having ζ in the desirable 0.4 to 0.7 range become unobtainable, reduce drastically the effectiveness of this compensator. On the other hand, better

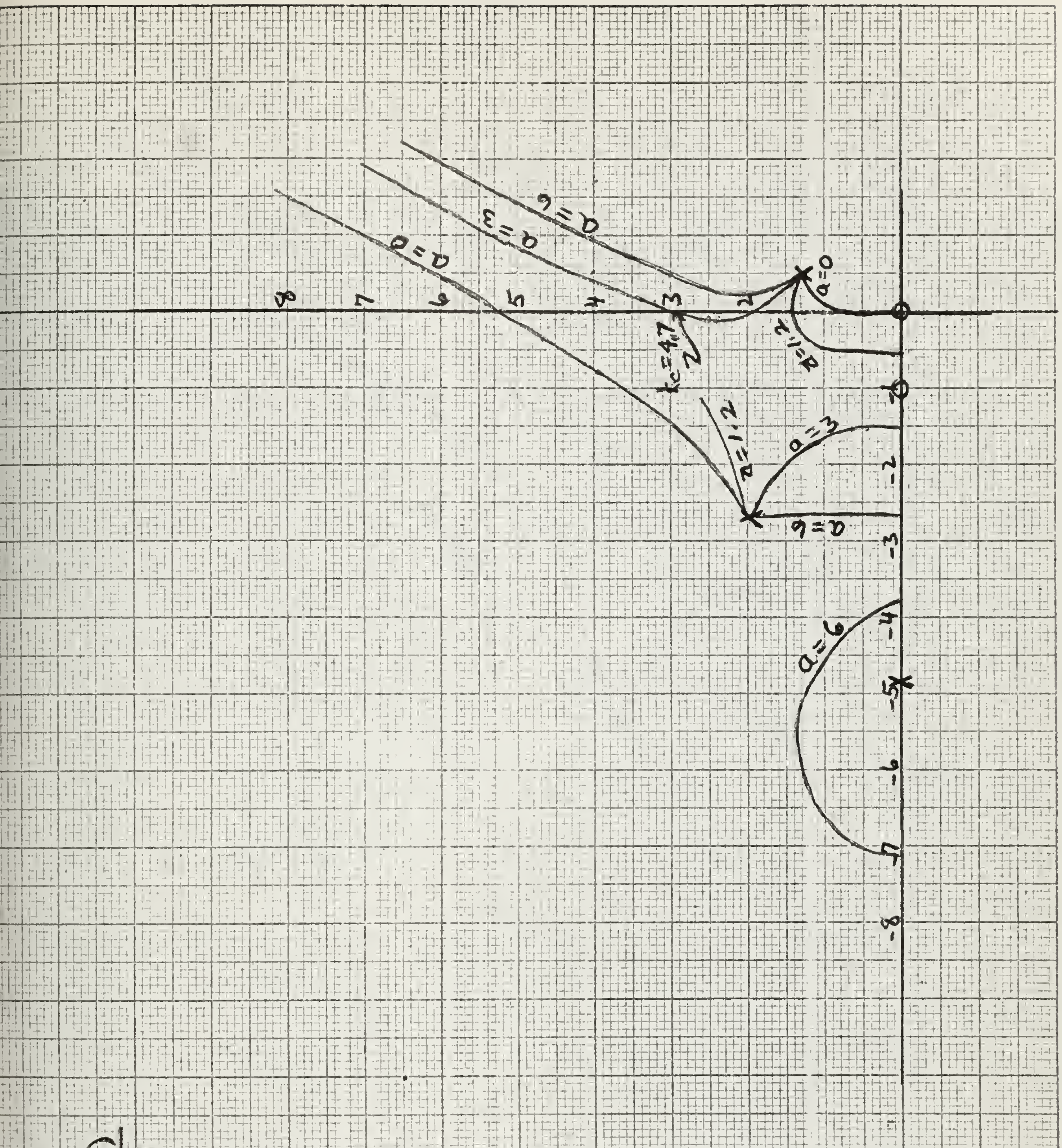
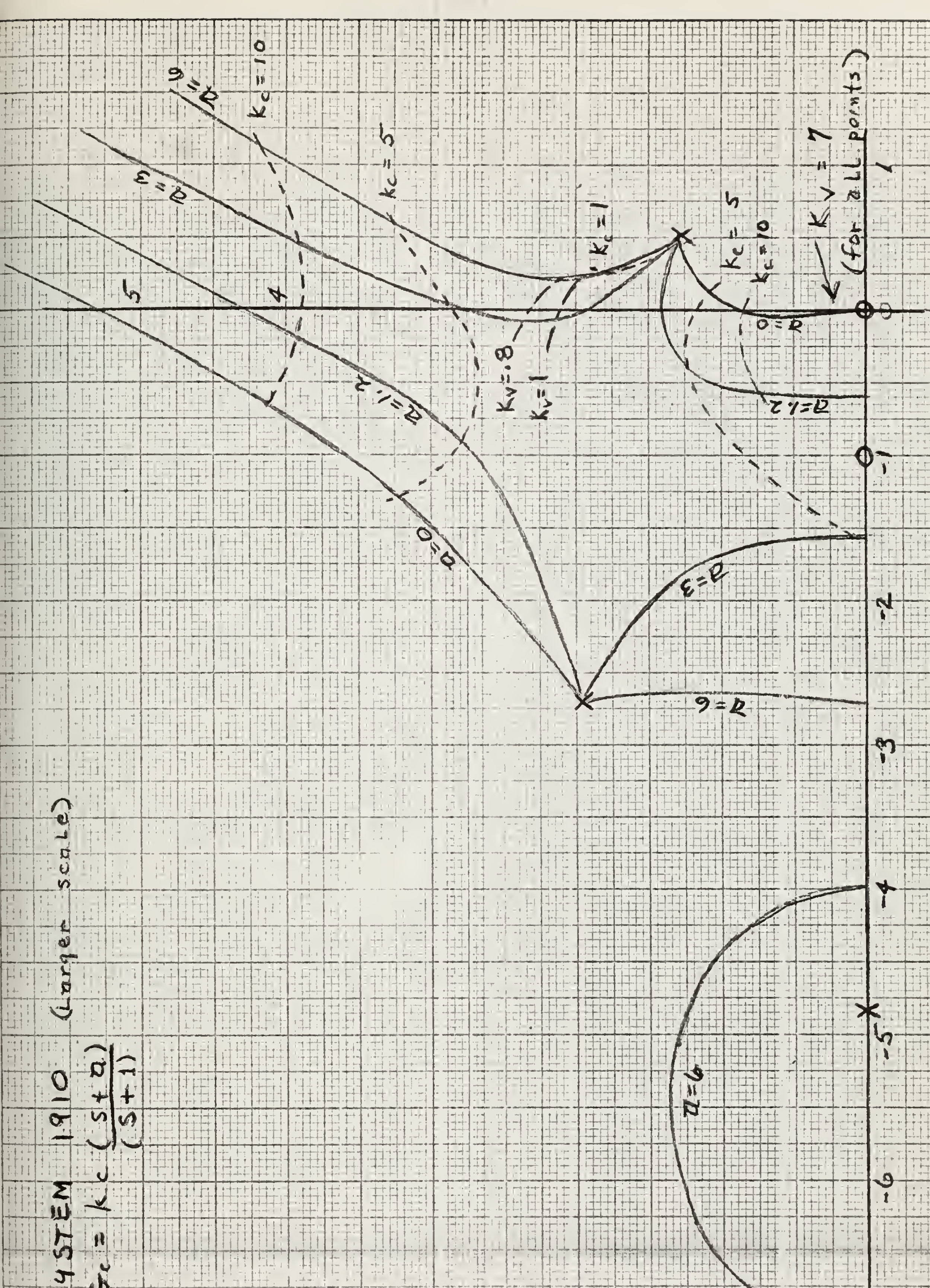
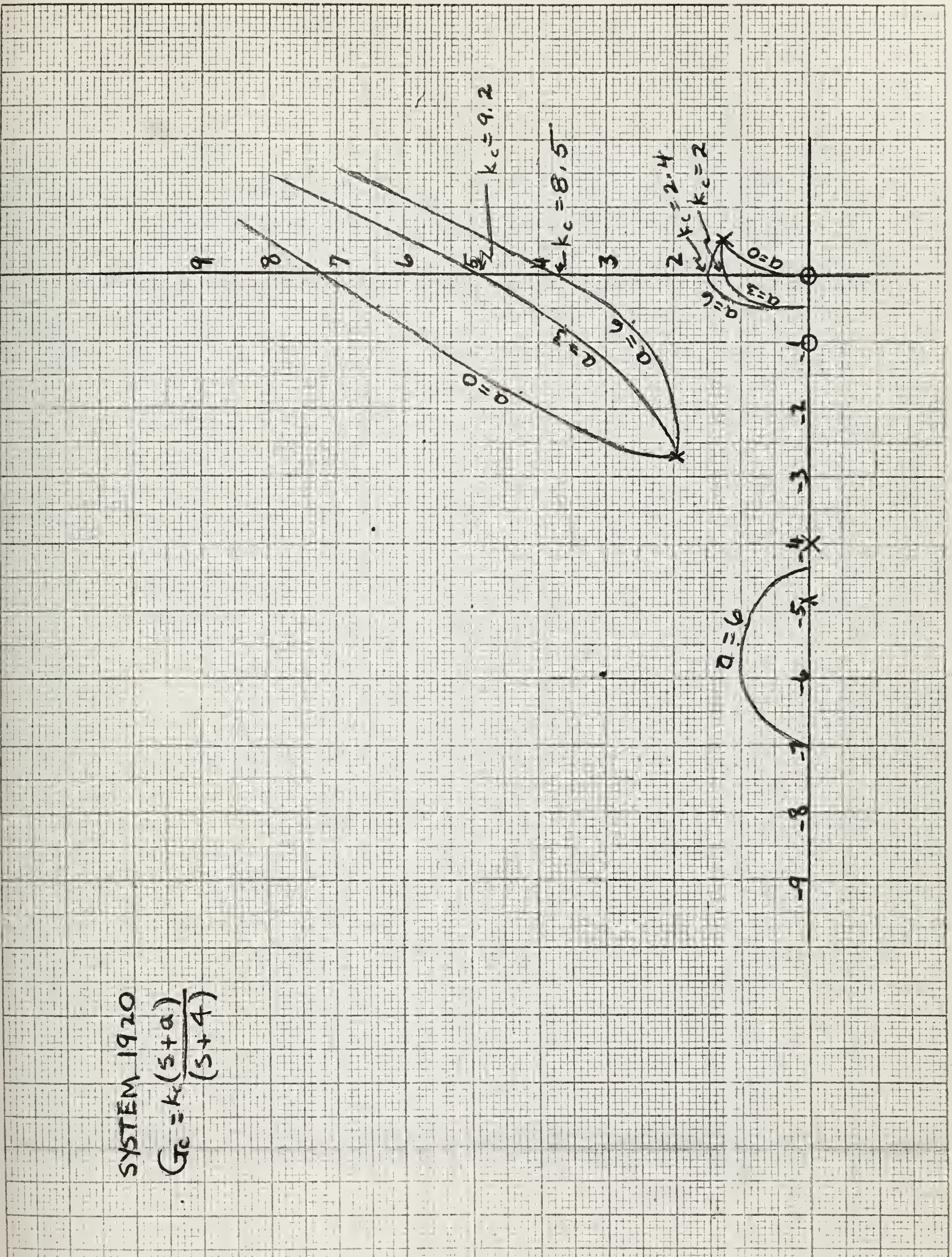

SYSTEMS 1910
$G_c = k_c \frac{(s+a)}{(s+1)}$
a=6
a=3
a=0
a=1.2
a=0
a=1.2
a=3
a=6
a=6
$k_c=47$
8
7
6
5
4
3
2
-1
-2
-3
-4
-5
-6
-7
-8

figure 8-3 (a)



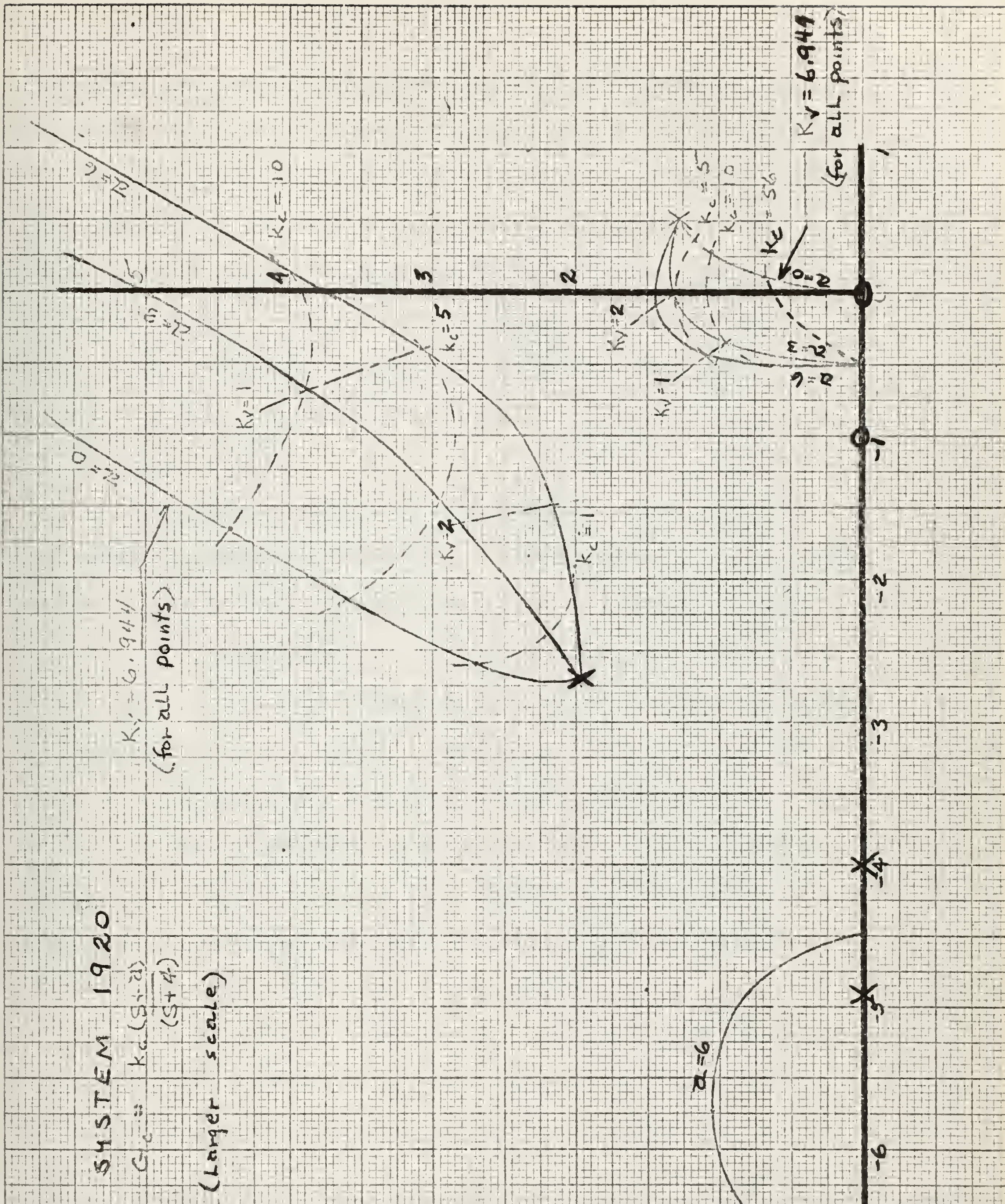
SYSTEM 1910 (Larger scale)
$G_c = k_c \frac{(s+a)}{(s+1)}$
a=6
a=3
a=1.2
a=0
a=6
a=3
a=1.2
a=0
a=6
$k_c=10$
$k_c=5$
$k_c=1$
$k_c=5$
$k_c=10$
$K_v=.8$
$K_v=1$
$K_v=7$ (for all points)
5
4
1
0
-1
-2
-3
-4
-5
-6
-7

figure 8-3 (b)



SYSTEM 1920
$G_c = k_c\frac{(s+a)}{(s+4)}$
$k_c = 9.2$
$k_c = 8.5$
$k_c = 2.4$
$k_c = 2$
a=0
a=3
a=6
a=0
a=3
a=6
a=6

figure 8-4 (a)



SYSTEM 1920
$G_c = K_c \frac{(S+a)}{(S+4)}$
(Larger scale)
$K_v = 6.941$ (for all points)
$K_v = 6.941$
(for all points)
$K_c = 10$
$K_c = 5$
$K_c = 1$
$K_v = 1$
$K_v = 2$
$a = 6$

figure 8-4 (b)

flexibility and consequently, a fair degree of effectiveness is available when the ratio is not much greater than unity. In this case there is considerable similarity between this compensator and "50" compensator. The available variation in ζ is similar and that in ω_n is slightly better: but the compensator's over all flexibility is still less due to the additional limitations on k_c and a arising from the need to maintain stability. These upper and lower limits on k_c are listed in table 8-1.

(2) Lead network.

The effectiveness of the lead network in compensating the basic system is quite similar to that of the lag network: however, here only complex roots having small values of ω_n are available. This fact can readily be verified by referring to figures 8-3 or 8-4, which show the root loci of both compensated systems plotted together. These illustrate the fact that a radical transition between the root loci caused by use of the lead and lag networks does not exist. As a matter of fact they seem to supplement each other. Hence, except when a is equal to 0, the lead network can be considered to represent the lag network with a limited to only small values; and the remarks made relative to the latter also apply to the former.

If a is equal to 0 the compensated system can not be stabilized using this compensator. However, for a not equal to 0, stability can be induced in the system provided k_c is greater than the minimum, k_{cr}. These values of k_{cr}, which depend on a are listed in table 8-1.

(3) "30" compensator with a not equal to 0.

The effectiveness of this compensator is very similar to that of the lag network: however, there is one significant difference which

makes this compensator somewhat more effective. Basically this difference is the fact that the complex root loci are oriented in such a fashion that the imaginary axis becomes the asymptote for _a_ sufficiently large. Thus, as shown by the root loci of figure 8-5, an upper limit on the value of k_c only exists when this large value of _a_ is exceeded. As a result the limiting value of _a_ (that which causes complete instability if exceeded) along with that of k_c are much larger than those observed for the lag network, and the effectiveness of this compensator is improved. Nevertheless, a lower limiting value of k_c always exists for this compensator and is listed in table 8-1.

As one might expect from the similarity of this and the lag network compensators, the methods of obtaining particular values of and ζ are also the same. Consequently, one is referred back to that discussion for the lag network for further information on the variation of these parameters.

(b) "40" compensator with _a_ not equal to 0.

The use of second derivative feedback combined with proportional feedback (the "40" compensator with _a_ not equal to 0) provides compensation which is somewhat more effective than that of the "30" compensator. In contrast to the latter, the root loci of figure 8-6 indicate that when _a_ is large the "40" compensator is always stable for k_c greater than k_{cr} (values of k_{cr} are listed in table 8-1) but when _a_ is small instability occurs. Thus it is possible to draw an analogy between these two compensators. The "40" compensator for _a_ small gives nearly the same poor effect noted for the "30" compensator with _a_ large. At the same time the "40" compensator with _a_ large provides a better stabilizing capability and therefore, better flexibility than the "30" compensator when its _a_ is small. In

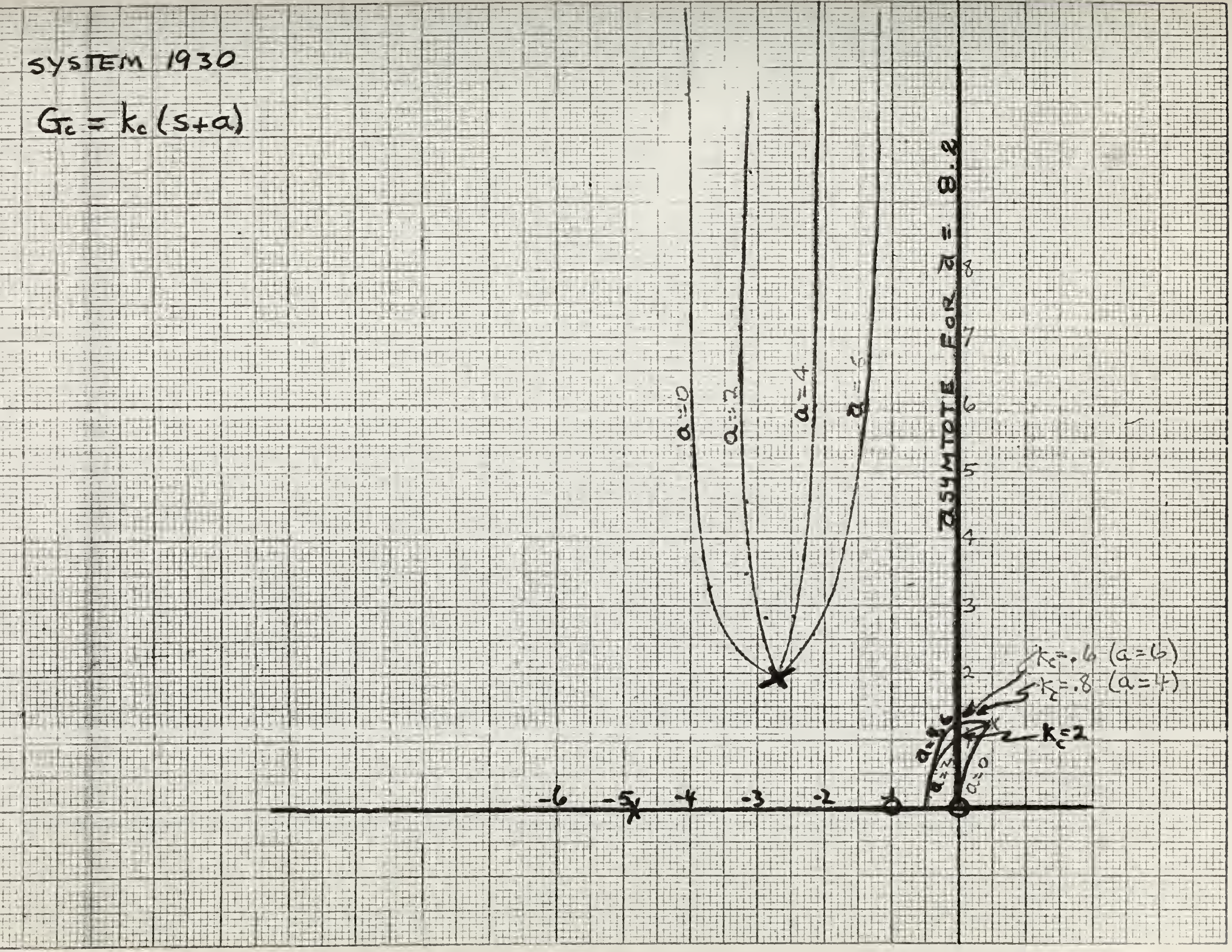

SYSTEM 1930
$G_c = k_c(s+a)$
a=0
a=2
a=4
ASYMTOTE FOR a = 8.2
-6
-5
-4
-3
-2
2
3
4
5
6
7
8
k_c=.6 (a=6)
k_c=.8 (a=4)
k_c=2
a=0

figure 8-5 (a)

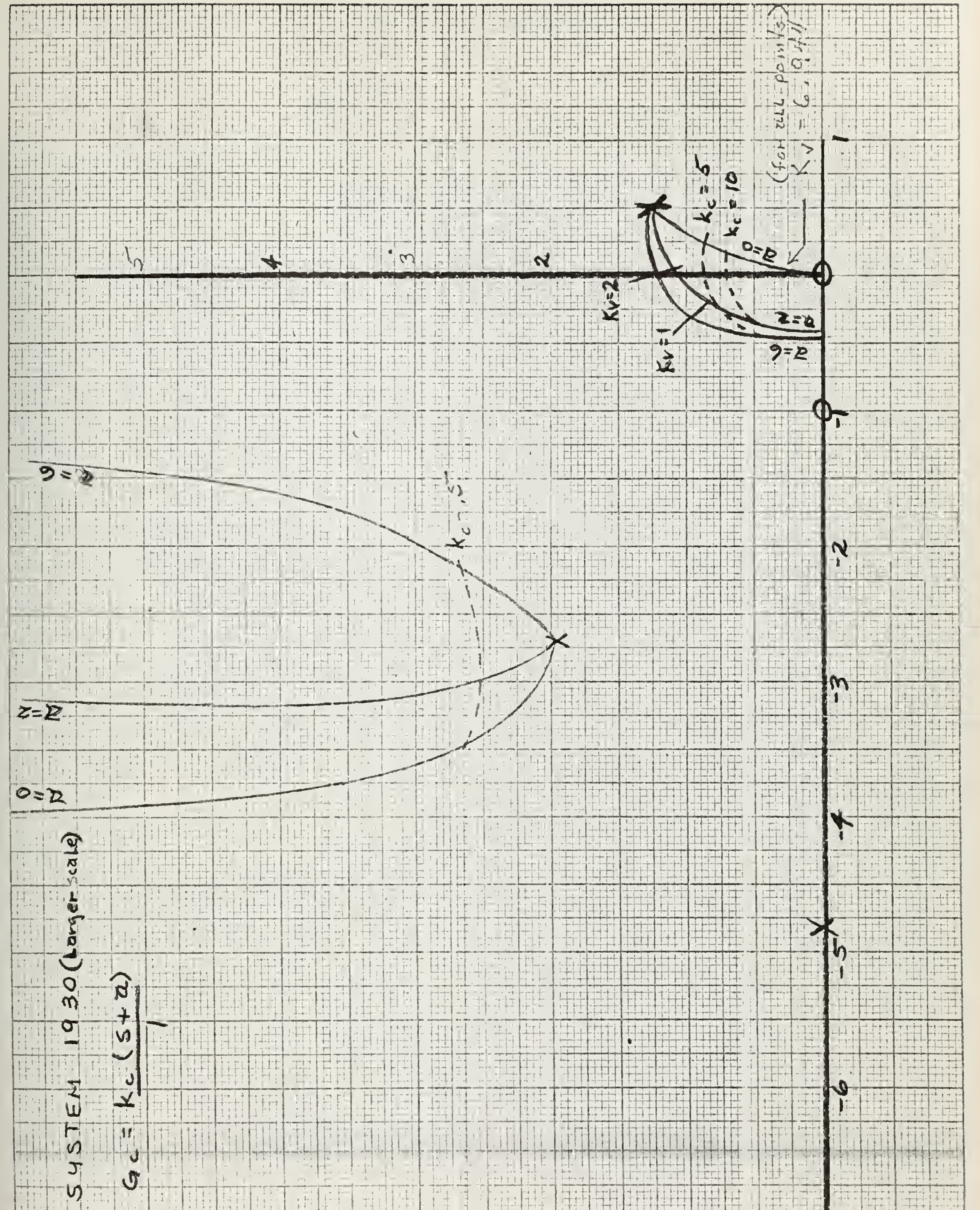


figure 8-5 (b)

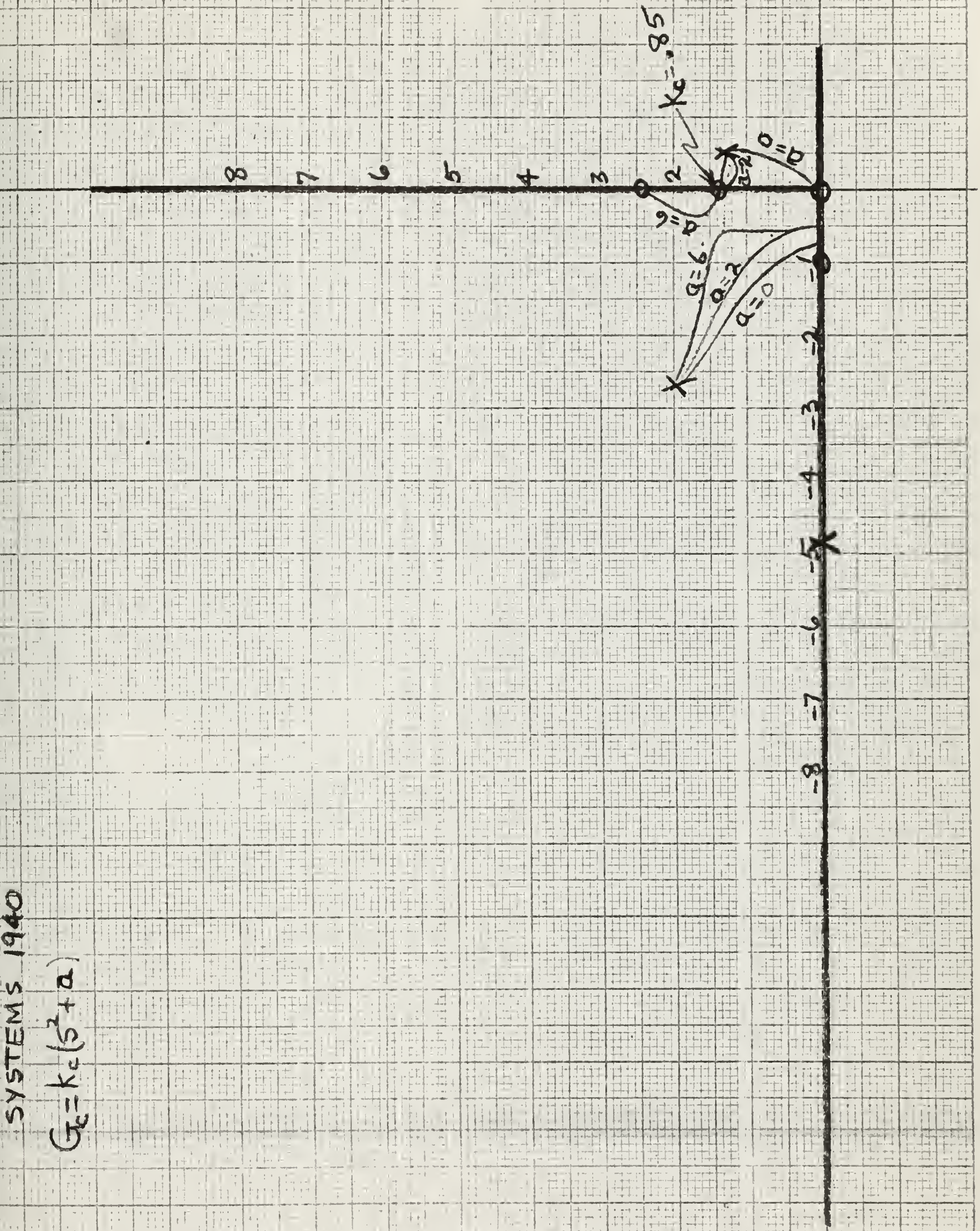


figure 8-6 (a)

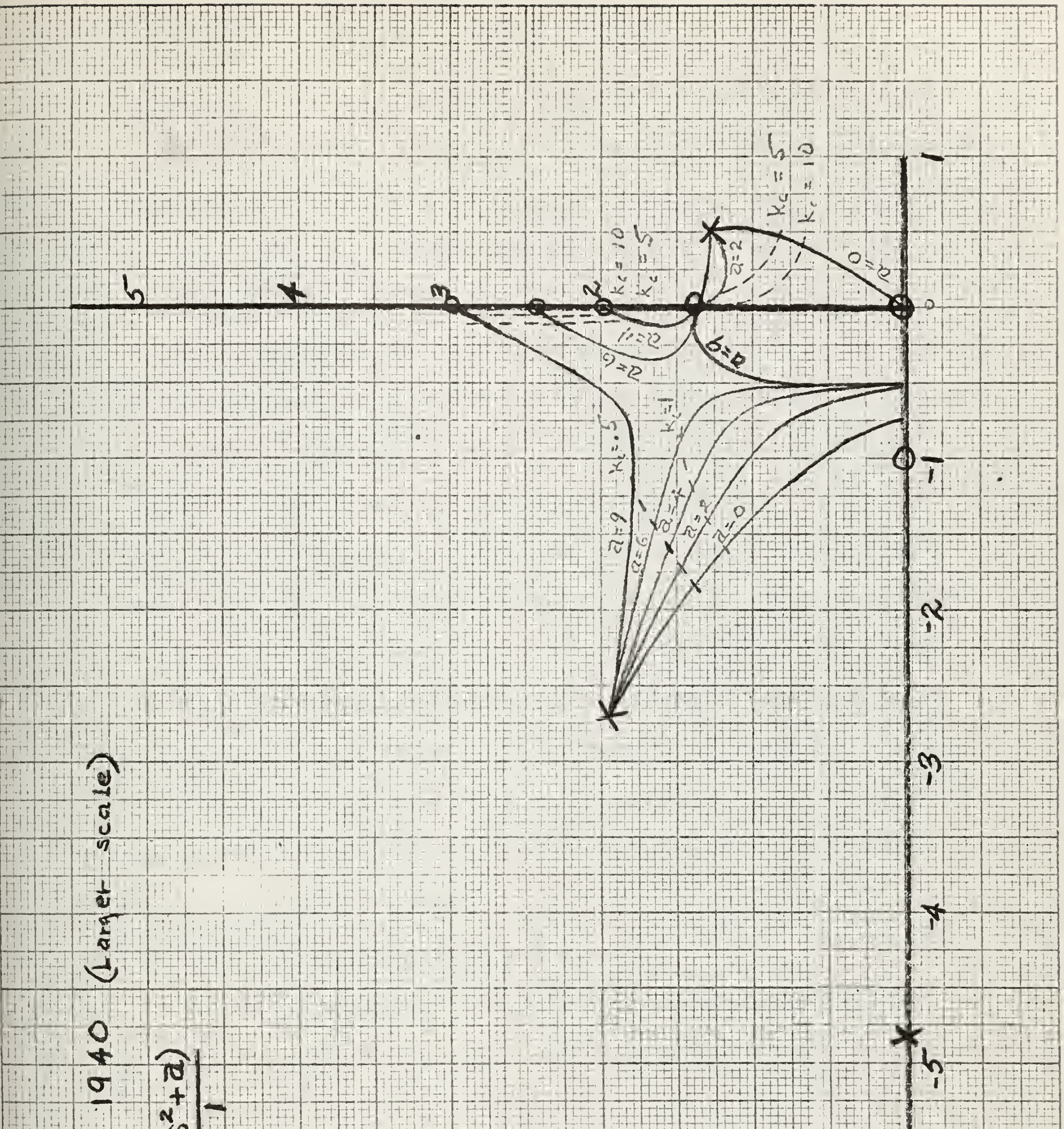


figure 8.6 (b)

particular, for this latter situation, ρ and ω_n may be varied in a fashion similar to that of the other more effective compensators; however, a wider variation in ω_n is available due to the fact that the upper limit on _a_ does not exist.

(5) "60" compensator.

Except for the fact that the root loci are more complicated, the effectiveness of the "60" compensator is very similar to that of the lag and lead networks combined. This is readily apparent in comparing the root loci of figures 8-3, 8-4 and 8-7. However, one significant difference does exist and this is the fact that stable compensation is only valid for small values of _a_. In the case of system 1960, instability occurs for values of _a_ greater than 2 and less than 0.1. Because of the close similarity in the stabilizing capacity and flexibility provided by the "60" compensator to that of the lag and lead networks, one is referred back to the discussion of either of the latter for further information with regards to the former compensator's effects.

D. Completely unsatisfactory compensators.

Two of the compensators investigated are considered to be completely unsatisfactory. This is due to the fact that they are unable to stabilize the system for any gain, k_c. These two compensators are:

(1) First derivative feedback

(2) Second derivative feedback.

The root loci showing the effect of these compensators on the basic system are shown in figure 8-5 and 8-6 respectively.

E. Normalization.

Normalization, which is desirable in order to extend these root

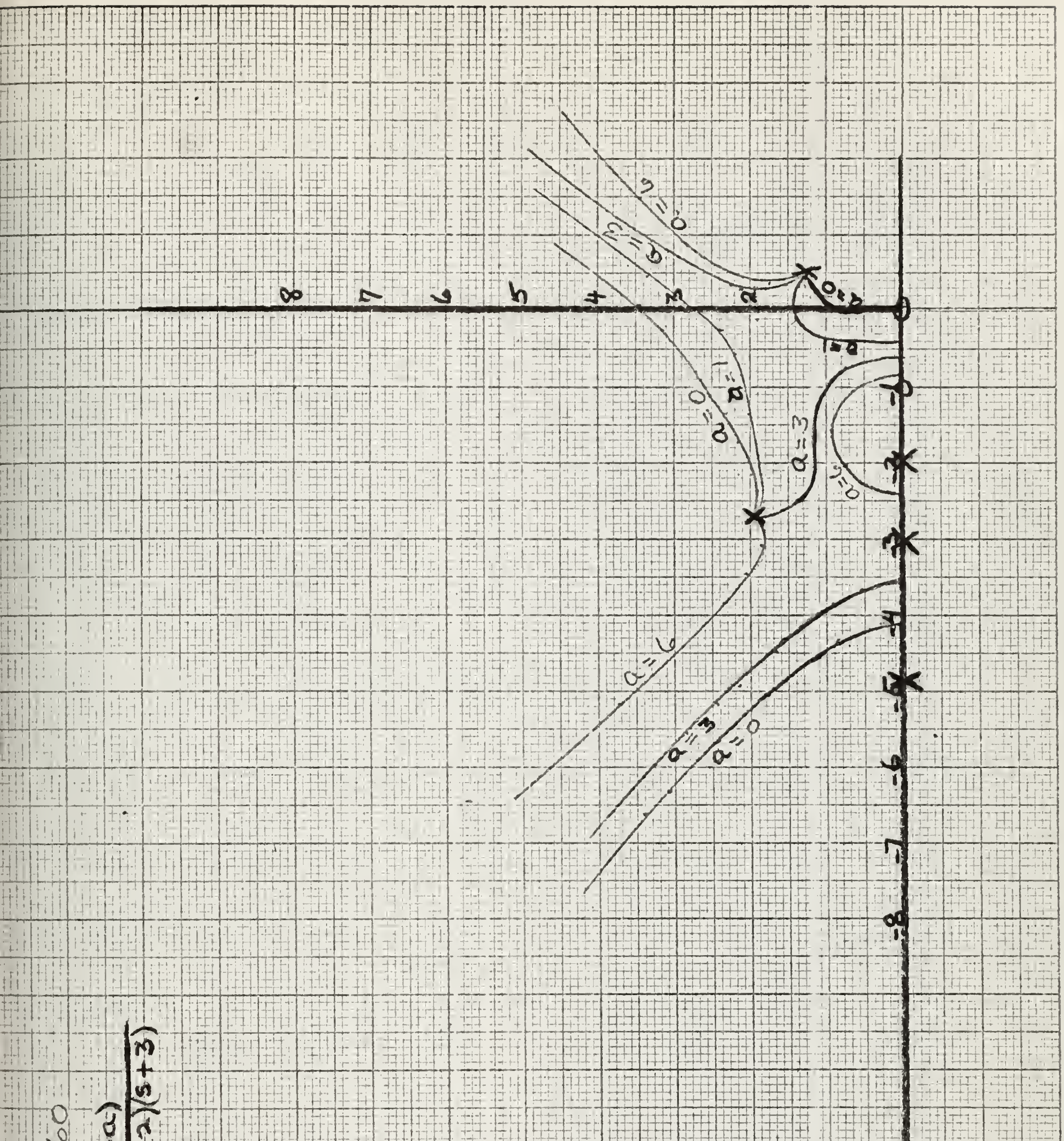
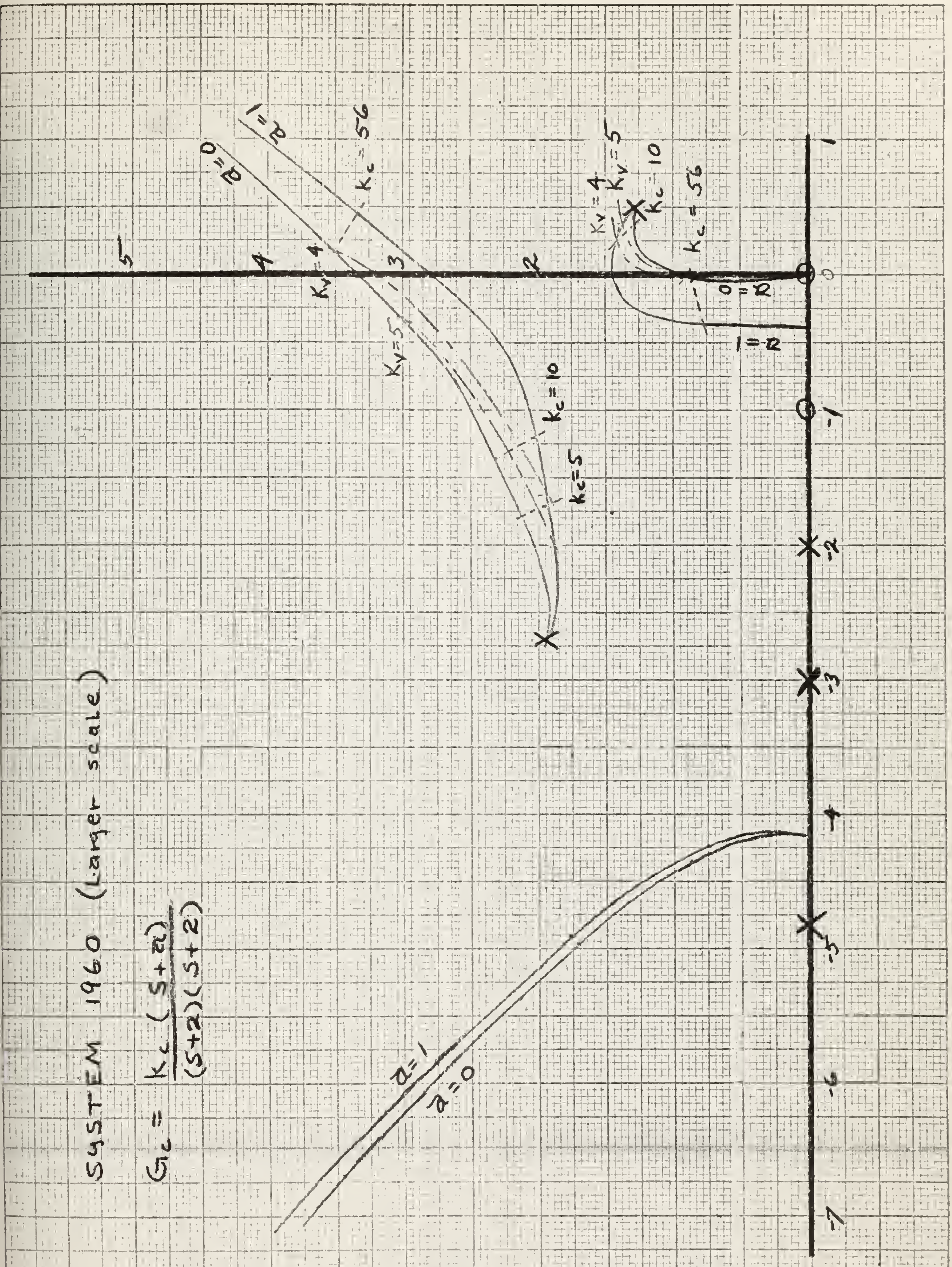


Figure 8-7 (a)



SYSTEM 1960 (Larger scale)
Gc = Kc (S+a) / (S+2)(S+2)
a=1
a=0
Kc=56
Kv=4
Kv=5
Kc=10
Kc=5
1
0
-1
-2
-3
-4
-5
-6
-7
2
3
4
5

figure 8-7 (b)

loci to other nearly similar systems has not been investigated to any great extent for this group.

TABLE 8-1

APPROXIMATE LIMITS OF STABILITY

Compensator	a	Lower limit k_{cr}	Lower limit K_v	Upper limit k_{cr}	Upper limit K_v
50	6	3.048	1.663		
50	7	2.540	1.699		
50	8	2.116	1.763		
50	9	2.000	1.700		
10	1.2	3.657	1.716		
10	2	3.048	1.327	9.100	0.509
10	3	2.7	1.0	4.380	0.685
20	3	5.266	1.855	20.000	0.58
20	6	2.540	1.905	7.584	0.780
20	7	2.116	1.944	6.320	0.800
20	8	2.0	1.84	5.266	0.835
20	9	1.764	1.849	4.389	0.884
20	10	1.55	1.80	3.657	0.945
30	2	2.116	1.763		
30	4	1.021	1.811		
30	6	0.709	1.757		
40	4	1.764	1.177		
40	6	0.90	1.45		
60	0.1	completely unstable			
60	1	18.871	2.181	32.609	1.455

. Analog Computer Checks.

A. General.

Analog computer checks were made on some of the systems using both lead or lag compensators ("10" and "20") or first and second derivative plus proportional type compensators ("30" and "40"). This was done on the following systems: 0100, 1000, 1100, 1800 and 1900. This would then cover the groups as follows:

Group I:	0100
Group IV:	1000
Group V:	1100
Group VI:	1800
Group VII:	1900

Tapes showing the servo output due to a step input are given in appendix D. It was found that these tapes confirmed completely the root loci.

B. Group I (0100) - Figures D-4,5,6.

This system was unstable for the uncompensated system. Only runs using a lead - lag type compensator were made or $G_c = k_c \frac{s+a}{s+b}$. This proved to be an effective compensator with the following general characteristics:

(1) When used as a lead network, the compensator was much more effective. The system could be made stable with a lag network up to a point; however, large values of k_c would have to be used.

(2) Increasing the values of $\underline{b}$ helped to stabilize the system.

(3) Increasing K_c improved the stability, but also decreased the gain of the system.

C. Group IV (1000) - Figures D-7, 8, 9, 10.

This system was initially stable to begin with; therefore, only a comparison for improved stability can be commented upon. In general,

the following characteristics were noted:

(1) The 2nd derivative feedback compensator made the system unstable.

(2) The system could be made stable or unstable with 1st derivative plus proportional feedback; however, the proportional factor, a, seemed to be the dominating influence. In general, it appeared to be an unsatisfactory compensator.

(3) The lag network was by far the most effective compensator used, and the greater the ratio of zero to pole, the better the response.

D. Group V (1100) - Figures D-11, 12, 13, 14, 15.

This system was initially unstable. The general characteristics of the compensated system were:

(1) The 2nd derivative feedback compensator was completely unsatisfactory.

(2) Stability could be attained with the 1st derivative plus proportional feedback compensator, but, again the value of a, the proportional component, was the dominating influence. Increasing k_c also improved the compensator.

(3) The lag network was again the better compensator. The larger the value of a and the smaller the value of b in $G_c = k_c \frac{s+a}{s+b}$, the more effective the compensator.

F. Group VI (1800) - Figures D-16, 17, 18, 19.

This system was initially unstable. The general characteristics of the compensated system were:

(1) The 2nd derivative feedback compensated system was unstable in the range of values checked.

(2) The 1st derivative plus proportional feedback was an

effective compensator, although the proportional component proved to have an important effect. Increasing the value of k_c tended to improve the stability.

(3) The lead-lag type compensator was an effective compensator. The higher the value of a and the lower the value of b, the more effective it became. Or the greater the zero-pole ratio, the better the compensator. Increasing the value of k_c tended to improve the system.

[illegible]. Group VII (1900) - Figures D-20, 21, 22, 23.

This system was initially unstable. The general characteristics of the compensated system were:

(1) The 2nd derivative feedback compensator was unsatisfactory.

(2) The 1st order plus proportional feedback was an effective compensator. However, some amount of proportional feedback was needed to stabilize the system. Increasing the value of k_c improved the stability in all cases checked.

(3) The lead-lag compensator could be either stable or unstable. In general, it was not as effective as the 1st derivative plus proportional type. Increasing the zero-pole ratio improved the stability initially, but then would make the system unstable. The higher the value of the pole, the more effective was the compensation.

[illegible]. Computer.

A block diagram of the servo system is shown in figure D-1 giving the defined quantities used in the computer diagrams. A sample computer setup is given in figure D-2 for the 1800 system with the compensator set up in figure D-3. Only slight modifications were needed for the other systems checked.

10. Conclusions.

A. Compensators which are above or below average.

It has not been the intent of this thesis to ascertain which of the compensators investigated is the best or the worst as much as it has been to comment upon the effect on the individual systems. However, it is possible from the previous discussion to pick the compensators that usually provide compensation which is more or less favorable than all the others. In particular these compensators are:

(1) Lag network - most favorable

(2) "40" compensator - least favorable.

In most of the systems investigated the lag network provided satisfactory stabilization and excellent flexibility. However, in some systems, such as the 0110, 0120, 1810 and 1820, the effectiveness of this compensator, while still good was limited to some extent in the size a may assume without instability occurring. At the same time, in none of the cases investigated did this compensator cause instability for all values of k_c and a. Thus, use of this compensator in motor input voltage feedback applications is almost certain to provide some favorable compensation if a and k_c are appropriately selected.

On the other hand, for most of the systems investigated the "40" compensator was prevalent as the most unfavorable one. Only in system 1940 (figure 8-6) did satisfactory compensation occur for a moderate magnitude of a. For all other systems, instability resulted for all, except large values of a. However, it is conceivable that stabilization could be induced if a was made large enough to predominate. One such case in which this results is shown in figure 7-7 for system 1860.

B. Normalization.

As mentioned previously in discussing the individual compensators, means by which the plotted root loci may be fitted to any feedback control system in the same group is necessary in order that full advantage may be taken of these curves. Actually only a limited amount of investigation has been conducted with regards to methods of normalization. In particular, only two groups, V and VI, included more than one system, which is necessary in order that an estimate of the effect of changing some function may be made.

Nevertheless, some degree of normalization is possible, of which the most significant is the grouping of the systems. As explained previously, the root loci of the systems can be grouped in accordance with both the number of excess poles in the G_p function and the compensator used; and the predominating section of the complex root loci will be nearly of the same shape depending on the compensator. Thus, assuming that the influence of the secondary sections of the complex and real root loci is not too significant, any other system may be similarly normalized by placing it in the correct group. Then it would be valid to assume that the predominating section of its root loci would also correspond with that of its group.

A comparison of root loci curves of the same group shows that indeed, correspondence does occur for ζ in the desired range of 0.4 to 0.7 approximately and ω_n small or moderate in magnitude, but there are differences. Depending on the situation there is considerable divergence of the root loci for small ζ . Also when ω_n is large a significant difference is observed. To a great extent the lack of correspondence when k_c is small is due to the difference in location of the uncompensated system's roots. Therefore, it is not sufficient

to assume that because a system falls into a particular group it can be expected to react accordingly: the designer must also take into account the difference in location of the uncompensated system's poles and realize that error exists when ω_n is large.

For most of the designated groups, the motor function is second order, type one with a pole located at -1 on the real axis; but it is conceivable that a similar system having a differently located motor function pole may also require investigation. Therefore, some normalization with respect to the motor function is desirable. Unfortunately the investigation has not been carried far enough to permit any general conclusions to be made in this connection.

Nevertheless, even if the motor function of the actual system is approximately the same as that of one of the investigated systems, in all probability the gains of the transfer functions would be different. Thus, normalization with respect to gain is very important. Unfortunately this normalization is difficult to accomplish. If it is assumed that k_a is the overall gain associated with the function G_aG_m, k_b is that for G_b, and k_c is the gain for G_c, then a corresponding value of k_c' may be obtained using the two conditions:

(1) $k_a\, k_b' = 100$

(2) $k_b'\, k_c' = 10k_c$

where k_c is the gain actually given on the root loci plots. Therefore:

$$k_c' = 0.1\, k_c\, k_a \tag{10-1}$$

and

$$k_b' = \frac{100}{k_a}$$

Thus, assuming that the function gains are the only difference between two systems - one actual and the other hypothetical for which the root loci plots are known - then the actual compensator gain, k_c', which is

required to obtain roots similar to those of the hypothetical system having a compensator gain, k_c, is given by equation (10-1); however, it would also be necessary to change the actual gain, k_b, to a new gain, k_b'.

C. K_v versus k_c.

Included on all the plotted root loci are contour curves showing the locus of dominating roots having constant K_v and also k_c. These contours were plotted for the purpose of lending some perspective to the root loci plots with regards to the gains and velocity lag errors to be expected.

A comparison of all the root loci plots reveals that in all cases the K_v of a system decreases as gain, k_c, increases. However, it is very rare when these two sets of contours parallel each other: thus, the relationship between them depends on the value of $\underline{a}$ and ζ.

When the values of K_v vary inversely with k_c, the utility of the compensator to the designer in meeting specifications is reduced. This is particularly true in the case where the maximum permissible steady state velocity lag error is specified. In these cases this specification may result in predominating roots having other than desirable values of ζ and ω_n.

D. Capability to interchange boxes G_a and G_m.

Soon after calculations for the root loci were begun, it became apparent that the functions, G_a and G_m, could be interchanged in the system block diagrams without effecting the loci. This is clear when one examines the equation of the loci:

$$\frac{D_a D_m N_b}{D_a D_b D_m + N_a N_b N_m} G_c = -1$$

[illegible]

Thus it is seen that the functions, or any parts thereof, could be interchanged and the equations would be identical, since each term involves the product of $D_a D_m$ or N_a or N_m. No term involves either function by itself. This fact could aid in matching some actual physical system to one of the groups involved in this investigation. For example, any pole which might be involved in the G_a function could be combined with the motor pole to make a quadratic function in the G_m box. Any motor gain could also be put into the G_a function for simplifying. Orginally, the 1600 and 1700 systems were included in these investigations to determine any possible effect of interchange. However, they were dropped when it became apparent that the two systems had identical loci.

APPENDIX A - A Digital Computer Program For Plotting Root Loci.

A. Introduction.

This appendix is for the purpose of explaining the digital computer program used in computing the plotted root loci. This program was used with the CONTROL DATA CORPORATION 1604 Digital Computer. The program is written using the fortran system.

B. Objective.

The objectives of this computer program are the following:

(1) Using the input information, calculate the basic characteristic equation for the feedback system being investigated.

(2) By actually varying the appropriate coefficients (by varying the gain) of the characteristic equation, calculate the individual points which together constitute the root locus of the specified feedback system.

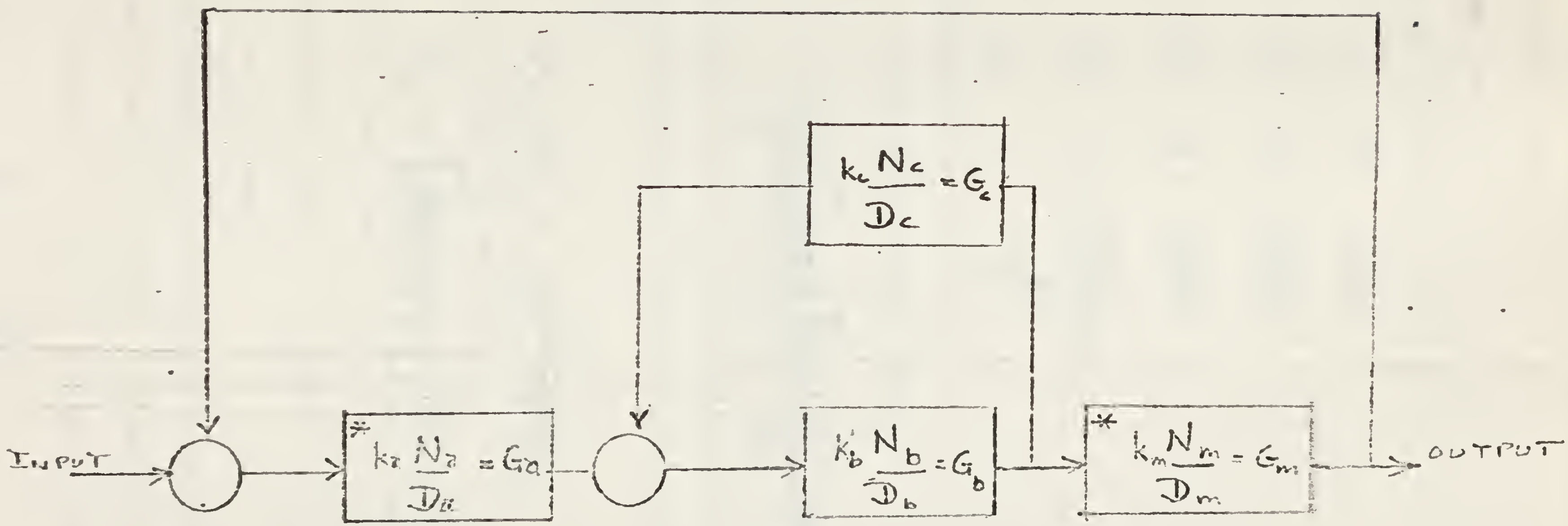
(3) Also, for each point of the root locus, calculate in accordance with Appendix B the corresponding value of K_v, which determines the steady state velocity lag error for all type one feedback systems.

(4) Record the above information in such a form that it facilitates manual plotting of the root locus points.

C. Systems for which applicable.

This program causes the digital computer to calculate the root locus for any system which can be arranged into the form of the system specified below. This system may be any feedback system whose characteristic equation is of degree less than 101. The variable can either be the gain of the open loop functions or the gain in one of the feedback functions. The block diagram of this system is shown in figure 1.

The form of the functions in the boxes labeled G_b, G_c, and G_m can



* Note: The functions in these blocks could be combined into one block without affecting the output signal

Figure 1

be in any one of a couple of forms. The preferred form would be to have these functions broken down to their simplest factors. Thus, if for example, G_b consisted of four subfunctions as follows:

$$G_{b_1} = \frac{1}{(S+a)}$$

$$G_{b_2} = \frac{(S+b)}{(S+C)}$$

$$G_{b_3} = \frac{(S+d)}{1}$$

$$G_{b_4} = \frac{(S+e)}{(S^2+fS+g)}$$

$$G_{b_5} = \frac{K}{1}$$

then the simplest form would be:

$$G_b = (G_{b_1})(G_{b_2})(G_{b_3})(G_{b_4})(G_{b_5})$$

$$= \left[\frac{1}{(S+a)}\right]\left[\frac{S+b}{(S+c)}\right]\left[\frac{S+d}{1}\right]\left[\frac{S+e}{S^2+fS+g}\right]\left[\frac{K}{1}\right]$$

However, another form which is permissible but not nearly as simple consists of the expanded products of any or all of the G_{b_1} function: that is if

$$G_{b_1} G_{b_2} = \frac{(S+b)}{(S^2+hS+j)}$$

then a permissible form would be

$$G_b = \left[\frac{S+b}{S^2+hS+j}\right]\left[\frac{S+d}{1}\right]\left[\frac{S+e}{S^2+fS+g}\right]\left[\frac{K}{1}\right]$$

and so forth. The purpose of this is to eliminate the tedious operation of manually multiplying the polynomial functions which the computer will do rapidly and accurately.

Nevertheless, if it is desired to use this program for a system which does not contain an inner feedback loop function then it is only necessary to assume a G_c function which is equal to zero - that is, assume

$$G_c = \frac{N_c}{D_c} = \frac{0}{1}$$

D. Program Operation.

The program consists of three major phases of operation - (1) input phase, (2) computation of the basic characteristic equation, and, (3) the computation of the root loci points. These three phases are explored below in more detail. The order in which they are listed is the same order in which they are accomplished. Each phase is completed before the next is commenced.

(1) Input phase.

During the input phase of operation the computer acquires all the input information required for the proper operation of the program including the system's parameters. This input information can be "read" into the computer either through the media of punched cards or magnetic tape. However, in either case, cards must be punched and if the magnetic tape input is desired, the cards must first be transferred to it. The specific information which must be supplied as the input to the computer will be covered in more detail later in part E of this appendix.

(2) Computation of the basic characteristic equation.

Upon the completion of "reading" in all the available input information, the computer commences the next phase of operation. This consists primarily of calculating the coefficients of the basic characteristic equation (where the variable, gain, is 1) using the system parameters obtained during the previous phase. Depending on the intended location of the variable, gain, there are three different methods for making this calculation, and each one gives a different basic characteristic equation. The desired method can be selected by use of the computer's selective jump switches as follows:

(a) All jump switches down - the characteristic equation's

coefficients are calculated assuming that the gain of either the G_a or G_m transfer box is variable.

(b) Jump switch number 1 up only - the characteristic equation's coefficients are calculated assuming that the gain in the G_c transfer box is variable.

(c) Jump switch number 3 up only - the coefficients of the characteristic equation are calculated assuming that the gain of the G_b transfer box is variable.

After calculating these coefficients of the basic characteristic equation the computer is ready to commence the third phase.

(3) Calculation of the points of the root locus.

The final phase of computer operation consists of computation of individual points which make up a root locus. This is done by a reiterative process consisting of three steps. In the first step those coefficients of the characteristic equation which are a function of the variable, gain, are increased by a factor which reflects the new gain. The second step is to solve for the roots of this new characteristic equation. Then for the third step these roots are printed on the chosen output media. Upon completion of this latter step, the computer increases the gain and corresponding gain dependent coefficients of the characteristic equation, and again reiterates the same three step process.

The gain increase provided in the third step of the last phase is exponential, and is determined by the program user through the initial inputs supplied to the computer in the input phase. In starting out with the characteristic equation of the second phase, the initial gain would be 1; but for the third phase this is multiplied by two factors - both specified in the input phase (for the case where

the gain in the [illegible] feedback loop in the [illegible], the initial [illegible] is set to [illegible] the [illegible] value [illegible] to the [illegible] value [illegible] the other two cases). [illegible] factor, SCALFAC, changes each value of gain by a constant factor. The second factor, BASE, is that which causes the gain to increase. The initial value of gain is:

$$K_1 = 1 \times \text{SCALFAC} \times \text{BASE}$$

Then each successive value of gain is obtained by multiplying the previous value by BASE again. Thus:

$$K_{(i+1)} = K_i \times \text{BASE} = K_{(i-1)} \times \text{BASE}^2$$
$$= 1 \times \text{SCALFAC} \times \text{BASE}^{(i+1)}$$

where $\underline{i}$ is the number of root locus points which have already been computed.

Upon solving for the roots of each new characteristic polynomial in these terms, the computer then records them in a manner explained in part [illegible] of this appendix. The media on which the computer records this information is to be determined by the program user. The program itself just calls for the computer to "print" in accordance with a prescribed format; and the media, such as magnetic tape, off-line printer, typewriter and so forth, must be specified to the computer prior to commencing the run.

In addition to specifying the output media, it is also necessary to specify, if other than a card reader, the input media. Upon starting the computer all of this input information is immediately assimilated by the computer.

The approximate running time of this computer program depends on several factors. The rate at which the characteristic polynomials are solved for their roots depends upon their degree. Also the number of iterations [illegible] influences greatly the running time. For a practical

example problem which consisted of an eighth order characteristic equation and approximately [illegible] points required an overall time of about four minutes. Completion of a program is indicated when the typewriter types [illegible].

[illegible]. Required Input Information.

In order that the root loci points may be calculated, it is necessary to supply the computer with certain input information. This program is designed to accept this information from punched cards; however, if desired, the information on the cards may be transferred to magnetic tape for subsequent input to the computer. The number of cards required and the information punched on each depends primarily on both the number of factors in each basic function box of the systems block diagram and the degree of each of these individual polynomial factors. Essentially, there is only one instruction or coefficient number to a card, thus permitting the deck to easily be changed as desired.

The input data is either of two basic types - computational parameters or system parameters. The computational parameters are input to the computer first. They have a threefold purpose:

1. To specify the number of root loci for which points are to be calculated.

2. To label each root locus.

3. To determine the extent and density of the root loci points.

A detailed discussion of each of the input computational parameters is contained below. The order in which they are discussed corresponds to that in which they are provided as input to the computer.

(1) TAG.

TAG is a floating point number in the range, 0 to 99999.99 (fortran format is F8.2), and it is for the purpose of providing for the labeling of the individual root loci. It is specifically designed for the case where more than one root locus is desired for a basic system, with each representing an increment in a constant coefficient of one of the system's transfer functions. Actually TAG is the label applied to the first root locus. Then for each successive root locus the label is increased by the same amount as the incremented coefficient. Thus it is advantageous to have the final digits of TAG reflect the initial value of this coefficient, and then its successive values will also be indicated in the corresponding labels. As an example of the use of TAG, consider the case where the initial value of the coefficient to be incremented is 7.5. Then if TAG is designated to be 1137.5 and the increment in the coefficient is 3.0, the computer will label the second root locus 1140.5, the third 1143.5 and so on.

(2) ANC, TOP.

The next two computational parameters, ANC and TOP will be discussed together in view of their close relationship and similar format. ANC precedes TOP and as mentioned in (1) above, prescribes the increase that is to be applied to the constant coefficient being changed for each root locus. TOP prescribes the maximum value to which this coefficient may be increased. Therefore, indirectly TOP sets forth the number of root loci which will be calculated. The selected coefficient will continue to be increased along with the root locus label and consequently, corresponding root loci will continue to be calculated until the coefficient's magnitude is greater than TOP (when this occurs, program operation terminates). Both ANC

and [illegible] fixed point number less than or equal to [illegible]. [illegible] format specification for them is F 7.2.

(3) [illegible], [illegible].

[illegible] and [illegible] specify the constant coefficient which is to be changed for each root locus computed. [illegible], which precedes [illegible] as an input, designates the specific transfer function by number in which this coefficient is located. In view of the fact that there are only four basic transfer functions, G_c, G_a, G_b and G_m, and each of these are divided into two polynomials, the numerator and denominator, [illegible] can be any fixed point number from 1 to 8. As an example, if [illegible] was specified as 3 the coefficient to be changed would be found in N_a while 4 would mean it was in D_a.

[illegible] specifies which term of this particular polynomial contains the coefficient to be changed. To be exact, [illegible] is the degree of the term and therefore can be any fixed point number from 0 to [illegible]. As an example, if [illegible] was specified to be 3 while [illegible] was [illegible], then the coefficient of the cubic term of [illegible] is the one that is changed for each root locus by the value of [illegible]. The fortran format for [illegible] is the same as that for [illegible], I 2.

(4) [illegible].

It is the purpose of [illegible] to obtain values of the variable, gain, which are less than 1 in magnitude. This computer program has built into it the value 1 as the initial and consequently, the minimum magnitude of the gain input, in many cases the section of the root locus which is of most interest occurs when the gain is less than unity. Thus, the solution to this dilemma is to scale down all values of gain by multiplying each by a constant factor which is less than unity, and this factor is called [illegible]. It may be any

floating point number less than $9 \times 10^{[illegible]}$. The fortran format is F 7.1. Accordingly, if it is desired that COMFAC be designated as 1×10^{-2}, then the computer card would read: .1E - 02 where E - 02 means the exponent or characteristic is - 02, and .1 is the mantissa.

(5) GAIN.

The input computational parameter, GAIN, provides the maximum limit for the variation in the gain of each root loci. When the magnitude of this varied gain exceeds that of GAIN, computation ceases for that particular root locus on the next increment in gain and commences for the next root locus. GAIN can be any floating point number less than or equal to $9 \times 10^{[illegible]}$. Its fortran format is identical to that of COMFAC, F 7.1.

(6) BASE.

The computational parameter, BASE, controls the number of points that are computed for each root locus by specifying the factor by which the gain is varied. As previously mentioned this variation can be represented as follows:

$$K_{i+1} = (COMFAC)(BASE)^{(i+1)}$$

Essentially, this expression indicates that each successive value of gain is obtained by multiplying the previous value by BASE. Thus, if BASE was 2.0, this would amount to doubling the last gain, whereas for BASE less than 1.0 a decrease in the last gain would occur. Actually a value of 1.20 for BASE has been found to be quite satisfactory, but the actual value used depends on the location of the variable gain. BASE can be any number less than or equal to 9999.99 and has a fortran format of F 7.2.

The second basic type of information - the system parameters -

constitute the final input to the computer. These are the actual coefficients found in the transfer function boxes of the system's block diagram. However, in order for the computer to comprehend the significance of each of these coefficients, it is necessary not only to group them but also to include with them some supplementary, amplifying information.

Essentially, the input system parameters are divided into eight groups. The basic grouping is provided by the four transfer function boxes of the general system's block diagram as shown in figure 1. Each of these boxes consists of two polynomials - one being the numerator of the transfer function and the other being the denominator. Therefore, if each of these polynomials is considered to be a group, there would be a total of eight instead of just four. This is actually what occurs. However, the polynomials of each transfer function are "read" in consecutively with the numerator first. The polynomials of the G_c transfer function are "read" in first with those of G_a, G_b and G_f following in that order. Thus, numbering each polynomial individually in the order in which it is "read" in, the input system coefficients fall into the following grouping:

a. polynomial 1 - N_c (the numerator of G_c)

b. polynomial 2 - D_c (the denominator of G_c)

c. polynomial 3 - N_a (the numerator of G_a)

d. polynomial 4 - D_a (the denominator of G_a)

e. polynomial 5 - N_b (the numerator of G_b)

f. polynomial 6 - D_b (the denominator of G_b)

g. polynomial 7 - N_f (the numerator of G_f)

h. polynomial 8 - D_f (the denominator of G_f)

In addition, because these polynomials may be in the factored form, it is also necessary to give the computer amplifying information about them. First of all it is necessary to state how many factors there are in the polynomial. Then, for each factor, it is also necessary to tell the computer its degree. The manner in which this supplementary information is to be included with the coefficients of the group can readily be shown by the following example. Consider the polynomial to be $(s^2)(s+6)$. This polynomial consists of two factors (each is enclosed in parenthesis) of varying degree. The input cards would read as follows:

2 ------ indicates the number of factors

2 ------ degree of the first factor

0.0 ------ coefficient of zero power term - first factor

0.0 ------ coefficient of first power term - first factor

1.0 ------ coefficient of second power term - first factor

1 ------ degree of the second factor

1.0 ------ coefficient of zero power term - second factor

6.0 ------ coefficient of first power term - second factor

where 2 would be the first and 6.0 would be the last cards to be "read" in for this polynomial. Immediately following the 6.0 for this polynomial would be the card telling the number of factors present in the next polynomial, and so on. One significant point to be noted about the supplementary information is the fact that it consists entirely of fixed point numbers having a fortran format designation of I 2. This means that these numbers can be any integer from 0 to 99.

The input coefficients are also shown in the above example. These can be any floating point number from 0.00 to 9999.99. The fortran format designation for these numbers is F 7.2. For every factor, the

coefficients are listed and that the lowest power term, the constant, comes first, the highest powered term last, and those in between in order of their increasing powers. It is very important that a separate card be present for each term. Thus for a fourth degree polynomial there would be five coefficient cards. In the case where a coefficient is zero, it is sufficient to represent this coefficient with a blank card.

Figure 2 shows the list of cards as they would be input to the computer for the system shown in figure 3. Those above the dotted line are computational parameters. Those below this line are system parameters. Also included in figure 2 is the detailed break down of the input cards by polynomial and basic functions for the system parameters. The column numbers at the top were included to indicate in which column of the data card each digit is to be punched.

F. Output Information.

There is an extensive amount of output information which must be made available to the user of this program. Primarily, as mentioned previously, the points of each root locus, the corresponding values of gain, and the label for each root locus make up the output information. However, there are two other groups of output information which, due to their potential usefulness to the program user, are also included as outputs. The first of these is the listing of the coefficients of the eight polynomial functions. These listed coefficients are those which result after the computer multiplies together all of the factors of the input polynomial; and thus, while not providing an exact check they do show the exact system for which the root loci are computed. The second group of information consists of statements, where appropriate in the output, which caution the

INPUT DATA

12345678 - Column number

1820.00	Computational parameters			
3.00				
3.00				
1				
0				
+.1E-02				
+.5E+02				
1.20				
1		N_c	G_c	System Parameters
0.00				
1.00				
1		D_c		
1				
4.00				
1.00				
1		N_a	G_a	
0				
10.00				
1		D_a		
0				
1.00				
1		N_b	G_b	
0				
10.00				
2		D_b		
1				
4.00				
1.00				
1				
3.00				
1.00				
1		N_m	G_m	
0				
1.00				
2		D_m		
1				
0.00				
1.00				
1				
1.00				
1.00				

Figure 2

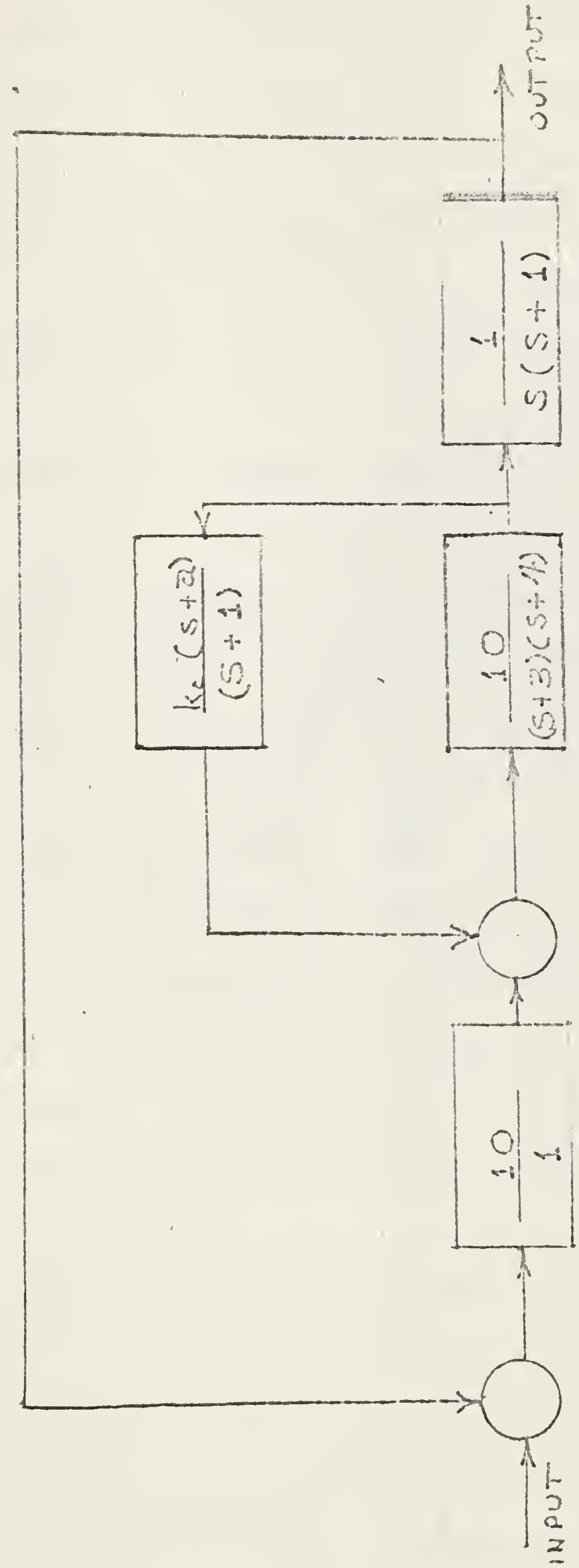


Figure 3

program user of the fact that a remainder greater in magnitude than 10^{-1} exists when the given roots are divided back into the characteristic polynomial; and therefore, indicate that these points can not be expected to fall exactly on the true root locus. The magnitude of this remainder is also listed.

All of the output information is recorded in accordance with a specific format as shown in figures 4 and 5. The former figure shows how the first page for the first root locus of a system would appear; the latter figure shows the termination of the first root locus and the commencement of the second for the system. Preceding the label of the first root locus, listed in the same order as they were "read" into the computer are the polynomial coefficients. After the root locus label, which is the statement "SYSTEM NUMBER 1820.00 FOLLOWS", are the groups of root locus points. In the row preceding each of these groups are listed two items. The first is the gain and is labeled KA, KB, or KC depending on whether k_a, k_b or k_c respectively is the variable. The second item is labeled KV and when divided into the input ramp signal, gives the steady state lag error for the type one servo system. Then listed three to the row are the actual points. The number following "s" is the real part and that following "j" is the imaginary part.

There are several significant comments which should be made regarding the listing of the points of a root locus in each group. First of all each point consists of two parts, a complex and an imaginary part. In some cases one of these parts may be zero, but still they will be listed. For each point which does not actually lie on the real axis, a conjugate complex point will also be listed in the same group; however, these two points will not be listed,

```
POLYNOMIAL 1
 .0000E+00 .1000E+01 .1000E+01

POLYNOMIAL 2
 .4000E+01 .5000E+01 .1000E+01

POLYNOMIAL 3
 .1000E+02

POLYNOMIAL 4
 .1000E+01

POLYNOMIAL 5
 .1000E+02

POLYNOMIAL 6
 .1200E+02 .7000E+01 .1000E+01

POLYNOMIAL 7
 .1000E+01

POLYNOMIAL 8
 .0000E+00 .1000E+01 .1000E+01


SYSTEM NUMBER  1820.00 FOLLOWS
KC              .000                 KV               8.333
S        -1.00   J       .00         S        .52  J    -1.96        S        .52  J      1.96
S        -4.52   J      1.96         S      -4.00  J      .00        S      -4.52  J     -1.96

KC              .001                 KV               8.333
S        -1.00   J       .00         S        .52  J    -1.96        S        .52  J      1.96
S        -4.52   J     -1.96         S      -4.00  J     -.00        S      -4.52  J      1.96

KC              .001                 KV               8.333
S        -1.00   J       .00         S        .52  J    -1.96        S        .52  J      1.96
S        -4.52   J      1.97         S      -3.99  J      .00        S      -4.52  J     -1.97

KC              .001                 KV               8.333
S        -1.00   J       .00         S        .52  J    -1.96        S        .52  J      1.96
S        -4.52   J      1.97         S      -3.99  J      .00        S      -4.52  J     -1.97

KC              .002                 KV               8.333
S        -1.00   J       .00         S        .52  J    -1.96        S        .52  J      1.96
S        -4.52   J      1.97         S      -3.99  J      .00        S      -4.52  J     -1.97
```

Figure 4

```
KC         27.174                      KV          8.333
S       -1.00   J      .00        S          .16   J    -.93        S           .16   J      .93
S       -1.45   J      .00        S        -5.43   J   16.75        S         -5.43   J   -16.75

KC         32.609                      KV          8.333
S       -1.00   J      .00        S          .14   J    -.87        S           .14   J      .87
S       -1.41   J      .00        S        -5.44   J  -18.30        S         -5.44   J    18.30

KC         39.130                      KV          8.333
S       -1.00   J      .00        S          .13   J    -.81        S           .13   J      .81
S       -1.37   J      .00        S        -5.45   J  -20.01        S         -5.45   J    20.01

KC         46.956                      KV          8.333
S       -1.00   J      .00        S          .12   J    -.76        S           .12   J      .76
S       -1.33   J      .00        S        -5.46   J  -21.88        S         -5.46   J    21.88

BELOW ROOT UNCERTAIN WITH REAL REMAINDER OF  .1E-01
BELOW ROOT UNCERTAIN WITH REAL REMAINDER OF -.2E-01
BELOW ROOT UNCERTAIN WITH IMAG REMAINDER OF  .2E-01
KC         56.348                      KV          8.333
S       -1.00   J      .00        S          .11   J    -.71        S           .11   J      .71
S       -1.29   J      .00        S        -5.46   J   23.93        S         -5.46   J   -23.93


SYSTEM NUMBER  1823.00 FOLLOWS
KC           .000                      KV          8.333
S       -1.00   J      .00        S          .52   J   -1.96        S           .52   J     1.96
S       -4.52   J     1.96        S        -4.00   J     .00        S         -4.52   J    -1.96

KC           .001                      KV          8.328
S       -1.00   J      .00        S          .52   J   -1.96        S           .52   J     1.96
S       -4.52   J    -1.96        S        -3.99   J    -.00        S         -4.52   J     1.96

KC           .001                      KV          8.327
S       -1.00   J      .00        S          .52   J   -1.96        S           .52   J     1.96
S       -4.52   J    -1.97        S        -3.99   J    -.00        S         -4.52   J     1.97

KC           .001                      KV          8.326
S       -1.00   J      .00        S          .52   J   -1.96        S           .52   J     1.96
S       -4.52   J    -1.97        S        -3.99   J     .00        S         -4.52   J     1.97
```

necessarily, adjacent to each other. Actually, there is no set order by which the points are listed in a group. If the points of a particular section of the root locus is listed first in the initial groups, there is no reason to expect this order of listing to continue for the latter groups. Points are listed in each group in the order in which they are obtained as roots of the characteristic polynomial.

One final comment regarding the listing of the points concerns the fact that the program does not cancel out any factors which appear in both the numerator and denominator of the system function, F_c. Therefore, this root is present as a factor of each characteristic polynomial, and appears as a point in each group of points for that particular root locus. But because the same point is repeated in each group, it is easily detected, and therefore it may readily be ignored. An example of this is the point 1.00 in figure 1.

G. Computer Program Instruction.

The fortran instructions which constitute the program for computing the root loci are included in this appendix. These instructions are listed in the proper sequence on pages A-20 through A-28.

```
    1 PROGRAM RTLOCUS
      DIMENSION A(20 0), B(100), C( 100),
     1D(100), POLY(8,100), CONDC(100),
     2DAM(100), VARDC(100), UABMDC(100),
     3ROOTS(200), RNAME(100).
      COMMON CRR, CRI, CPR, CPI, ROOTR, ROOTI, C, D
 6390 FORMAT(F8.2)
      READ 6390, TAG
      READ 801, ANC,  TOP
      READ 7, LEVER, NORDER
 6391 FORMAT(E7.1)
      READ 6391, SCA LFAC, GAIN
      READ 801, BASE
    4 DO 243 INDEC = 1, 8
    5 DO 6 K = 1, 100
    6 POLY(INDEC, K) = 0
    7 FORMAT(I2)
    8 READ 7, NUMBER , N
  801 FORMAT(F7.2)
      NIC = N + 1
      READ 801,(POLY(INDEC,K), K=1,NIC)
    9 IF(NUMBER - 1) 9191, 9191, 10
 9191 NUB = NIC
      GO TO 24
   10 CALL EQUAT2(A, POLY,INDEC)
   11 DO 12 K = 1, 100
   12 POLY(INDEC,K) = 0
   13 READ 7, M
      MIC = M + 1
  130 READ 801, (POLY(INDEC,K), K=1, MIC)
   14 ITH = 2
   15 CALL EQUAT2 (B, POLY,INDEC)
   16 CALL MULTPL (A, B, A)
   17 IF(NUMBER - ITH) 23, 23, 18
   18 DO 19 K = 1,100
   19 POLY(INDEC, K) = 0
   20 READ 7, N
      NIC = N + 1
  200 READ 801, (POLY(INDEC, K), K = 1, NIC)
   21 ITH = ITH +1
   22 GO TO 15
   23 NUB = 100
 2222 IF(A(NUB)) 2224, 2223,2224
 2223 NUB = NUB -1
      GO TO 2222
 2224 CONTINUE
      CALL EQUAT3(PO LY, INDEC, A)
 2225 FORMAT(/11HPOLYNOMIAL I1)
  241 FORMAT(8E10.4)
   24 PRINT 2225, INDEC
  243 PRINT 241, (POLY(INDEC, K),K =1,NUB)
 2401 GO TO 25
 2402 TAG = TAG + ANC
   25 IEXP = -1
 1043 FORMAT(/// 14HSYSTEM NUMBER F8.2, 8H FOLLOWS)
      PRINT 1043, TAG
   29 CALL EQUAT2 (A, POLY,3)
   30 CALL EQUAT2 (B, POLY,5)
   31 CALL MULTPL (A, B, A)
   32 CALL EQUAT2 (B, POLY,7)
   33 CALL MULTPL (A, B, A)
   34 CALL EQUAT2 (B, POLY,2)
   35 CALL MULTPL (A, B, A)
   36 CALL EQUAT1 (UABMDC, A)
   37 CALL EQUAT2 (A, POLY,4)
   38 CALL EQUAT2 (B, POLY,8)
   39 CALL MULTPL (A, B, A)
   40 CALL EQUAT1 (DAM, A)
   41 CALL EQUAT2 (B, POLY,6)
   42 CALL MULTPL (DAM, B, A)
   43 CALL EQUAT2 (B, POLY,2)
      UABMDC2 = UABMDC(2)
   44 CALL MULTPL (A, B, A)
```

```
      CALL EQUAT1(RN AME, A)
      CALL EQUAT2(B,  POLY, 1)
      CALL MULTPL(DA M, B, A)
   48 CALL EQUAT2 (B, POLY,5)
   49 CALL MULTPL (A, B, A)
      CALL EQUAT1(DA M, A)
      IF(SENSE SWITC H 1)      7161,  7262
 7262 IF(SENSE SWITC H 3) 7363, 746 4
 7161 CALL ADD(UABMDC, RNAME, CONDC)
      CALL EQUAL1(VA RDC, DAM)
 5101 CALL EQUAL1(UABMDC, CONDC)
      CALL BRNULI(UABMDC, ROOTS, N)
      VKV = CONDC(1)/(CONDC(2) - UABMDC2)
      AKA = 0.0
      CALL PRINT(ROOTS, N, AKA, VKV)
      GO TO 4701
 7464 CALL ADD(DAM, RNAME, CONDC)
      CALL EQUAL1(VA RDC, UABMDC)
      GO TO 4701
 7363 CONTINUE
      CALL ADD(UABMD C, DAM, VARDC)
      CALL EQUAT1 (C ONDC, RNAME)
 4701 DO 471 I =  1, 100
  471 VARDC(I) = VARDC(I) * SCALFAC
   52 CALLEQUAL1(C, VARDC)
   53 DO 54 I = 1, 100
   54 VARDC(I) = VARDC(I) * BASE
   55 IEXP = IEXP + 1
   56 CALL ADD(CONDC, C, UABMDC)
      DC1 = UABMDC(1)
      DC2 = UABMDC(2)
   61 CALL BRNULI(UABMDC, ROOTS, N)
      VKV = DC1/(DC2 - UABMDC2)
      AKA =(BASE ** IEXP) * SCALFAC
      CALL PRINT(ROOTS, N, AKA, VKV)
      IF(AKA - GAIN) 62, 63, 63
   62 GO TO 52
   63 IF(ANC) 6301,  6309, 6301
 6301 INORDER = NORDER + 1
       POLY(LEVER, INORDER) = POLY(LEVER, INORDER) + ANC
       IF(ABSF(POLY(LEVER,INORDER))-TOP)2402,2402,6309
 6309 STOP
      END
      SUBROUTINE MULTPL (A, B, E)
      DIMENSION A(100), B(100), C(100), D(100), E(100)
      COMMON CRR, CRI, CPR, CPI, ROOTR, ROOTI, C, D
      N = 100
  651 IF(A(N)) 654, 652, 654
  652 N=N - 1
  653 GO TO 651
  654 M= 100
  655 IF(B(M)) 658, 656, 658
  656 M = M - 1
  657 GO TO 655
  658 NTOTAL  = M + N
  659 IDELTA = 0
  660 DO 661 I = 1, 100
      D(I) = 0
  661 C(I) = 0
      DO 663 I=1, N
      DO 662 J = 1, M
      IP = J + IDELTA
  662 D(IP) = A(I) * B(J)
      CALL ADD(C, D, C)
  663 IDELTA = IDELTA + 1
  664 CALL EQUAT1(E, C)
      RETURN
      END
      SUBROUTINE ADD(A, B, E)
      DIMENSION A(100), B(100), E(100)
      DO 672 I = 1, 100
      E(I) = A(I) + B(I)
  672 B(I) = 0
```

```
      RETURN
      END
      SUBROUTINE EQUAT2 (A, POLY, INDEC)
      DIMENSION A(100), POLY(8, 100)
      DO 690  I = 1,100
  690 A(I) = POLY(INDEC, I)
      RETURN
      END
      SUBROUTINE EQUAT3 (POLY, INDEC, A)
      DIMENSION A(100), POLY(8, 100)
      DO 700 I = 1, 100
      POLY(INDEC, I) = A(I)
  700 A(I) = 0
      RETURN
      END
      SUBROUTINE PRINT(RESULT,N, GAIN, VKV)
      DIMENSION RESULT(200)
      IF(SENSE SWITCH 1) 5053, 5054
 1050 FORMAT(3HKC F15.3,20X,3HKV F15.3)
 5051 FORMAT(3HKA F15.3,20X,3HKV F15.3)
 5052 FORMAT(3HKB F15.3,20X,3HKV F15.3)
 5054 IF(SENSE SWITCH 3 )5055,5056
 5055 PRINT 5052,GAIN, VKV
      GO TO 5057
 5056 PRINT 5051, GAIN, VKV
      GO TO 5057
 5053 PRINT 1050,GAIN, VKV
 5057 ORDER = N
      COUNT = ORDER/3.0
      IF(COUNT - 1.0) 1149, 1150, 1051
 1149 MORE = N
      KOUNT = 0
      GO TO 1061
 1150 KOUNT = 6
      GO TO 1056
 1051 OUNT = 34.0
 1052 IF(OUNT - COUNT) 1055, 1055, 1054
 1054 OUNT = OUNT - 1.0
      GO TO 1052
 1055 KOUNT = OUNT
      KOUNT = KOUNT * 6
 1056 DO 1058 J=1, KOUNT, 6
      J5 = J+5
 1057 FORMAT(3HS  F10.2, 3H  JF10.2, 7X, 3HS  F10.2, 3H  JF10.2, 7X,
     13HS  F10.2, 3H  JF10.2 )
 1058 PRINT 1057, (RESULT(JN), JN =J,J5)
      MORE = N-(KOUNT/2)
 1061 IF(MORE - 1) 1062, 1063, 1065
 1062 FORMAT(/)
      PRINT 1062
      RETURN
 1063 KT1 = KOUNT + 1
      KT2 = KOUNT + 2
 1064 FORMAT(3HS  F10.2, 3H  JF10.2)
      PRINT 1064, RESULT(KT1),RESULT(KT2)
      GO TO 1062
 1065 KT1 = KOUNT + 1
      KT4 = KOUNT + 4
 1066 FORMAT(3HS  F10.2, 3H  JF10.2, 7X, 3HS  F10.2, 3H  JF10.2)
      PRINT 1066, (RESULT(KT), KT=KT1, KT4)
      GO TO 1062
      END
      SUBROUTINE BRNULI(A, ANSWER, NEL)
       DIMENSION CRR(129),CRI(129),XR(4),XI(4),FXR(4),FXI(4),SR(3),SI(3)
       DIMENSION KAPPA(10),CPR(129),CPI(129),ROOTR(128),ROOTI(128)
      DIMENSION ANSWER(200), A(100)
      COMMON CRR, CRI, CPR, CPI, ROOTR, ROOTI, C, D
      INT = 1
      CALL CONVRT(A)
      NEL = 100
 9996 IF(A(NEL)) 9998, 9997, 9998
 9997 NEL = NEL - 1
      GO TO 9996
```

```
9998 NEL = NEL - 1
     N = NEL
     DO 9995 I = 1,100
9995 CRR(I) = A(I)
  65 CONTINUE
   1 CONTINUE
   5 CONTINUE
   2 CONTINUE
  50 CONTINUE
  66 CONTINUE
     IMAX = 25
     NUM = 3
     DEL = 0.2
     RATIO = 5.0
     ALTER = 1.000001
     MODE = 1
     EP1 = 0.0000000000000000000001
     EP2 = 0.00000001
     EP3 =           0.0000001
     EP4 = 0.00000001
      MODE=MODE+1
   7 CONTINUE
  52 CONTINUE
   4 CONTINUE
     SR(1) = -0.5
     SI(1) = 0.0
     SR(2) = 0.5
     SI(2) = 0.0
     SR(3) = 0.0
     SI(3) = 0.0
  67 CONTINUE
   8 CONTINUE
 608 CONTINUE
      NP1=N+1
     IF(SENSE SWITCH 2)71, 51
  71  FORMAT(40H THE COEFFICIENTS OF THE POLYNOMIAL ARE )
 501  PRINT 71
      PRINT                  68,(CRR(J),J=1,NP1)
  51 CONTINUE
     DO 9990 I = 1,129
9990 CRI(I) = 0.0
 502  DO 6 I=1,NP1
      CPR(I)=CRR(I)
   6  CPI(I)=CRI(I)
      CALL COMAG(CRR(1),CRI(1),C1,KE)
      DO 305 K=1,N
      GO TO(9,609),MODE
  53  FORMAT(      2X, 6H ROOT  4X, 7HITERANT 6X,
 531          2(10HREAL  PART  10X, 10HIMAG  PART  10X),11HROOT IS IN /
 532               2X, 6HNUMBER 4X, 7HNUMBER  6X,
 533   2(10H OF  ROOT  10X),2(10H OF  R(Z)  10X),        11HA RADIUS OF)
   9  PRINT                  53
 609  I=1
      NP1=N+2-K
      NN=NP1-1
      IF(K+1-N)13,12,11
  11  CALL DIVD(-CRR(2),-CRI(2),CRR(1),CRI(1),XR(1),XI(1),KE)
      K1=1
      K2=1
      GO TO 16
  12  AR=CRR(1)
      AI=CRI(1)
      BR=CRR(2)
      BI=CRI(2)
      CR=CRR(3)
      CI=CRI(3)
      K1=1
      K2=1
      M4=2
      GOTO  123
  14  XR(1)=DBARR
      XI(1)=DBARI
      GO TO 16
```

```
  13  DO 15 J=1,3
      XR(J)=SR(J)
  15  XI(J)=SI(J)
      K1=1
      K2=3
  16  M1=1
      M2=1
      M3=1
      M4=1
 700  DO 717 L=K1,K2
 701  ZR=XR(L)
 702  ZI=XI(L)
      CALL POLYNOM(NP1,ZR,ZI,CRR,CRI,RR,RI,KE)
 710  FXR(L)=RR
 711  FXI(L)=RI
      CALL COMAG(RR,RI,PMAG,KE)
 713  RAD=ALTER*(PMAG/C1)**(1.0/FLOATF(NN))
 714  GO TO (715,19,179),M3
 715  GO TO(716,718),MODE
  55  FORMAT(     2(I6, 4X),  4E20.11,  5X,  E10.4  )
 716  PRINT 55, K,I,ZR,ZI,RR,RI,RAD
 718  POLYO=POLYN
      POLYN=PMAG
      IF(PMAG)717,300,717
 717  I=I+1
      IF(K+1-N)17,300,300
  17  GO TO (18,200),M1
  18  VAL=DEL*POLYN
      DBARR=RAD
      DBARI=0.0
  19  K1=4
      K2=4
      M1=2
      M3=1
 101  ABARR=XR(1)-XR(3)
 102  ABARI=XI(1)-XI(3)
 103  BBARR=XR(2)-XR(3)
 104  BBARI=XI(2)-XI(3)
 105  AMIBR=XR(1)-XR(2)
 106  AMIBI=XI(1)-XI(2)
 107  CALL MULT(ABARR,ABARI,BBARR,BBARI,TA,TB,KE1)
 108  CALL MULT(TA,TB,AMIBR,AMIBI,DENR,DENI,KE2)
      CALL COMAG(DENR,DENI,TA,KE)
      CALL COMAG(XR(3),XI(3),T4,KE)
      IF(TA-EP1*T4)110,110,111
 110  CALL DERIV(NP1,XR(3),XI(3),CRR,CRI,DR,DI,K50)
      CALL COMAG(RR,RI,TR,KE)
      CALL COMAG(DR,DI,TD,KE)
      IF(TR-EP4*TD)192,171,171
 111  DELAR=FXR(1)-FXR(3)
 112  DELAI=FXI(1)-FXI(3)
 113  DELBR=FXR(2)-FXR(3)
 114  DELBI=FXI(2)-FXI(3)
 115  CALL MULT(BBARR,BBARI,DELAR,DELAI,TA,TB,KE5)
 116  CALL MULT(ABARR,ABARI,DELBR,DELBI,TC,TD,KE6)
 117  CALL DIVD(TA-TC,TB-TD,DENR,DENI,AR,AI,KE7)
 118  CALL MULT(ABARR,ABARI,TC,TD,T1,T2,KE8)
 119  CALL MULT(BBARR,BBARI,TA,TB,T3,T4,KE9)
 120  CALL DIVD(T1-T3,T2-T4,DENR,DENI,BR,BI,KE10) -
      CR=FXR(3)
      CI=FXI(3)
 123  CALL MULT(BR,BI,BR,BI,T1,T2,KE11)
 124  CALL MULT(AR,AI,CR,CI,T3,T4,KE12)
 131  TA=T1-4.0*T3
 132  TB=T2-4.0*T4
      CALL CSQRT(TA,TB,TC,TD)
 147  T1=-BR+TC
 148  T2=-BI+TD
 149  T3=-BR-TC
 150  T4=-BI-TD
      CALL COMAG(T1,T2,TA,KE14)
      CALL COMAG(T3,T4,TB,KE15)
 153  IF(TA-TB)154,168,168
```

```
154   TA=TB
155   T1=T3
156   T2=T4
168   GO TO (157,159),M4
157   IF(TA)161,161,158
158   CALL COMAG(2.0*CR,2.0*CI,TB,KE16)
      IF(TB-RAD*TA)159,159,180
159   CALL DIVD(2.0*CR,2.0*CI,T1,T2,DBARR,DBARI,KE17)
      GO TO (161,14),M4
161   XR(4)=XR(3)+DBARR
162   XI(4)=XI(3)+DBARI
      TR=ABSF(XR(4))
      TI=ABSF(XI(4))
      IF(TR)167,167,140
140   IF(TI)167,167,169
169   IF(TR-TI)164,167,163
163   IF(TI-EP2*TR)165,167,167
165   XI(4)=0.0
      GO TO(503,504),MODE
 56   FORMAT(40H ITERANT ALTERED TO BE PURE REAL NUMBER.)
503   PRINT 56
504   GO TO 167
164   IF(TR-EP2*TI)166,167,167
166   XR(4)=0.0
      GO TO(505,167),MODE
 57   FORMAT(45H ITERANT ALTERED TO BE PURE IMAGINARY NUMBER.)
505   PRINT 57
167   GO TO 700
180   CALL DIVD(T1,T2,TA ,0.0,T1,T2,K20)
      CALL DIVD(CR,CI,TB ,0.0,CR,CI,K21)
      CALL MULT(CR,CI,RAD,0.0,CR,CI,K22)
      GO TO(506,507),MODE
 58   FORMAT(87H ITERANT IS OUTSIDE CIRCLE WHICH BOUNDS A ROOT.  INTERP
 581OLATE ITERANT TO EDGE OF CIRCLE.)
506   PRINT 58
507   GO TO 159
171   CALL COMAG(ABARR,ABARI,T1,KR1)
         ENA(1)     STA(ITA)  ENA(3)      STA(ITB)
      IF(T1-EP3*T4)174,174,172
172   CALL COMAG(BBARR,BBARI,T2,KR2)
         ENA(2)     STA(ITA)  ENA(3)      STA(ITB)
      IF(T2-EP3*T4)175,175,173
173   CALL COMAG(AMIBR,AMIBI,T3,KR3)
         ENA(1)     STA(ITA)  ENA(2)      STA(ITB)
      IF(T3-EP3*T4)174,174,111
174      ENA(1)     STA(K1)   STA(K2)     ENA(2)    STA(M3)   SLJ(178)
175      ENA(2)     STA(K1)   STA(K2)     ENA(3)    STA(M3)
178   XR(K1)=XR(K1)*(1.0+2.0*EP3)
      XI(K1)=XI(K1)*(1.0+2.0*EP3)
      GO TO(508,509),MODE
 60   FORMAT(11H ITERANTS X  I1,6H AND X  I1,
 601  41H ARE TOO CLOSE TOGETHER.  ALTER ITERANT X  I1,1H.)
508   PRINT 60,ITA,ITB,ITA
509   GO TO 700
179   POLYO=PMAG
      GO TO 19
200   IF(POLYN-VAL)201,201,210
201   VAL=DEL*POLYN
202   LIM=I+NUM
203   M2=2
      CALL DERIV(NP1,XR(4),XI(4),CRR,CRI,DR,DI,K60)
      CALL COMAG(RR,RI,TR,KE)
      CALL COMAG(DR,DI,TD,KE)
      IF(TR-EP4*TD)192,210,210
210   DLT=RATIO*POLYO/POLYN
211   IF(1.0-DLT)220,220,212
212   CALL MULT(DBARR,DBARI,DLT,0.0,DBARR,DBARI,K30)
215   LIM=LIM+1
      GO TO (510,511),MODE
 72   FORMAT(120H POLYNOMIAL HAS INCREASED IN MAGNITUDE TOO MUCH WITH CU
 721RRENT STEP. THEREFORE REDUCE CURRENT STEP.                          )
510   PRINT 72
511   GO TO 161
```

```
220   GO TO (221,231),M2
221   IF(I-IMAX)250,250,222
222   GO TO(512,513),MODE
 62   FORMAT(69H MAXIMUM NUMBER OF ITERATIONS REACHED WITHOUT REDUCING
 621P(Z) BY DELTA.)
512   PRINT 62
513  RETURN
231   IF(I-LIM)250,250,300
250   DO 251 L=1,3
      XR(L)=XR(L+1)
      XI(L)=XI(L+1)
      FXR(L)=FXR(L+1)
251   FXI(L)=FXI(L+1)
      GO TO 101
192   GO TO(514,300),MODE
 69   FORMAT(19H FIRST DERIVATIVE ( E17.9,E20.9,    55H) INDICATES THAT
 691 ITERANT IS SUFFICIENTLY CLOSE TO ROOT.)
514   PRINT 69,DR,DI
300   DO 302  J=2,NN
      CALL MULT(ZR,ZI,CRR(J-1),CRI(J-1),TR,TI,K40)
      CRR(J)=TR+CRR(J)
302   CRI(J)=TI+CRI(J)
 63   FORMAT(59H THE COEFFICIENTS OF THE REDUCED POLYNOMIAL ARE AS FOLL
 631OWS )
      ROOTR(K)=ZR
      ROOTI(K)=ZI
      GO TO(303,305),MODE
303   PRINT               63
 68   FORMAT(1H 6E18.9)
304   PRINT               68,(CRR(J),J=1,NN)
      PRINT 68, (CRI(J),J=1,NN)
305   CONTINUE
      GO TO (515,516),MODE
515   PRINT 75
516  CONTINUE
 80  CONTINUE
      DO 306 I=1,N
      CALL POLYNOM(N+1,ROOTR(I),ROOTI(I),CPR,CPI,RR,RI,KE)
     ANSWER (INT) = ROOTR(I)
     INT = INT + 1
     ANSWER(INT) = ROOTI(I)
     INT = INT + 1
     IF(ABSF(RR)- 0.01) 8001, 8001, 8002
8001 IF(ABSF(RI) - 0.01) 306, 306, 8003
8999 FORMAT( 44HBELOW ROOT UNCERTAIN WITH REAL REMAINDER OF E7.1)
8002 PRINT 8999,RR
8012 FORMAT(E7.1)
     GO TO 8001
8998 FORMAT( 44HBELOW ROOT UNCERTAIN WITH IMAG REMAINDER OF E7.1)
8003 PRINT 8998, RI
8013 FORMAT(E7.1)
306  CONTINUE
 83  CONTINUE
 75   FORMAT(1H1)
     RETURN
999   CONTINUE
     RETURN
      END
      SUBROUTINE    DERIV(N,ZR,ZI,CR,CI,DR,DI,KER)
      DIMENSION CR(129),CI(129)
         ENA(0)      STA(DR)    STA(DI)    STA(RR)    STA(RI)ENA(1)STA(KER).
      DO 2 J=1,N
      CALL MULT(ZR,ZI,RR,RI,TRR,TRI,K1)
      CALL MULT(ZR,ZI,DR,DI,TDR,TDI,K2)
         RSO(K1)    AJP1(L+2) RSO(K2)    AJP(1)     ENA(2) STA(KER)SLJ(3).
  1   DR=TDR+RR
      DI=TDI+RI
      RR=TRR+CR(J)
  2   RI=TRI+CI(J)
  3   CONTINUE
      END
      SUBROUTINE    POLYNOM(N,ZR,ZI,CR,CI,RR,RI,KER)
      DIMENSION CR(129),CI(129)
```

```
      ENA(0)     STA(RR)    STA(RI)    ENA(1)     STA(KER)
   DO 2 J=1,N
   CALL MULT(ZR,ZI,RR,RI,TR,TI,K1)
      RSO(K1)    AJP(1)     ENA(2)     STA(KER)   SLJ(3)
 1 RR=TR+CR(J)
 2 RI=TI+CI(J)
 3 CONTINUE
   END
   SUBROUTINE CSQRT(XR,XI,YR,YI)
      CON(SQ2=1.4142135624).
      LDA(XR)    LDQ(XI)    AJP2(L+1)  LAC(XR)    QJP2(L+1)  LQC(XI)
      STA(A)     STQ(B)     QJP1(1)    STQ(Q)     SLJ4(8) +STA(P)SLJ(5).
 1    AJP1(2)    LDA(B)     SLJ4(8)    +FDV(SQ2)  STA(P)
      SSK(XI)    SLJ(L+2)   LAC(P)     +STA(Q)    SLJ(5)
 2    THS(B)     SLJ(3)     STQ(S)     FDV(B)     STA(T)     STA(R)SLJ(4)
 3    STA(S)     LDA(B)     FDV(A)     STA(T)     LDA(1.0)   STA(R)SLJ(4)
 7 Y=SQRTF(X)
 8    SLJ(*)     STA(X)     SLJ(7)
 4    LDA(T)     FMU(T)     FAD(1.0)   SLJ4(8)    +FAD(R)    FDV(2.0)
      SLJ4(8)    +STA(T)    LDA(S)     SLJ4(8)    +FMU(T)    STA(P)
      LDA(XI)    FDV(2.0)   FDV(P)     STA(Q)
 5    SSK(XR)    SLJ(6)     LDA(Q)     LQC(P)     AJP2(L+1)  LAC(Q)
      SSK(XI)    LDQ(P)     STA(YR)    STQ(YI)    SLJ(L+3)
 6    LDA(P)     LDQ(Q)     STA(YR)    STQ(YI)
   END
   SUBROUTINE COMAG(XR,XI,Z,KER)
      LDA(XR)    LDQ(XI)    AJP2(L+1)  LAC(XR)    QJP2(L+1)  LQC(XI)
      STQ(T)     +THS(T)    LLS(48)    QJP(3)     STQ(T)     FDV(T)
      STA(H)     FMU(H)     FAD(1.0)   STA(H)
   Y=SQRTF(H)
    -FMU(T)      +EXF7(141B)SLJ(L+2)   ENQ(1)     SLJ(3)     ENQ(2)
 3    STQ(KER)   STA(Z)
   END
   SUBROUTINE   DIVD(XR,XI,YR,YI,ZR,ZI,KER)
   CALL PROD(XR,XI,YR,-YI,B1,B2,PR,PI,DR,DI)
      LDA(B2)    AJP1(1)    ENA(3)     SLJ(3)
 2    ENA(2)     SLJ(3)
 1 T=DR*DR+DI*DI
      LDA(B1)    -FDV(B2)   +EXF7(141B)SLJ(2)     STA(B1)
      LDA(PR)    FDV(T)     -FMU(B1)   +EXF7(141B)SLJ(2)     STA(ZR)
      LDA(PI)    FDV(T)     -FMU(B1)   +EXF7(141B)SLJ(2)    STA(ZI)ENA(1)
 3    STA(KER)
   END
   SUBROUTINE   MULT(XR,XI,YR,YI,ZR,ZI,KER)
   CALL PROD(XR,XI,YR,YI,B1,B2,PR,PI,D1,D2)
      LDA(B2)    -FMU(B1)   +EXF7(141B)SLJ(1)     STA(B1)
      LDA(PR)    -FMU(B1)   +EXF7(141B)SLJ(1)     STA(ZR)
      LDA(PI)    -FMU(B1)   +EXF7(141B)SLJ(1)     STA(ZI)
      ENA(1)     STA(KER)   SLJ(L+2)
 1    ENA(2)     STA(KER)
   END
   SUBROUTINE   PROD(XR,XI,YR,YI,B1,B2,PR,PI,DR,DI)
   CALL NORM(XR,XI,B1,AR,AI)
   CALL NORM(YR,YI,B2,DR,DI)
   PR=AR*DR-AI*DI
   PI=AI*DR+AR*DI
   END
   SUBROUTINE   NORM(A1,A2,B1,S1,S2)
1A    SLJ(1)     +SEV7(70000B)         ZRO(0)     +ZRO(4000B)ZRO(0)
 1    LDA(1A+1)  LDQ(A1)    QJP2(L+1)  LQC(A1)    STL(E)     LDQ(A2)
      QJP2(L+1)  LQC(A2)    LDL(1A+1)+THS(E)      SLJ(L+2)   LDA(E)
     +AJP1(L+2)  STA(S1)    STA(B1)    SLJ(L+5)   +ADD(1A+2) STA(B1)
      LDA(A1)    FDV(B1)    STA(S1)    LDA(A2)    FDV(B1)    +STA(S2)
   END
  SUBROUTINE DIVIDE(ANN, BET, E, R)
  DIMENSION ANN(100), BET(100), D(100), E(100), R(2)
  COMMON CRR, CRI, CPR, CPI, ROOTR, ROOTI, C, D
  DO 1000 JL=1,100
1000 E(JL) =0.0
  M = 100
1 IF(ANN(M)) 3, 2, 3
2 M = M-1
  GO TO 1
```

```
  3 M = M-1
 31 N = 100
  4 IF(BET(N)) 6, 5, 6
  5 N = N-1
    GO TO 4
  6 N = N - 1
    CALL CONVRT(ANN)
    CALL CONVRT(BET)
 61 INTH = (N-M+1)
    DO 7 K = 1, INTH
    E(K) = BET(K) / ANN(1)
    DO 7 I = 1, N+1
    J = I+K-1
    D(J) = E(K)*ANN(I)
    BET(J) = BET(J) - D(J)
  7 D(J) = 0.0
    CALL CONVRT(E)
    NA = N+1
    R(1) = BET(NA)
    R(2) = BET(N)
    ZERO = 0.0
    RETURN
    END
    SUBROUTINE EQUAL1(A, B)
    DIMENSION A(100), B(100)
    DO 1 J = 1, 100
  1 A(J) = B(J)
    RETURN
    END
    SUBROUTINE EQUAT1 (A, B)
    DIMENSION A(100) , B(100)
    DO 680 I = 1, 100
    A(I) = B(I)
680 B(I) = 0
    RETURN
    END
    SUBROUTINE CONVRT(ABLE)
    DIMENSION ABLE(100), BAKER(100)
    N = 100
  1 IF(ABLE(N)) 3, 2, 3
  2 N = N-1
    GO TO 1
  3 N = N - 1
 31 NOC = N + 1
 32 DO 4 I = 1, NOC
    J = N-I + 2
  4 BAKER(J) = ABLE(I)
    DO 5 I = 1, NOC
    ABLE(I) = BAKER(I)
  5 BAKER(I) = 0.0
    RETURN
    END
    END
```

Appendix B - Derivation of K_v.

The usual representation for steady state error for a unity feedback system in response to a ramp input is:

$$\mathcal{E}\Big|_{t=\infty} = \frac{\omega_i}{K_v}$$

where K_v is derived from the open loop transfer function, F_o. However, due to the block algebra manipulations used in deriving the locus equations for this investigation, there is no longer a function, F_o, as such, since it is not represented as a unity feedback system. However, to maintain convention it is proposed to continue the use of a "K_v" to give an indication of steady state error in response to a ramp input. This "K_v" is derived as follows:

$$\mathcal{E}(s) = \Theta_i(s) - \Theta_o(s) = \Theta_i(s)\left[1 - \frac{\Theta_o(s)}{\Theta_i(s)}\right]$$

and $\quad F_c(s) \triangleq \dfrac{\Theta_o(s)}{\Theta_i(s)}$

therefore,

$$\mathcal{E}(s) = \Theta_i(s)\left[1 - F_c(s)\right]$$

From the block algebra in section 1, it is seen that:

$$F_c(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{\dfrac{N_{abm}}{D_{abm} + N_{abm}}}{1 + \dfrac{D_{am}\,N_{bc}}{D_c(D_{abm} + N_{abm})}}$$

$$F_c(s) = \frac{D_c\,N_{abm}}{D_c(D_{abm} + N_{abm}) + D_{am}\,N_{bc}}$$

If we assume F_c to be a polynomial over a polynomial:

$$F_c(s) = \frac{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}{\left[s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0\right] + \left[s^p + c_{p-1}s^{p-1} + \cdots c_1 s + c_0\right]}$$

where $\quad n \leq m$

If the system is a servo, then

$$F_c(s)\Big|_{s=0} = 1 = \frac{a_0}{b_0 + c_0}$$

but

$$\mathcal{E}(s) = \Theta_i(s)\left[1 - F_c(s)\right]$$

$$\mathcal{E}(s) = \Theta_i(s)\left[1 - \frac{s^n + \cdots a_1 s + a_0}{(s^m + \cdots b_1 s + b_0) + (s^p + \cdots + c_1 s + c_0)}\right]$$

$$\mathcal{E}(s) = \Theta_i(s)\left[\frac{s^m + \cdots + b_1 s + b_0 + s^p + \cdots + c_1 s + c_0 - s^n - \cdots - a_0}{s^m + \cdots b_1 s + b_0 + s^p + \cdots c_1 s + c_0}\right]$$

and

$$\mathcal{E}\Big|_{t=\infty} = s\,\mathcal{E}(s)\Big|_{s=0}$$

Thus setting all s terms of power greater than 1 to zero:

$$\mathcal{E}\Big|_{t=\infty} = s\,\Theta_i(s)\left[\frac{(b_1 + c_1 - a_1)s + b_0 + c_0 - a_0}{(b_1 + c_1)s + (b_0 + c_0)}\right]$$

However, from above, it was found that, for a servo:

$$\frac{a_0}{b_0 + c_0} = 1$$

or $\quad b_0 + c_0 - a_0 = 0$

therefore $\quad \mathcal{E}\Big|_{t=\infty} = \dfrac{s\,\Theta_i(s)\left[b_1 + c_1 - a_1\right]s}{(b_1 + c_1)s + (b_0 + c_0)}$

if $\quad \Theta_i(s) = \dfrac{\Theta_i}{s}$ (step), $\quad \mathcal{E}\Big|_{t=\infty} = 0$

If $\quad \Theta_i(s) = \dfrac{\omega_i}{s^2}$ (ramp), $\quad \mathcal{E}\Big|_{t=\infty} = \dfrac{\omega_i\,(b_1 + c_1 - a_1)}{b_0 + c_0}$

Now by definition,

$s^n + \cdots + a_1 s + a_0$ represents $D_c N_{abm}$

$s^m + \cdots + b_1 s + b_0$ represents $D_c(D_{abm} + N_{abm})$

$s^{p} + \cdots + C_1 S + C_0$ represents $D_{am} N_{bc} = k_c (D_{am} N_b N_c')$

therefore: C_0 can be written $k_c C_0'$

C_1 can be written $k_c C_1'$

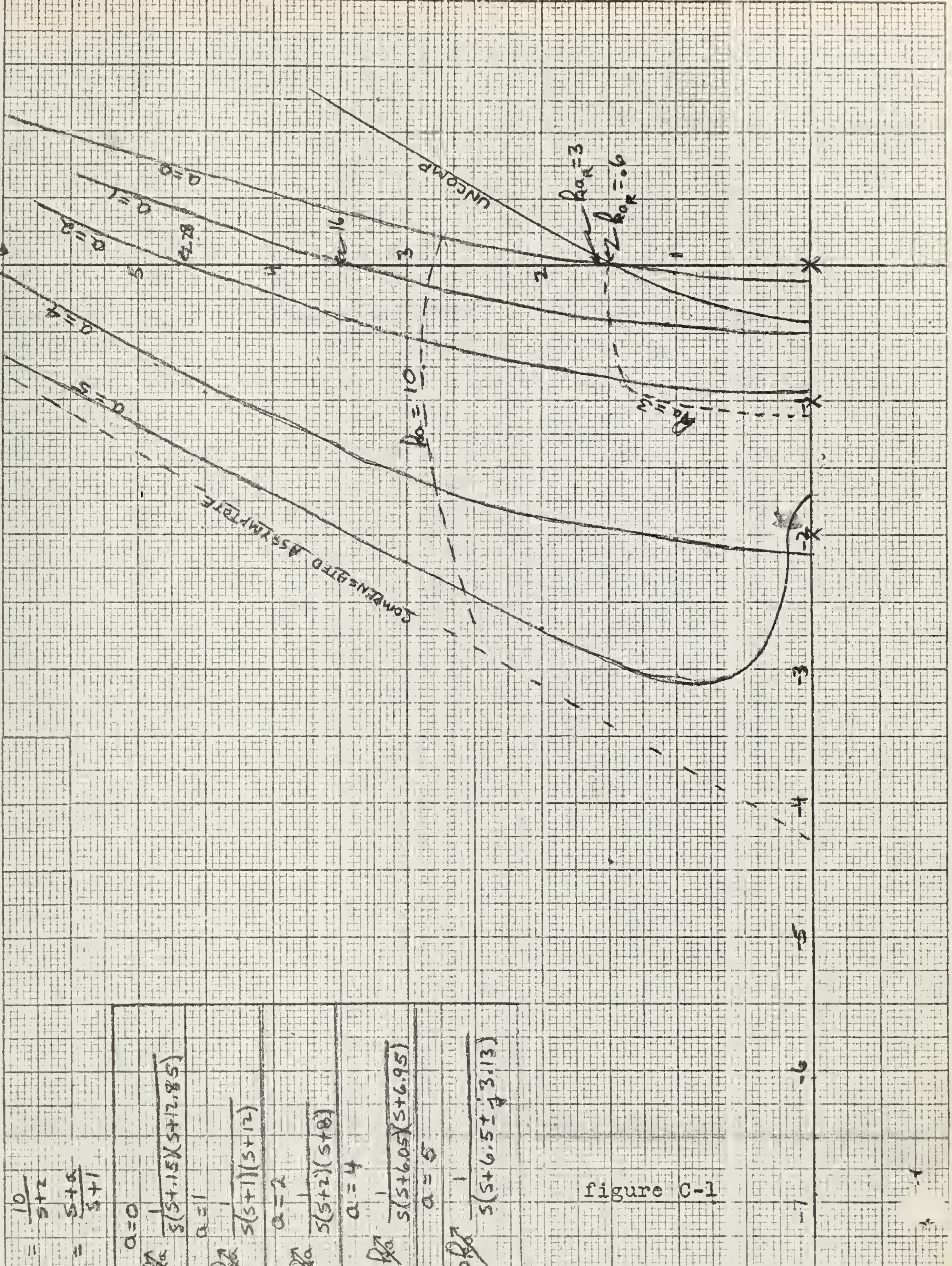
Thus,
$$\mathcal{E}\Big|_{t=\infty} = \frac{\omega_i (b_1 + k_c C_1' - a_1)}{b_0 + k_c C_0'} \triangleq \frac{\omega_i}{K_v}$$

for $\omega_i = Bt$

or
$$K_v = \frac{b_0 + k_c C_0'}{b_1 + k_c C_1' - a_1}$$

It is the above expression that has been represented throughout this investigation as K_v.

APPENDIX C



$G_B = \frac{10}{s+2}$
$G_c = \frac{s+a}{s+1}$
a=0 $10k_a \frac{1}{s(s+.15)(s+12.85)}$
a=1 $10k_a \frac{1}{s(s+1)(s+12)}$
a=2 $10k_a \frac{1}{s(s+2)(s+8)}$
a=4 $10k_a \frac{1}{s(s+6.05)(s+6.95)}$
a=5 $10k_a \frac{1}{s(s+6.5 \pm j3.13)}$
UNCOMP
COMPENSATED ASSYMPTOTE
$k_a = 48.5$
$k_a = 10$
$k_{aR} = 3$
$k_{aR} = .6$
a=0
1=0
2=0
4=0
5=0

figure C-1

$$G_B = \frac{10}{s+2}$$

$$G_C = \frac{s+a}{s+2}$$

$a=0$ $10k_a \dfrac{(s+2)}{s(s+1)(s+.3)(s+13.7)}$ $a=1$ $10k_a \dfrac{(s+2)}{s(s+1)(s+1.1)(s+12.9)}$
$a=2$ $10k_a \dfrac{\cancel{(s+2)}}{s(s+1)\cancel{(s+2)}(s+12)}$
$a=4$ $10k_a \dfrac{(s+2)}{s(s+1)(s+4.76)(s+9.24)}$
$a=5$ $10k_a \dfrac{(s+2)}{s(s+1)(s+7 \pm j2.24)}$
$a=10$ $10k_a \dfrac{(s+2)}{s(s+1)(s+7 \pm j7.41)}$

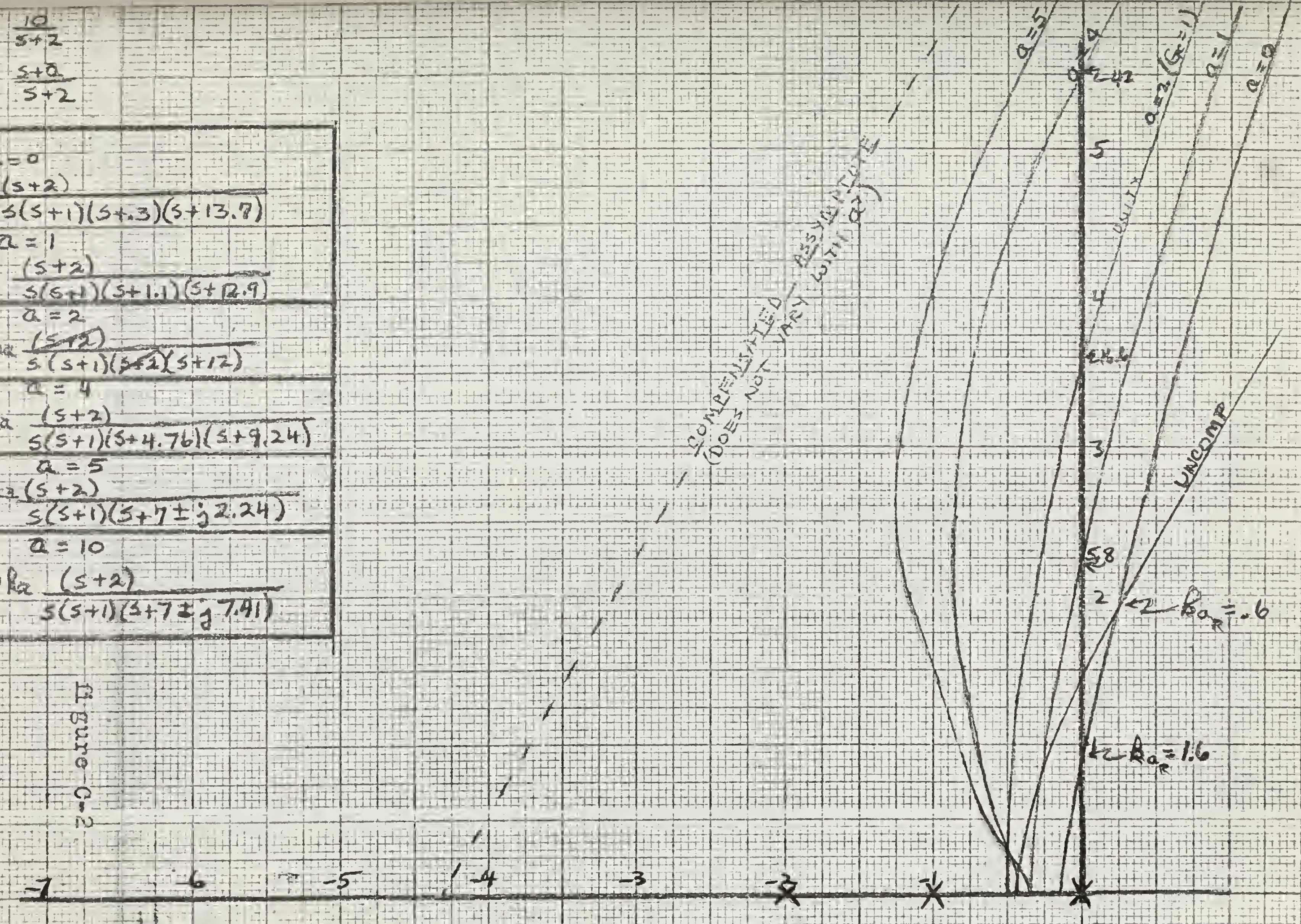


figure C-2

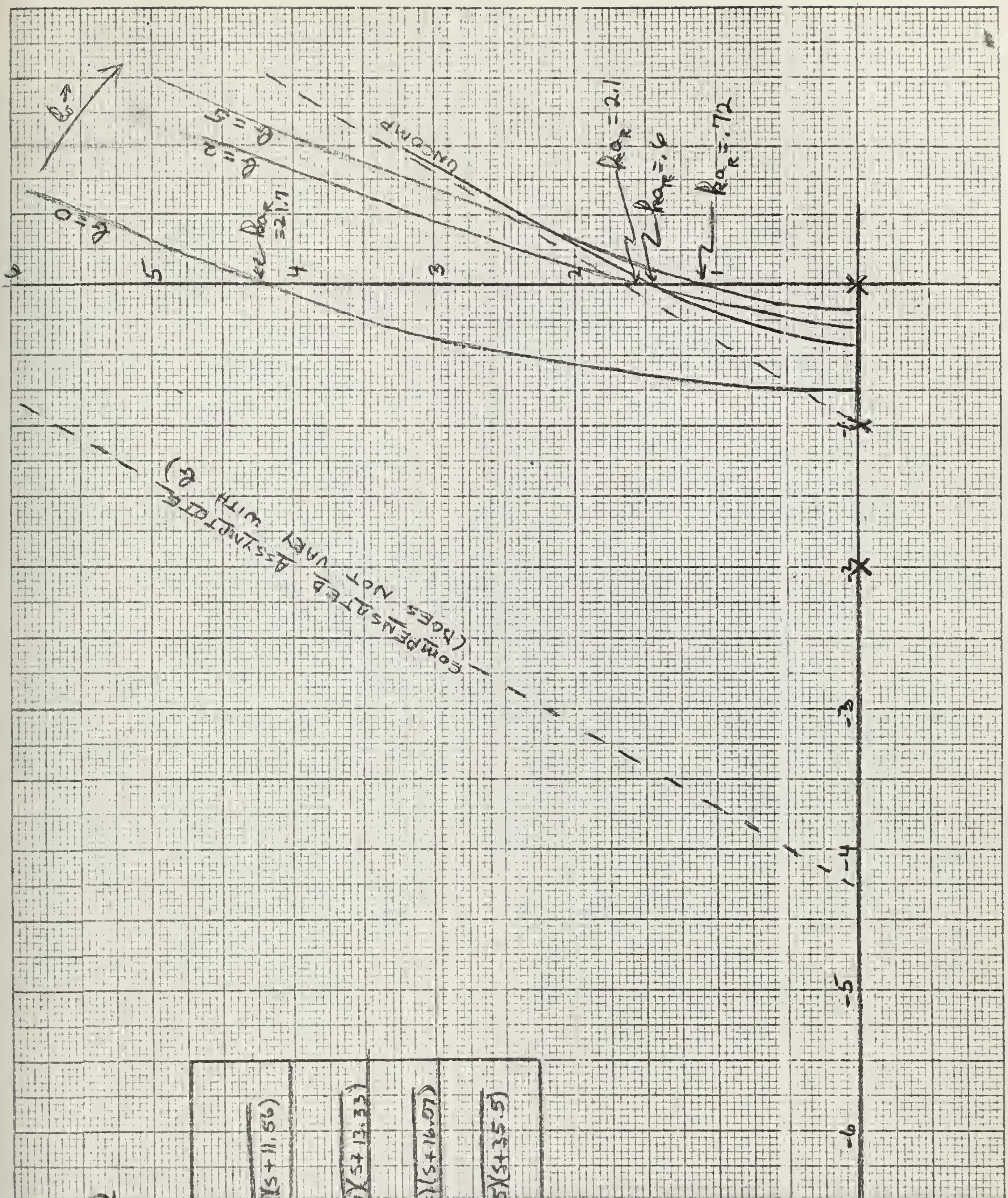

G_a = 10/(s+2)
G_c = (s+.5)/(s+b)
b=0 10Ka s/s(s+1)(s+.44)(s+11.56)
b=2 10Ka (s+2)/s(s+1)(s+.67)(s+13.33)
b=5 10Ka (s+5)/s(s+1)(s+.9)(s+16.07)
b=25 10Ka (s+25)/s(s+1)(s+1.5)(s+35.5)
b→
b=0
b=2
b=5
UNCOMP
Ka_R = 21.7
Ka_R = 2.1
Ka_R = .6
Ka_R = .72
COMPENSATED ASSYMPTOTE (DOES NOT VARY WITH b)
1
2
3
4
5
6
-1
-2
-3
-4
-5
-6

figure C-3

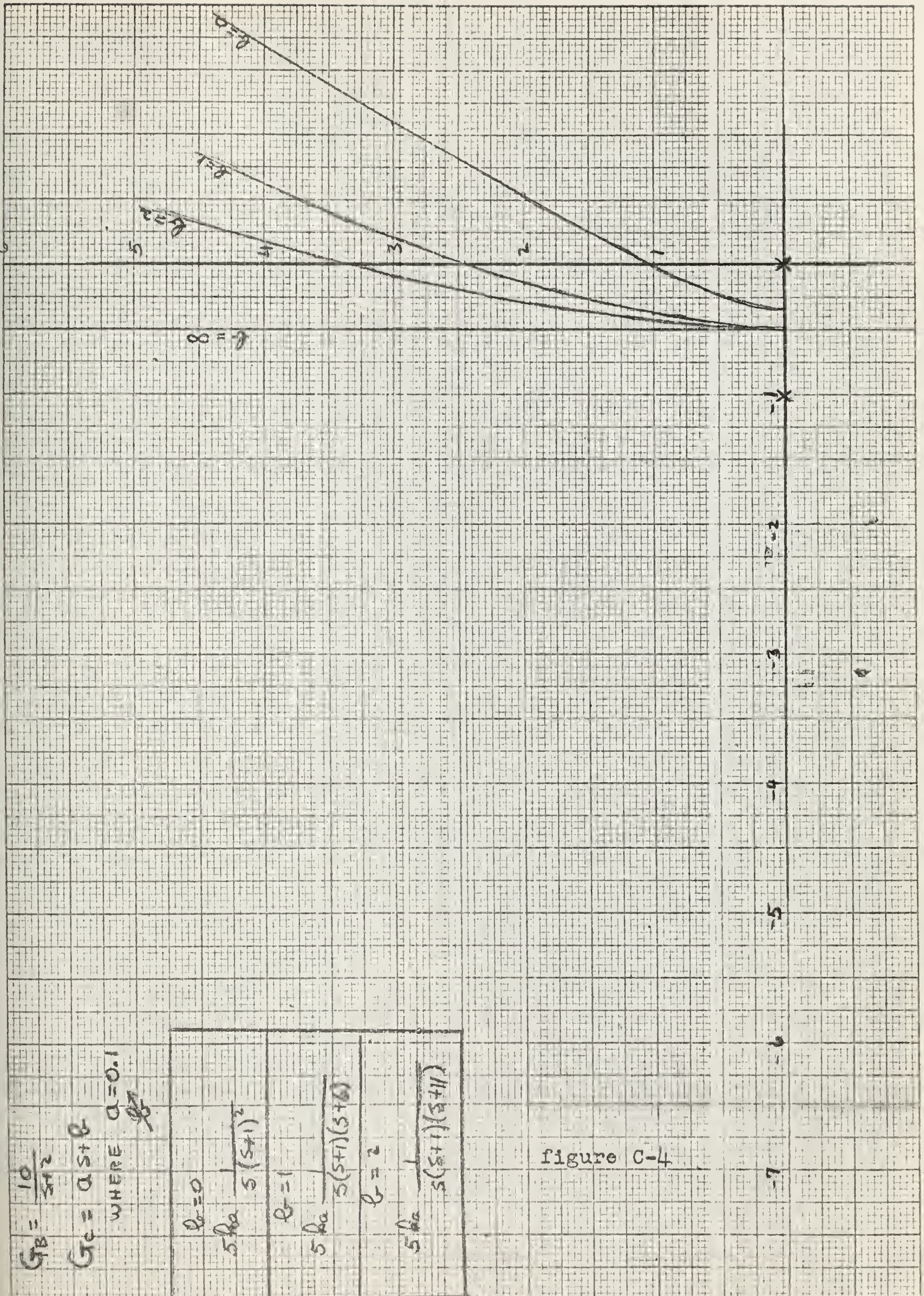
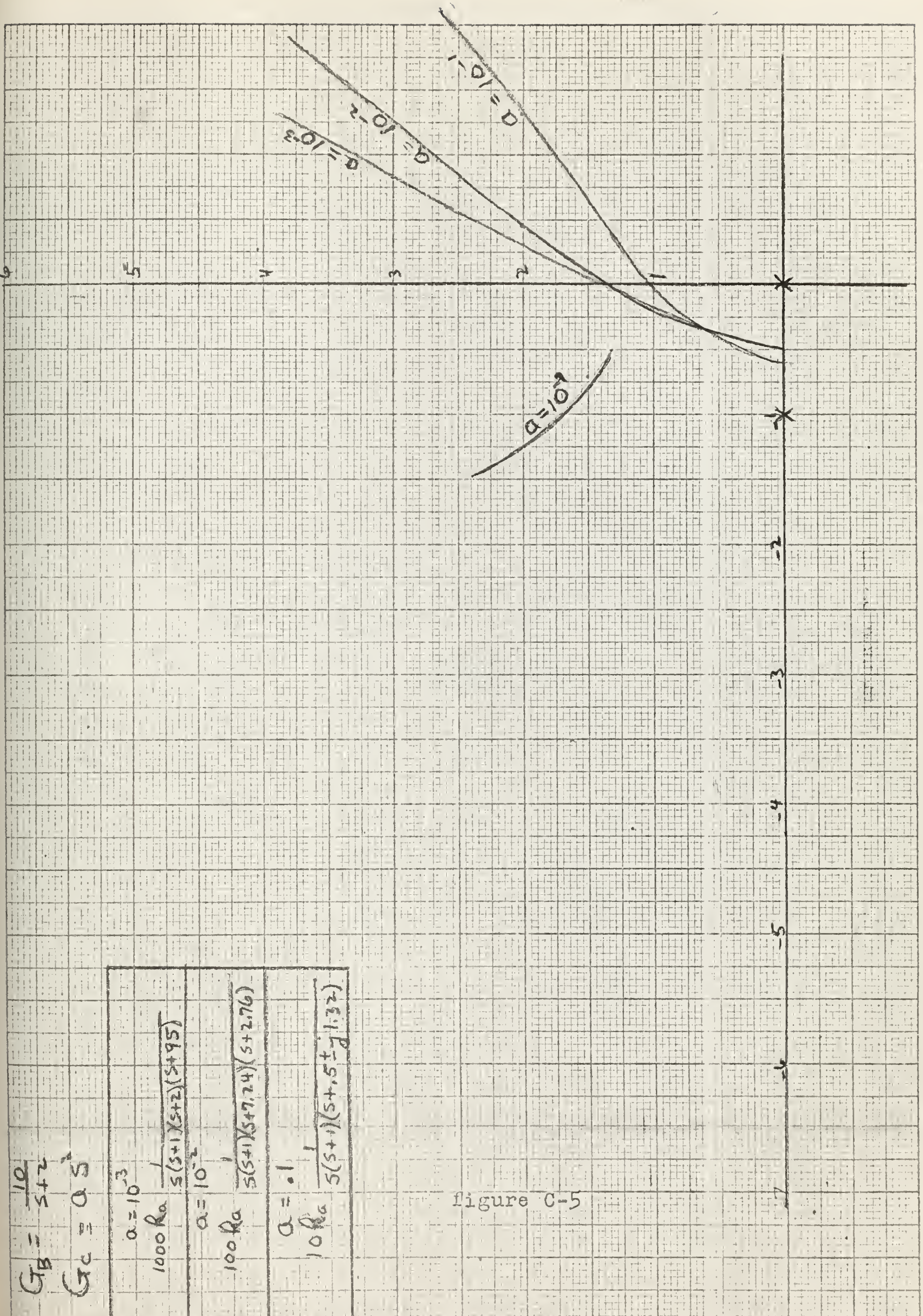
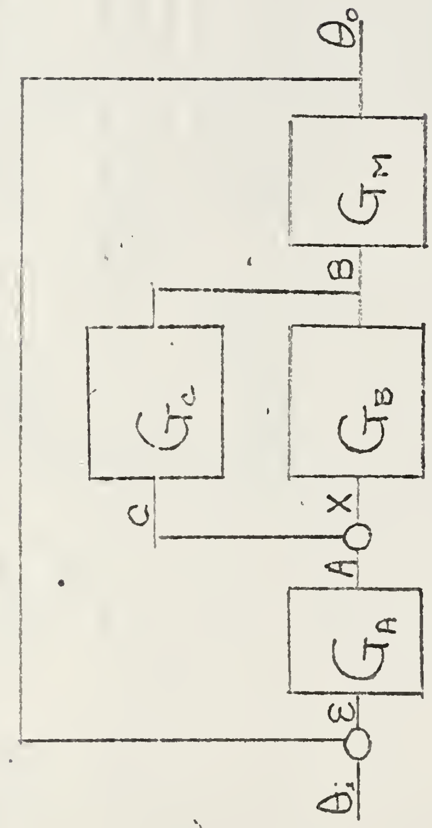


figure C-4



G_B = 10/(s+2)
G_c = a s^3
a = 10^-3
1000 K_a 1/(s(s+1)(s+2)(s+95))
a = 10^-2
100 K_a 1/(s(s+1)(s+7.24)(s+2.76))
a = .1
10 K_a 1/(s(s+1)(s+.5 ± j1.32))
a = 10^-1
a = 10^-2
a = 10^-3
a = 10^-4

figure C-5



SERVO BLOCK DIAGRAM
FOR COMPUTER SETUP

figure D-1

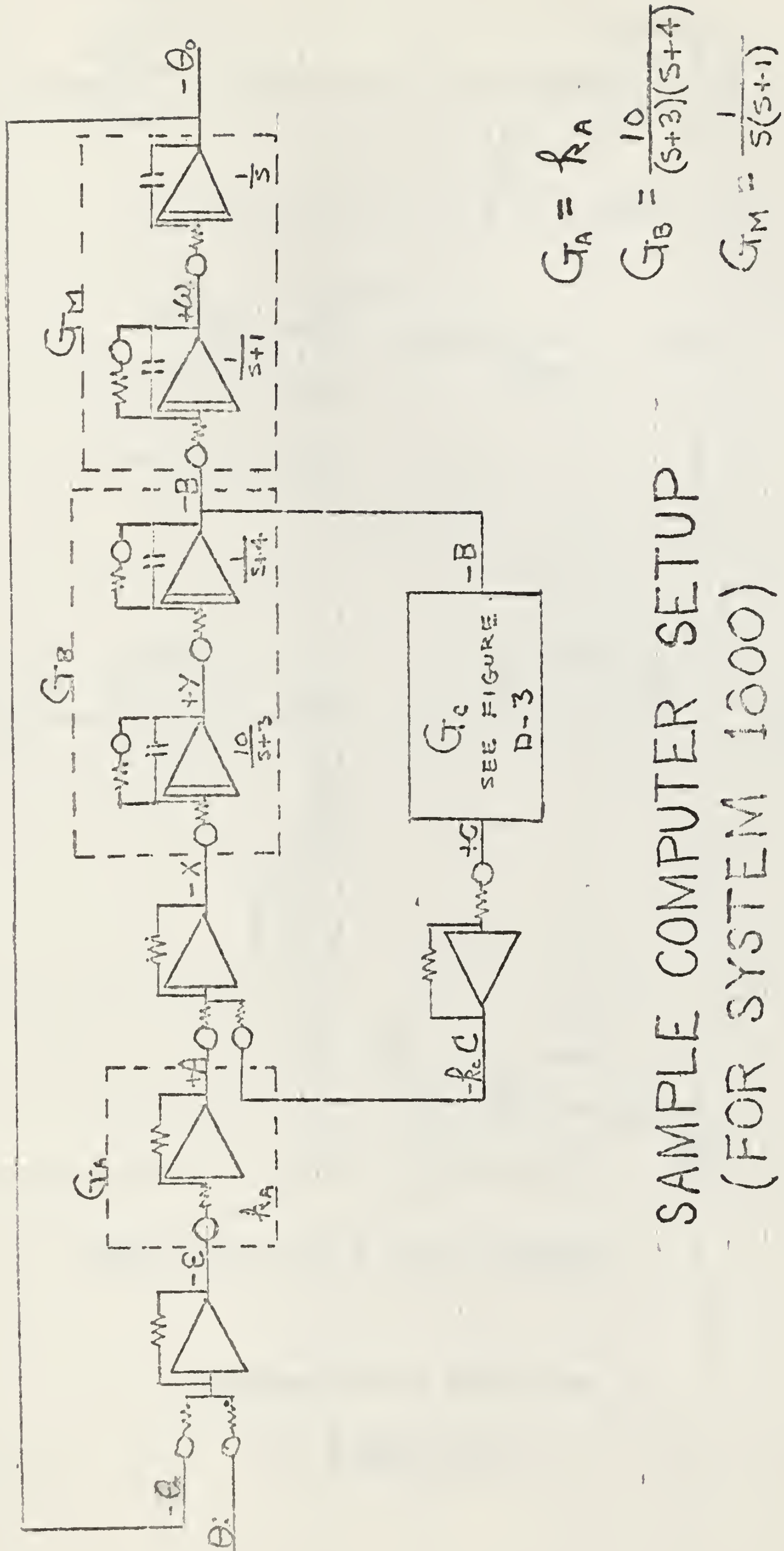
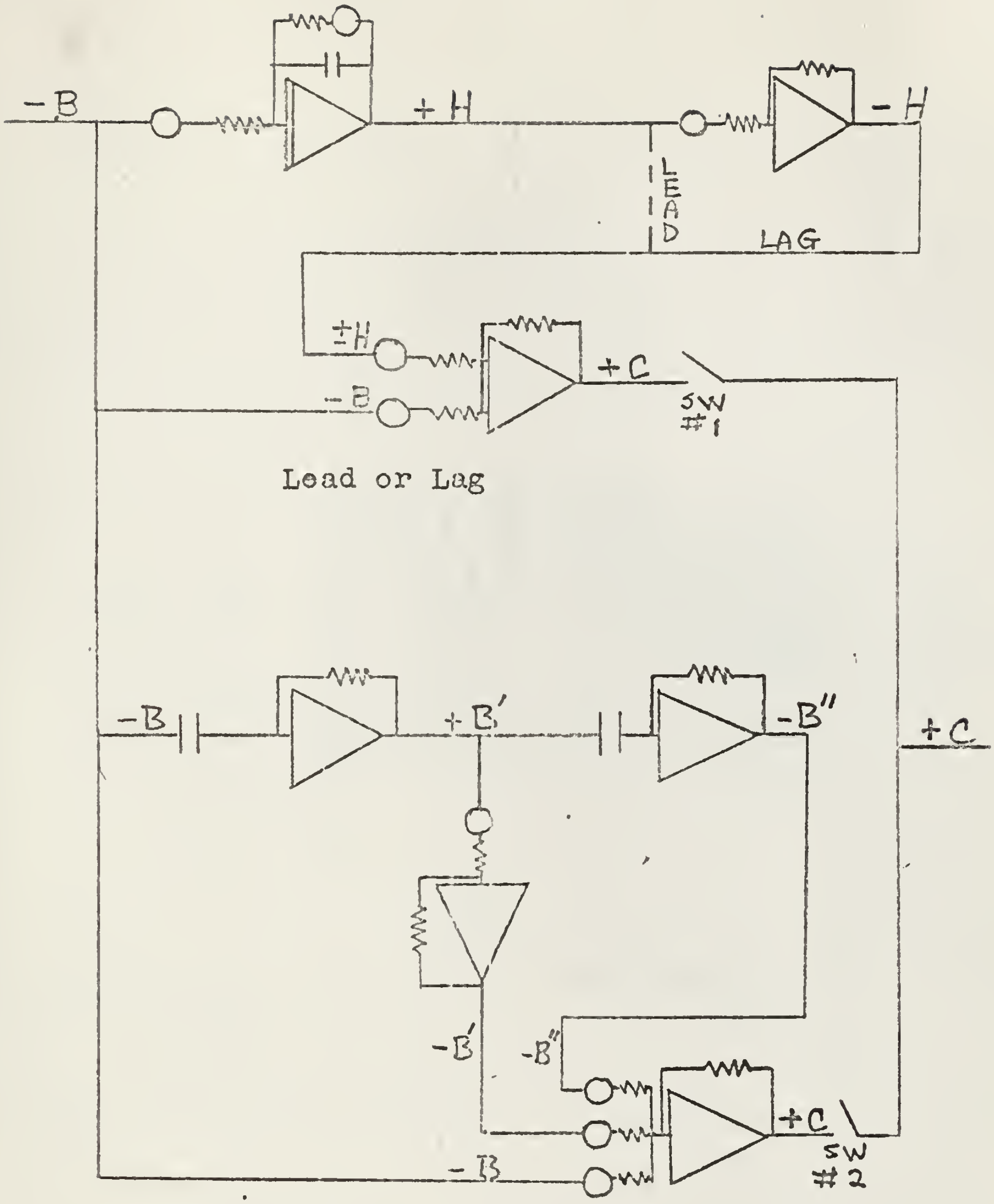

SAMPLE COMPUTER SETUP
(FOR SYSTEM 1000)
$G_A = k_A$
$G_B = \frac{10}{(s+3)(s+4)}$
$G_M = \frac{1}{s(s+1)}$
G_c
SEE FIGURE D-3
G_A
G_B
G_M
$-\theta_o$
θ_i
$-\varepsilon$
k_A
$+A$
$-X$
$+Y$
$-B$
$+\omega$
$\frac{10}{s+3}$
$\frac{1}{s+4}$
$\frac{1}{s+1}$
$\frac{1}{s}$
$+C$
$-k_c C$

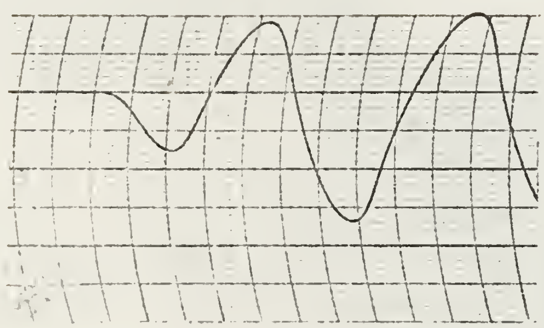
figure D-2



Derivative plus proportional

Compensator Function G_c

Figure D-3



0100

UNCOMPENSATED

figure D-4

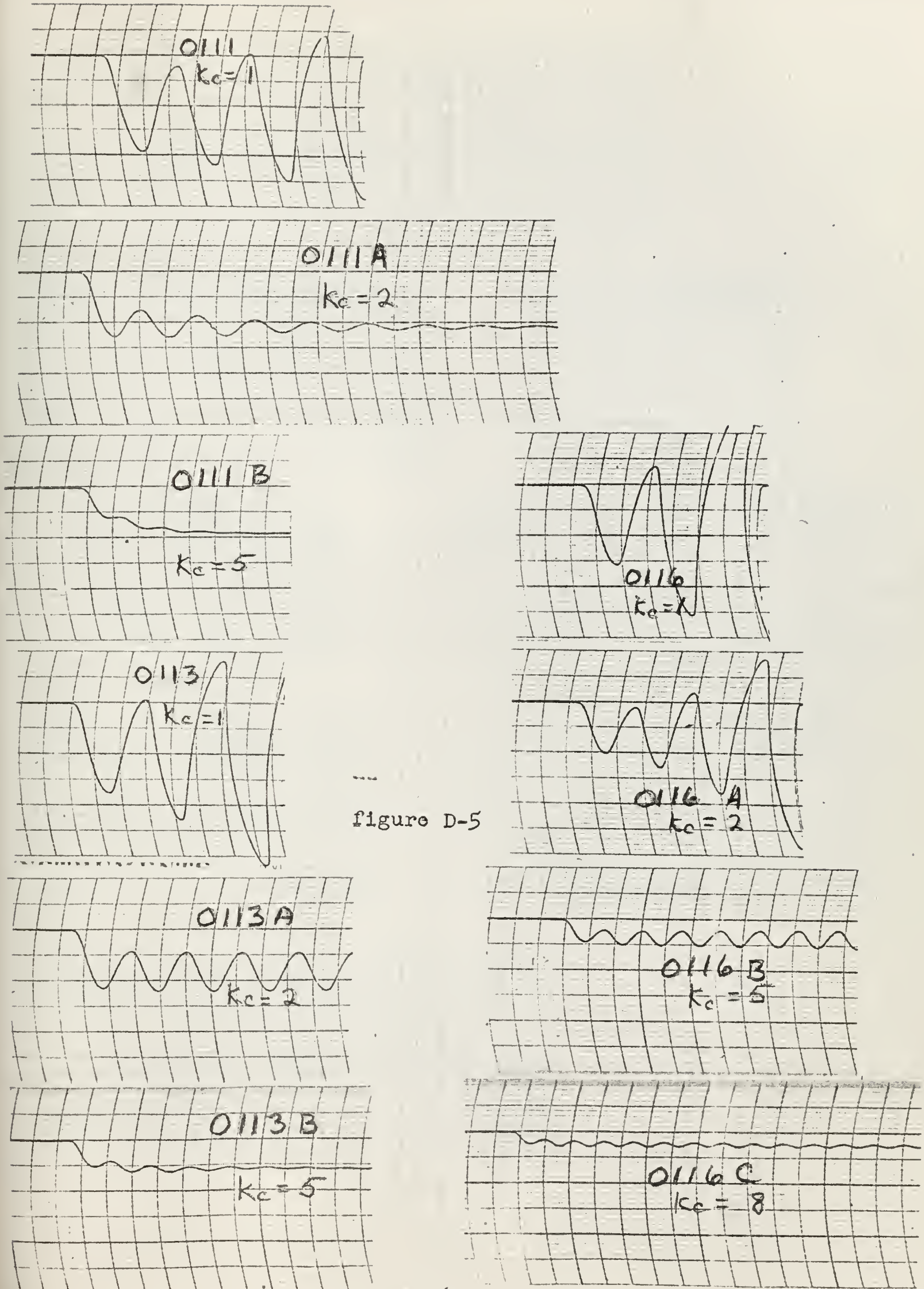

0111
Kc=1
0111 A
Kc=2
0111 B
Kc=5
0116
Kc=1
0113
Kc=1
0116 A
Kc=2
0113 A
Kc=2
0116 B
Kc=5
0113 B
Kc=5
0116 C
Kc=8

figure D-5

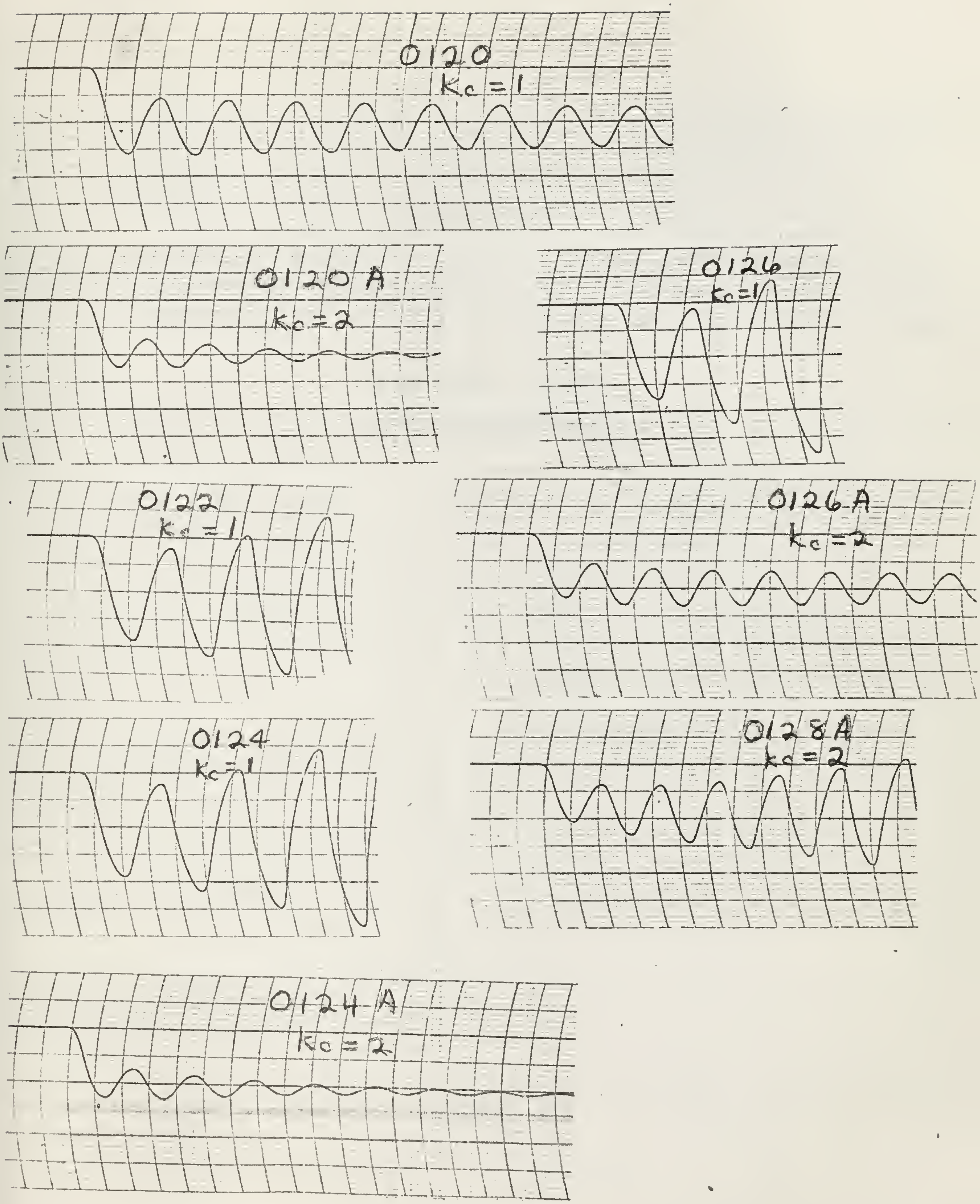
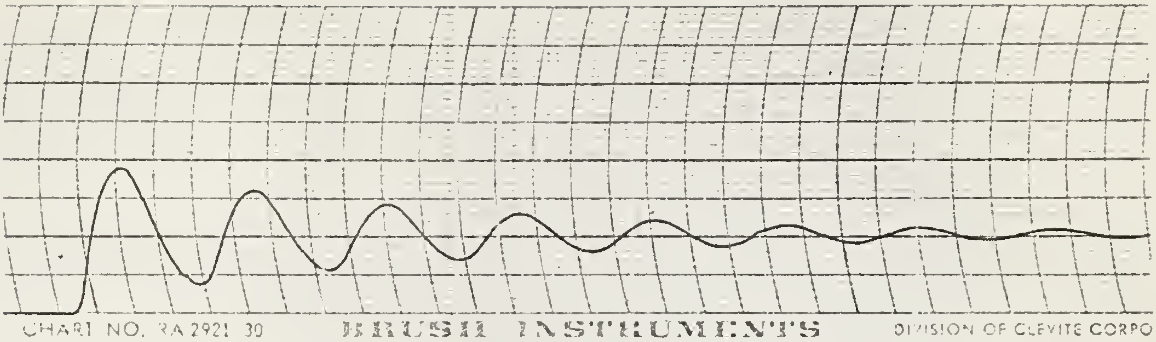

0120
Kc = 1
0120 A
Kc = 2
0126
Kc=1
0122
Kc = 1
0126 A
Kc = 2
0124
Kc = 1
0128 A
Kc = 2
0124 A
Kc = 2

figure D-6



SYSTEM 1000
-UNCOMPENSATED-

figure D-7

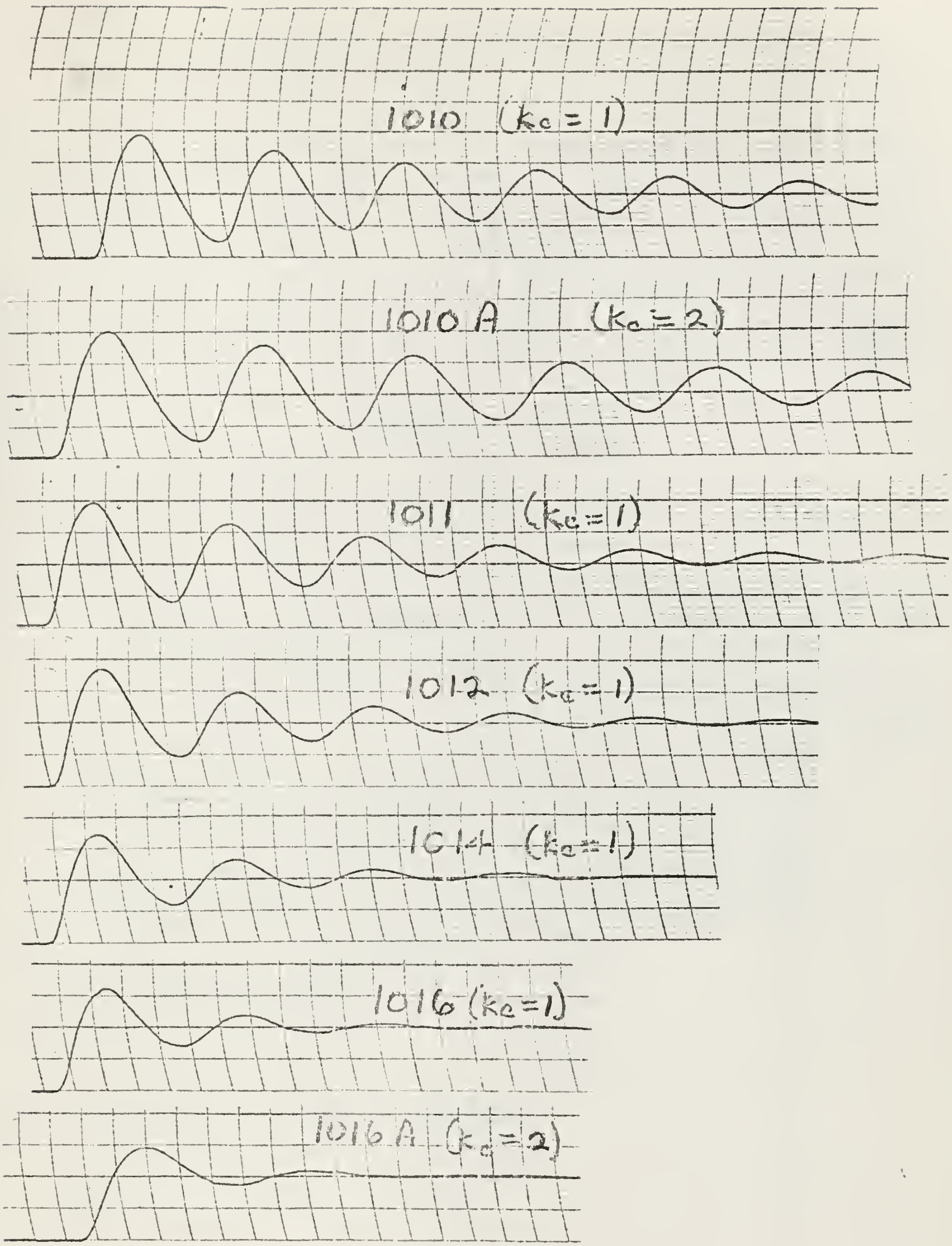

1010 (kc = 1)
1010 A (kc = 2)
1011 (kc = 1)
1012 (kc = 1)
1014 (kc = 1)
1016 (kc = 1)
1016 A (kc = 2)

figure D-8

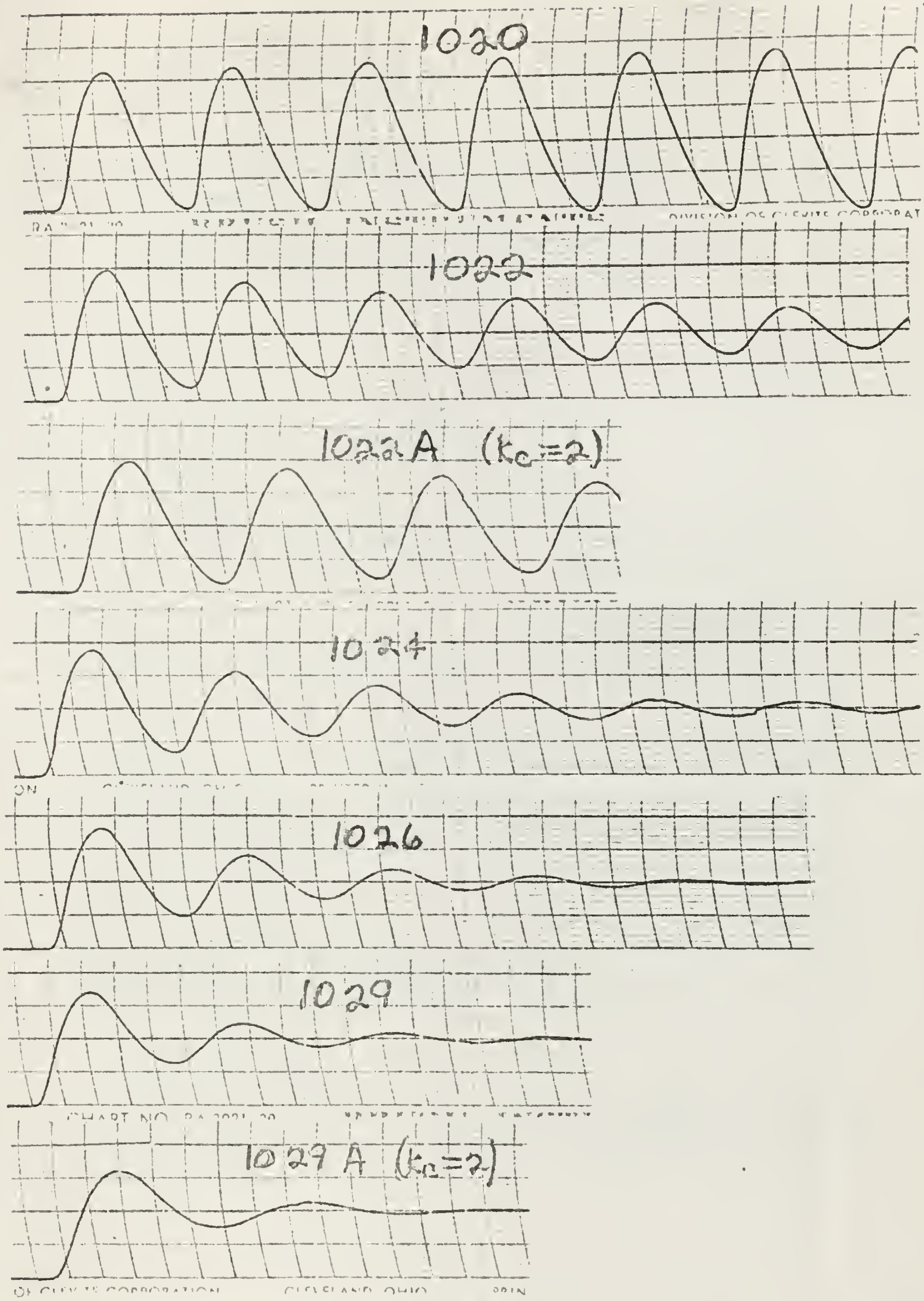


figure D-9

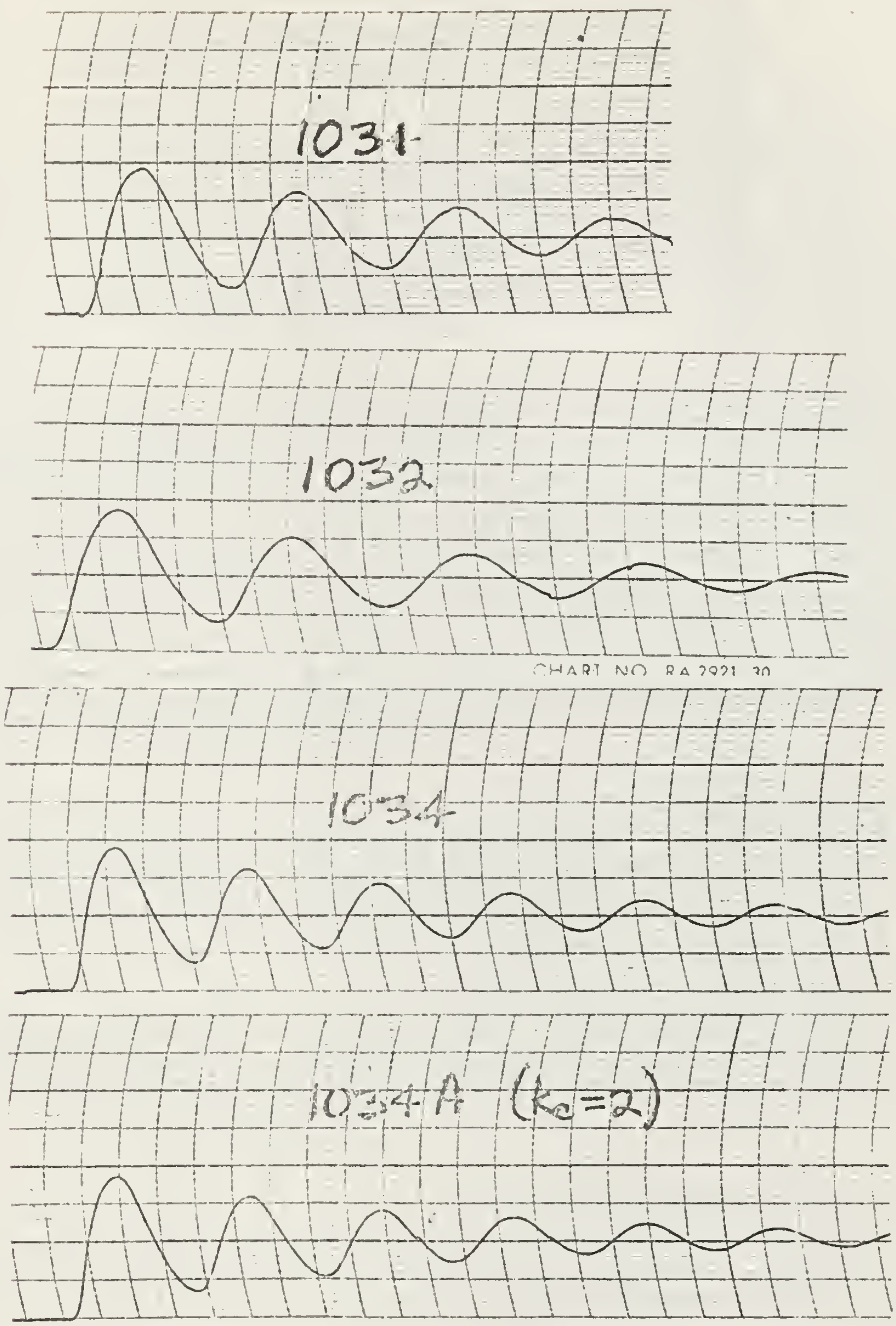
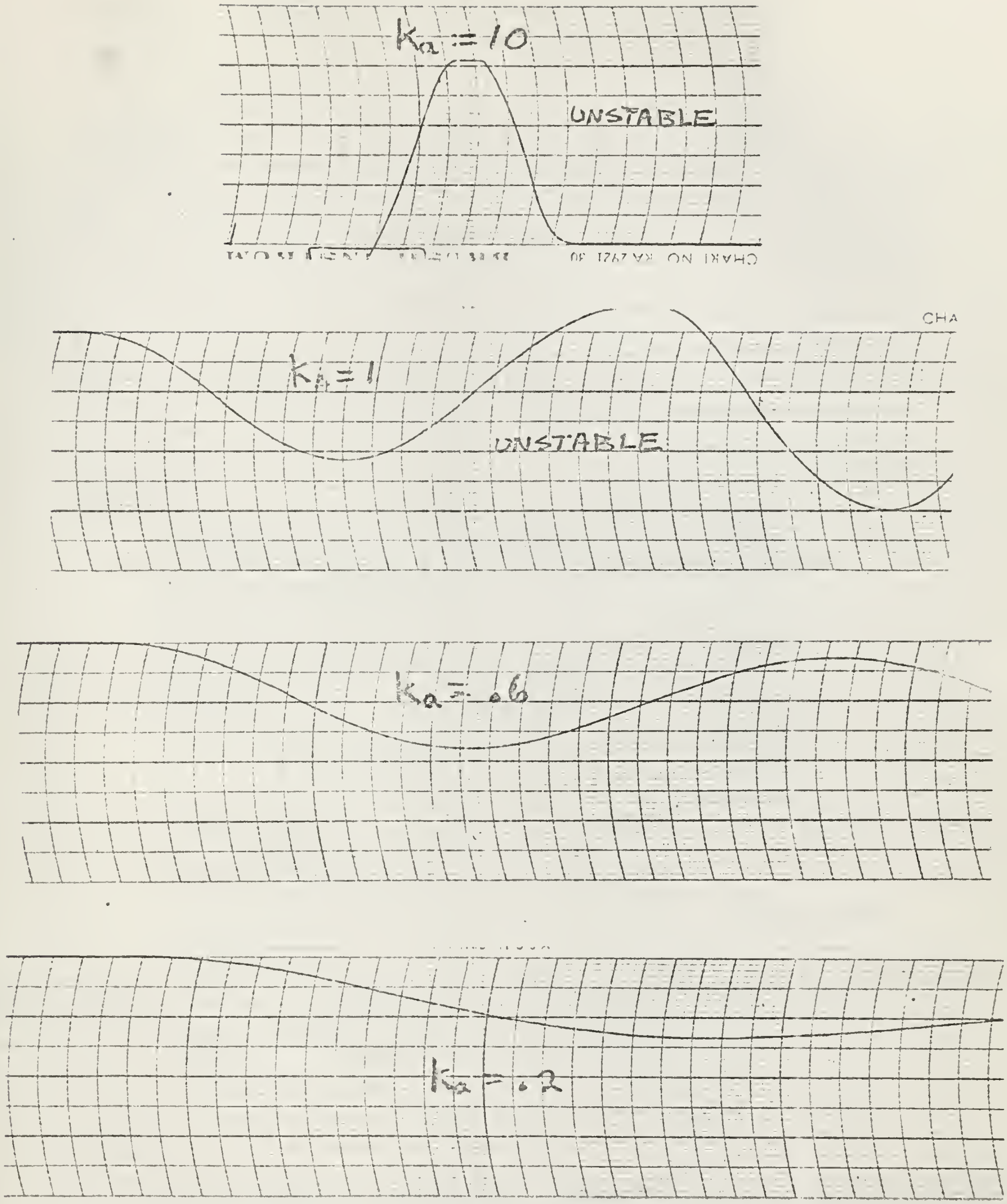


figure D-10



SYSTEM 1100

-UNCOMPENSATED

figure. D-11

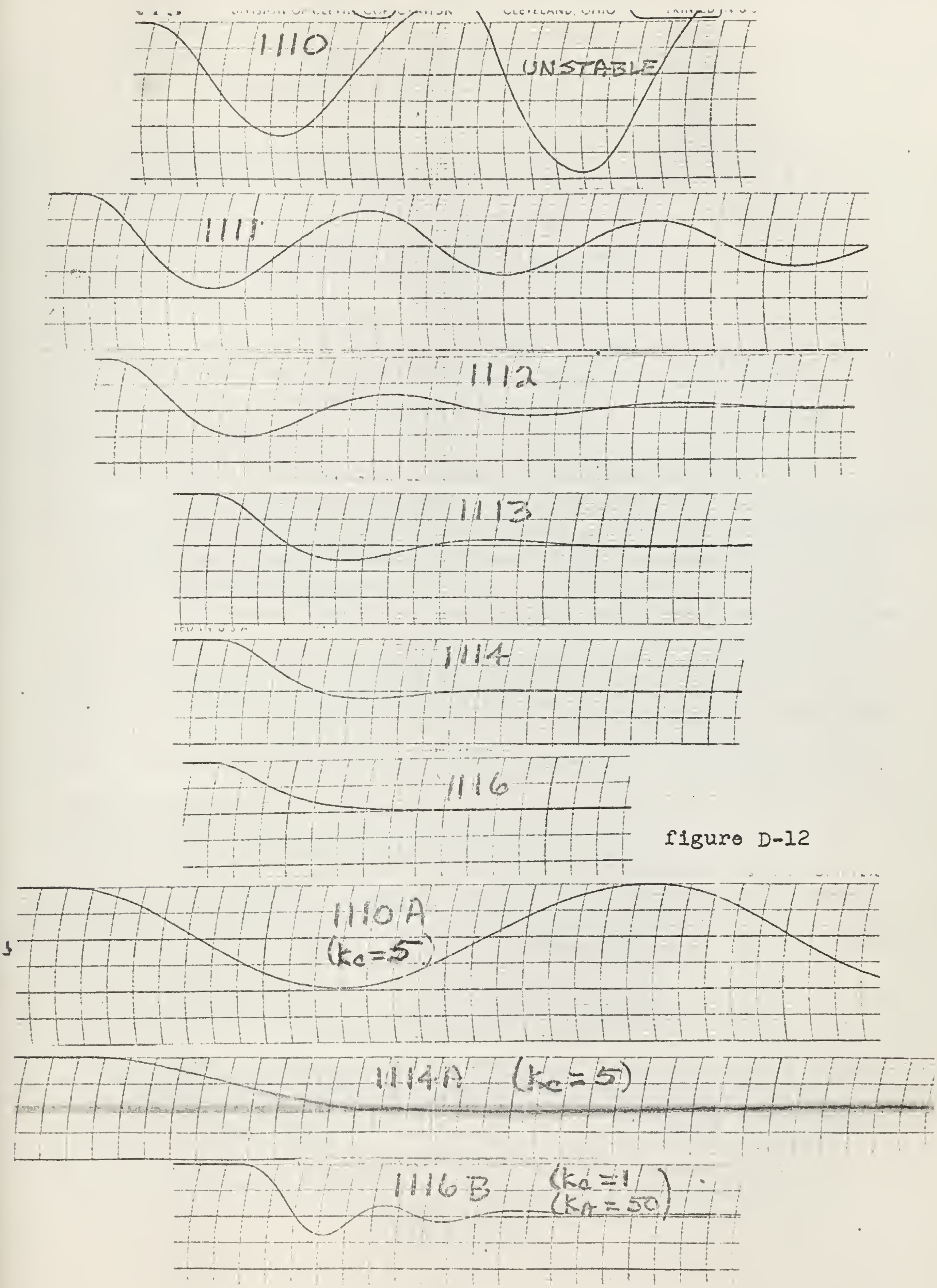

1110
UNSTABLE
1111
1112
1113
1114
1116
1110 A
($k_c=5$)
1114A ($k_c=5$)
1116B
($k_c=1$)
($k_A=50$)

figure D-12

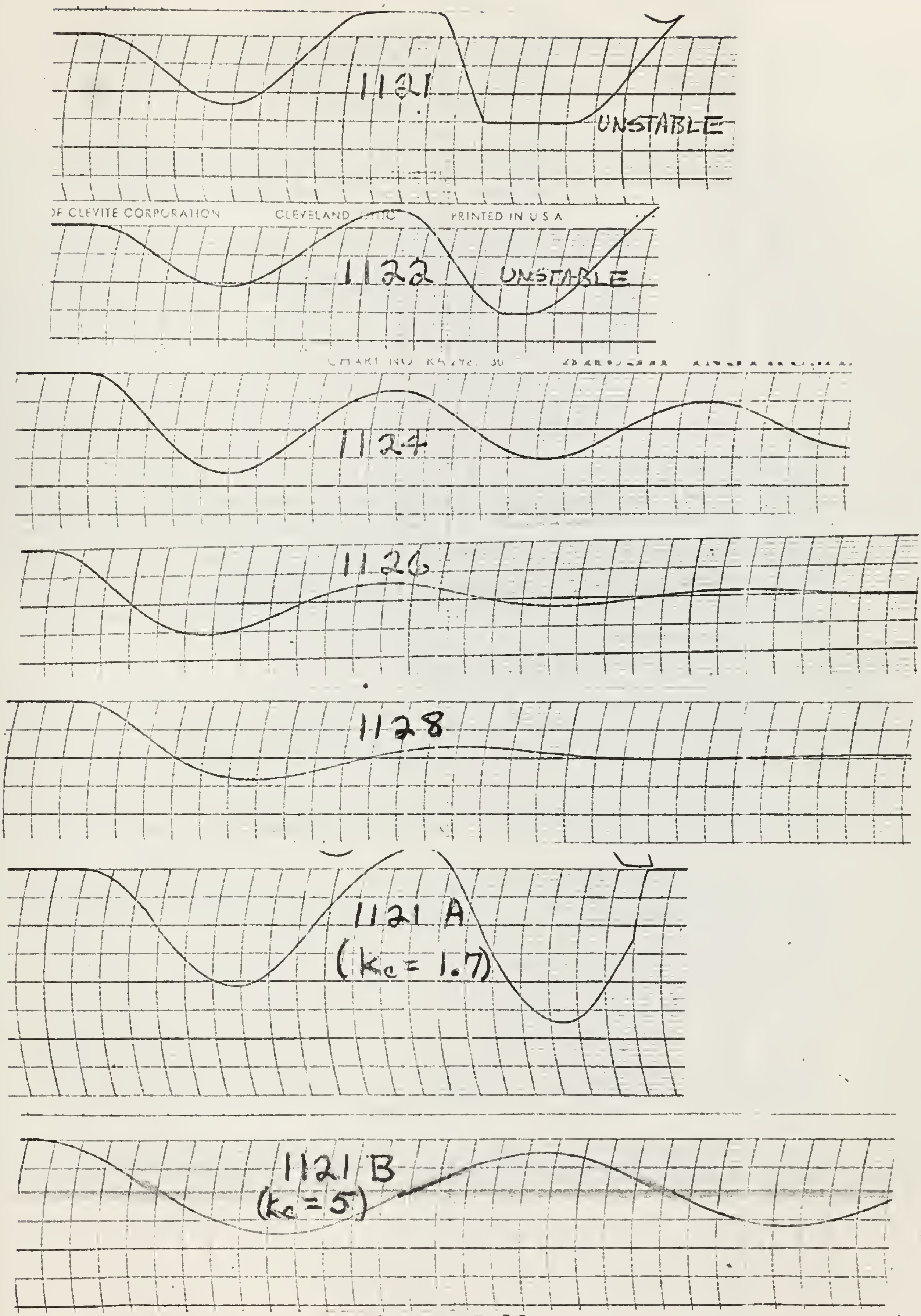

1121
UNSTABLE
CLEVITE CORPORATION
PRINTED IN U S A
1122
UNSTABLE
1124
1126
1128
1121 A
(Kc = 1.7)
1121 B
(Kc = 5)

figure D-13

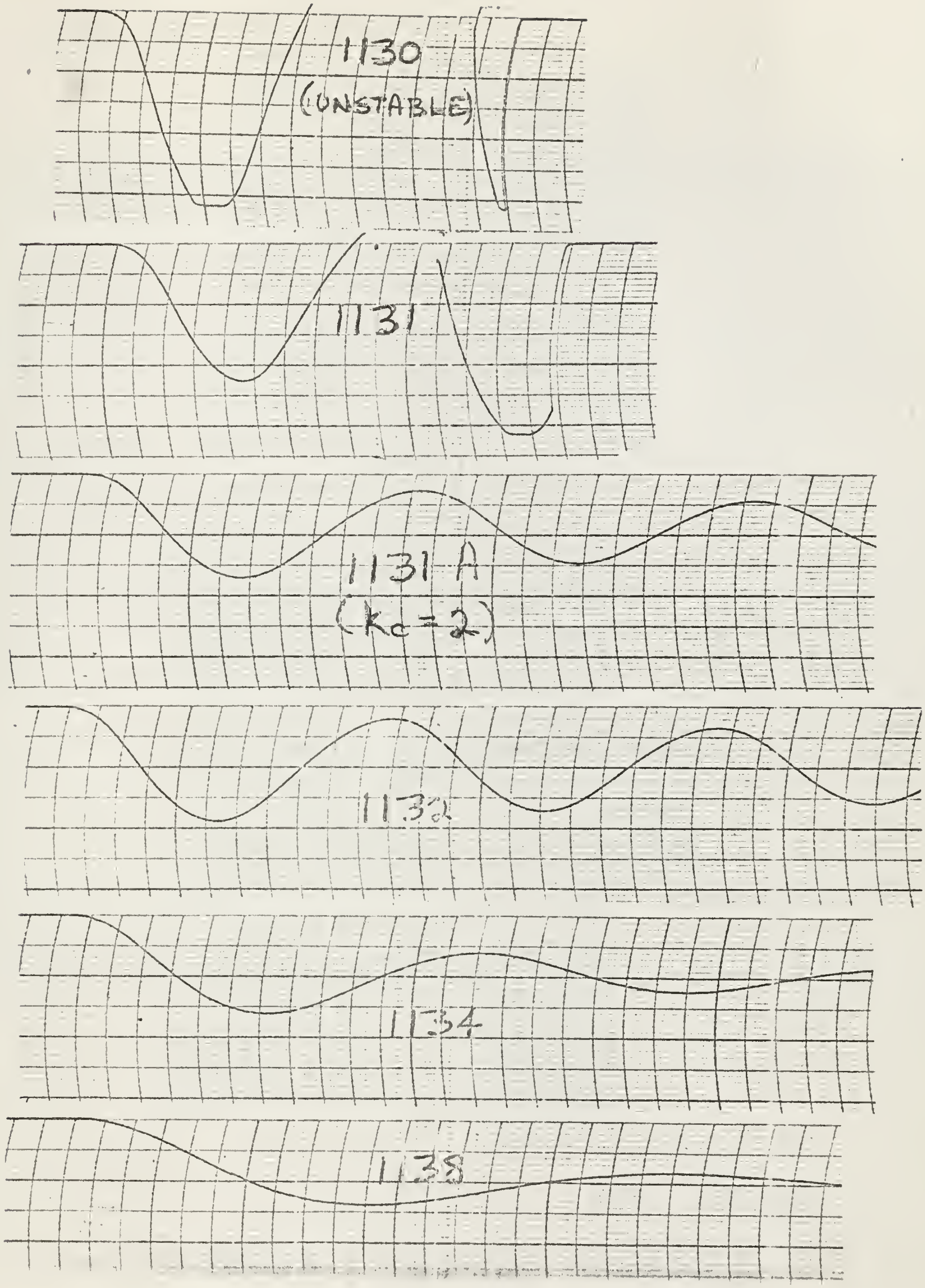
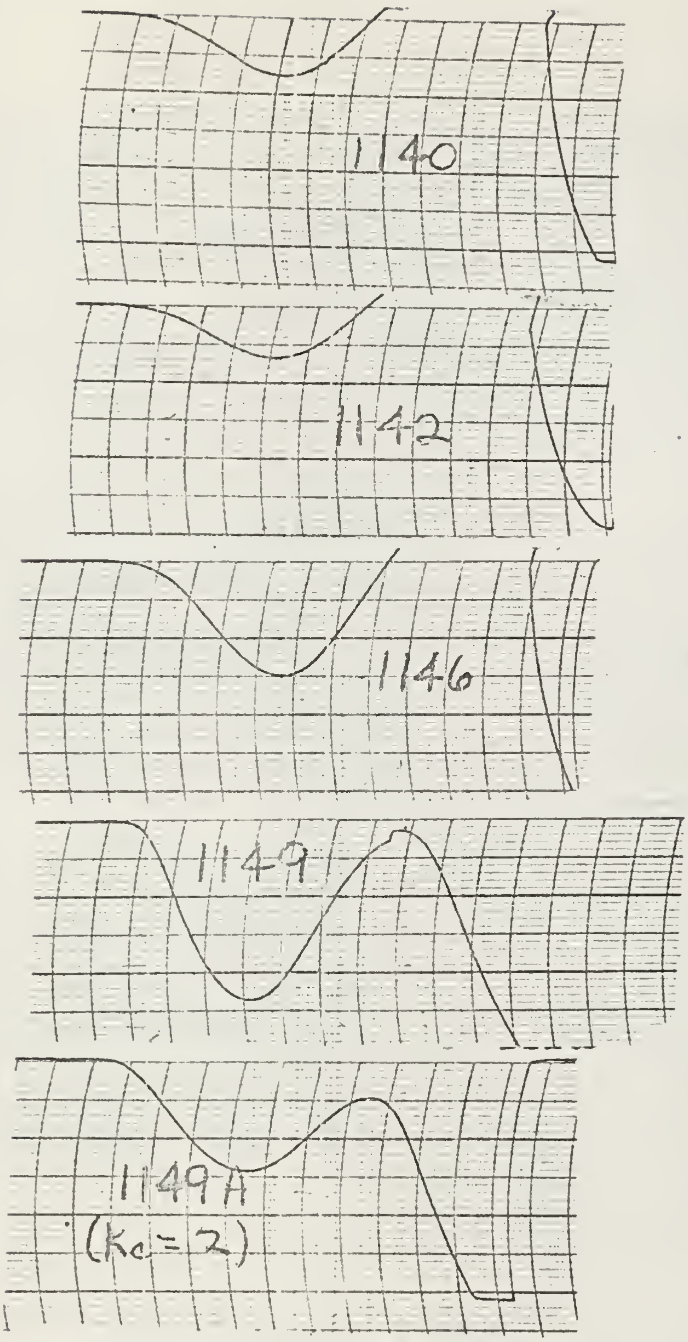


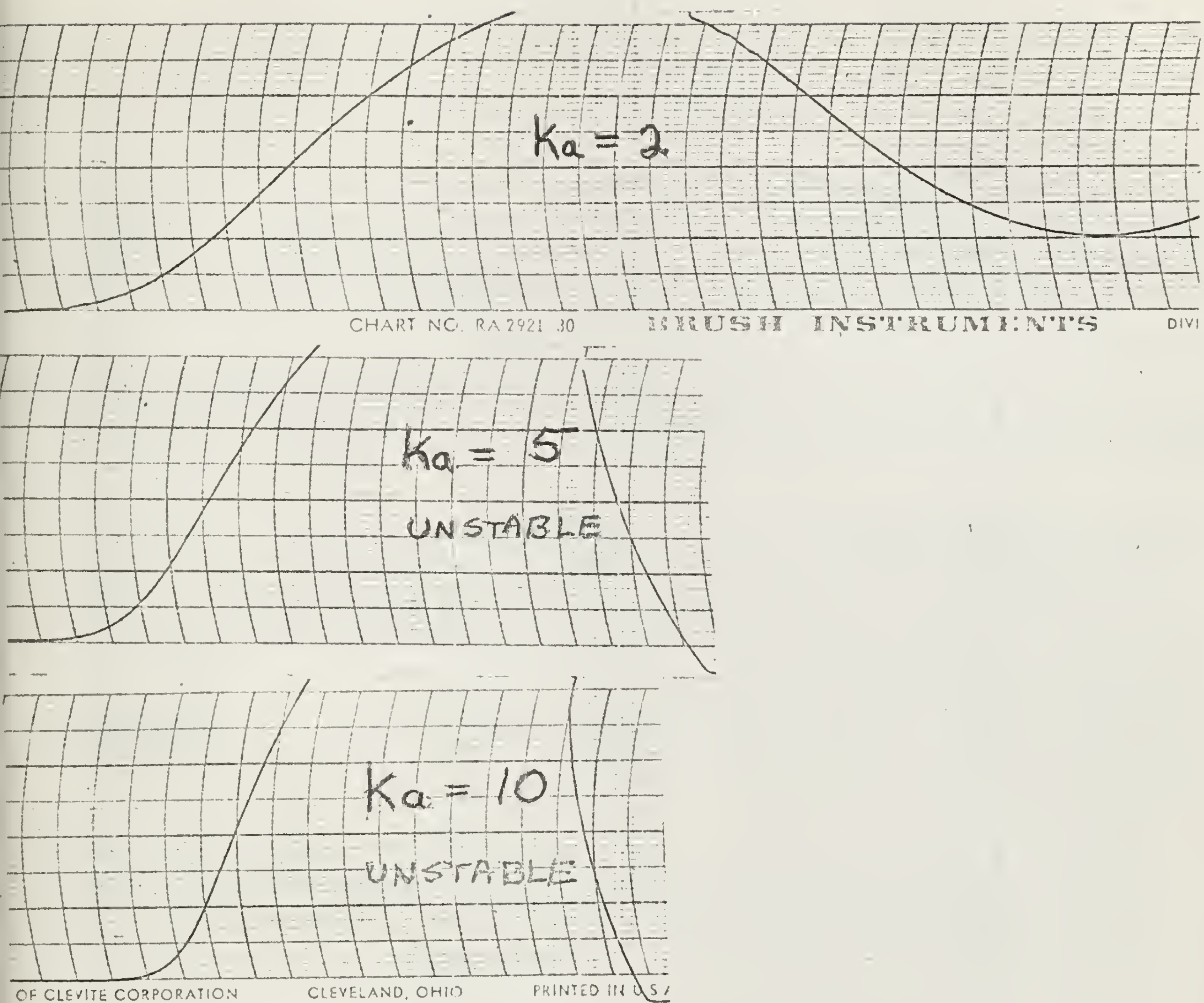
figure D-14



SYSTEMS 11·40 SERIES

NOTE: ALWAYS UNSTABLE

figure D-15



SYSTEM 1800

-UNCOMPENSATED-

figure D-16

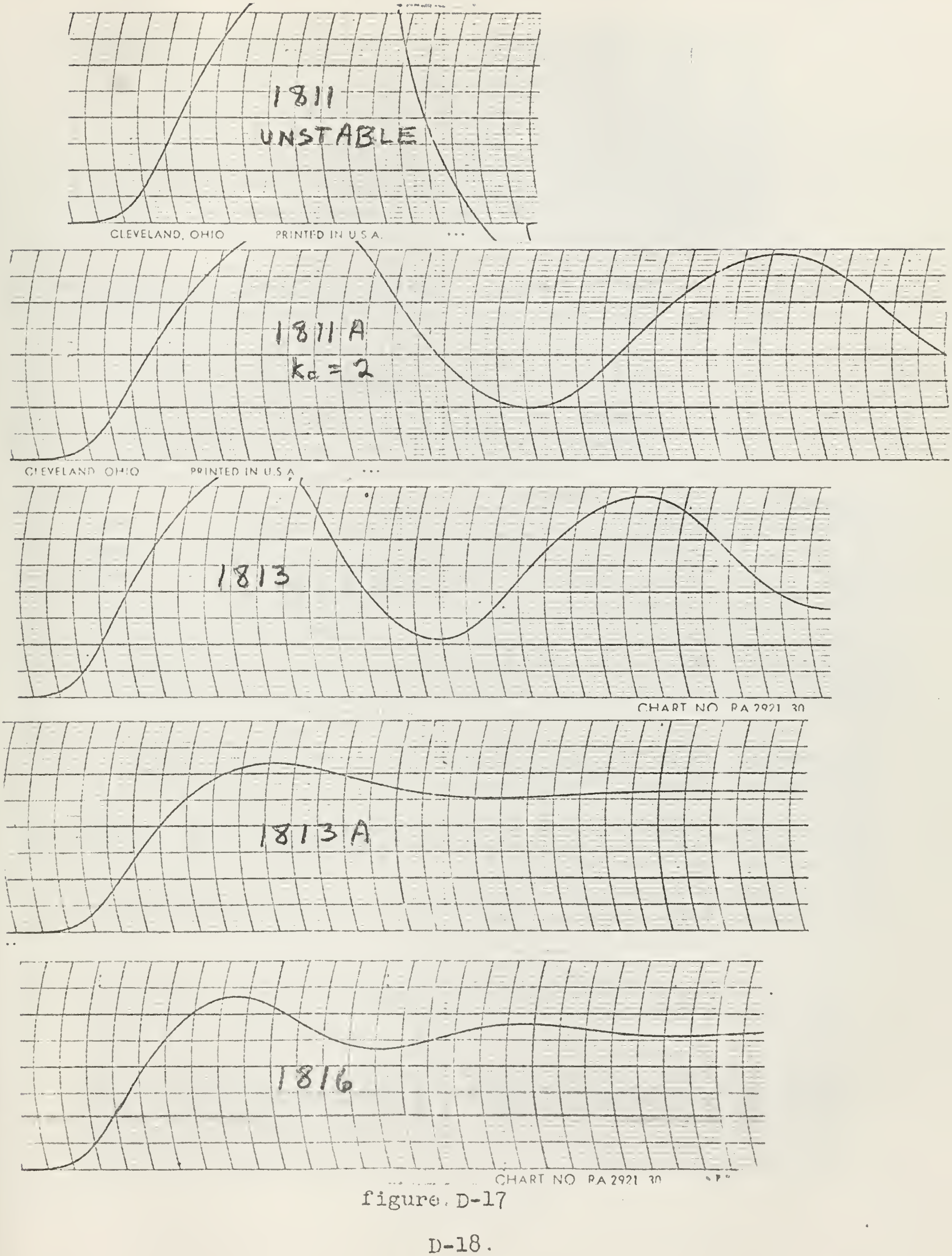

1811
UNSTABLE
CLEVELAND, OHIO
PRINTED IN U.S.A.
1811 A
ka = 2
CLEVELAND OHIO
PRINTED IN U.S.A.
1813
CHART NO. PA 2921 30
1813 A
1816
CHART NO. PA 2921 30

figure. D-17

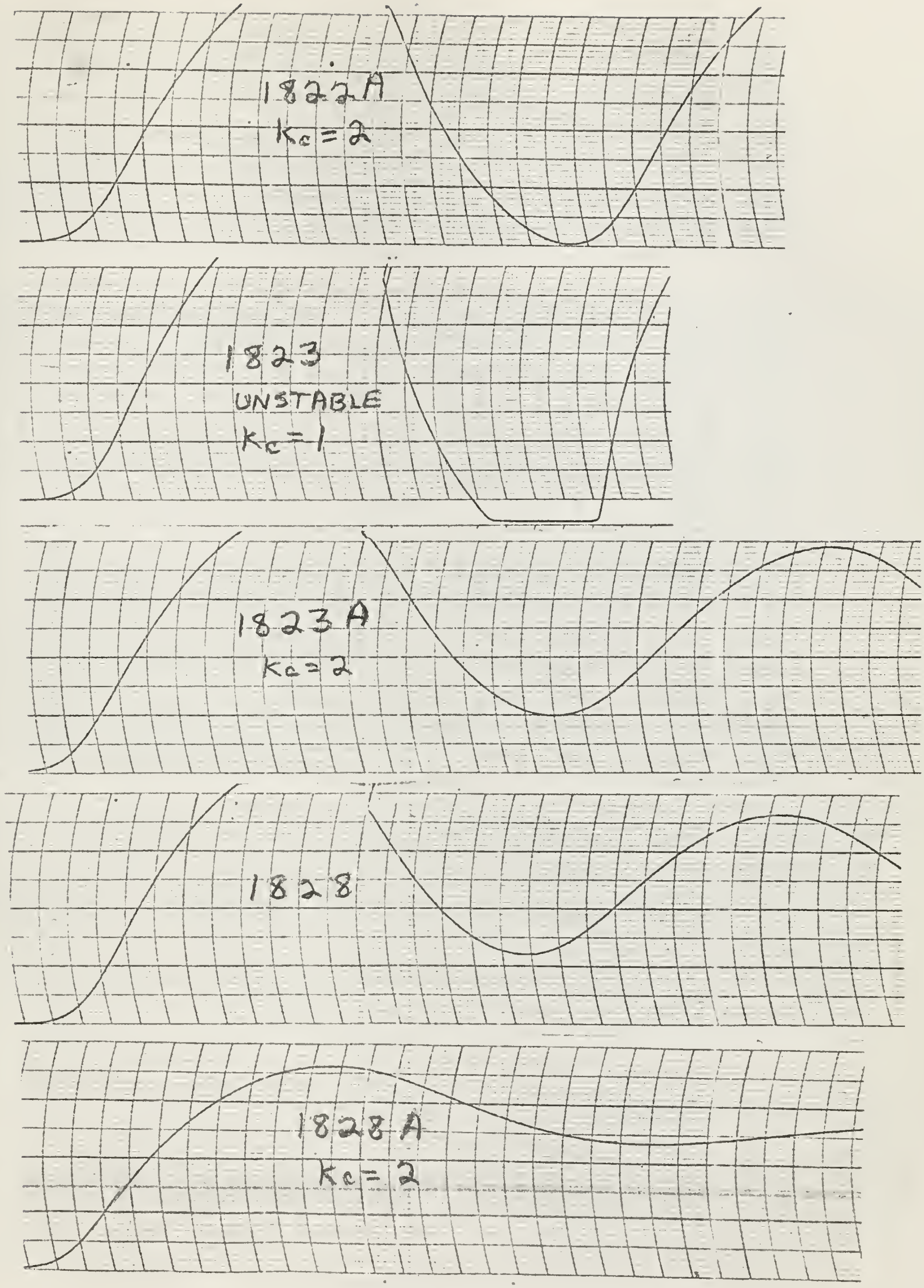


figure D-18

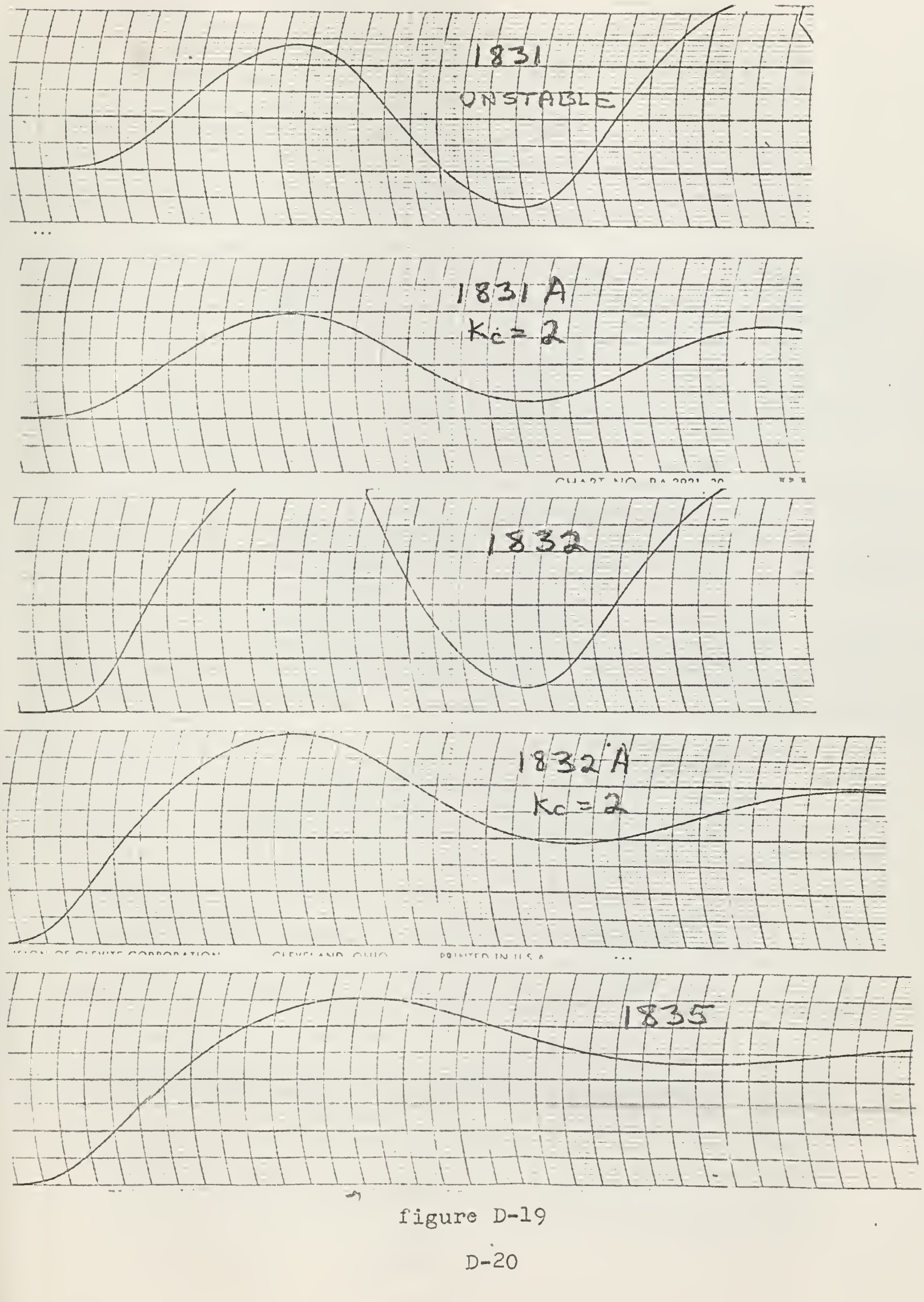


figure D-19

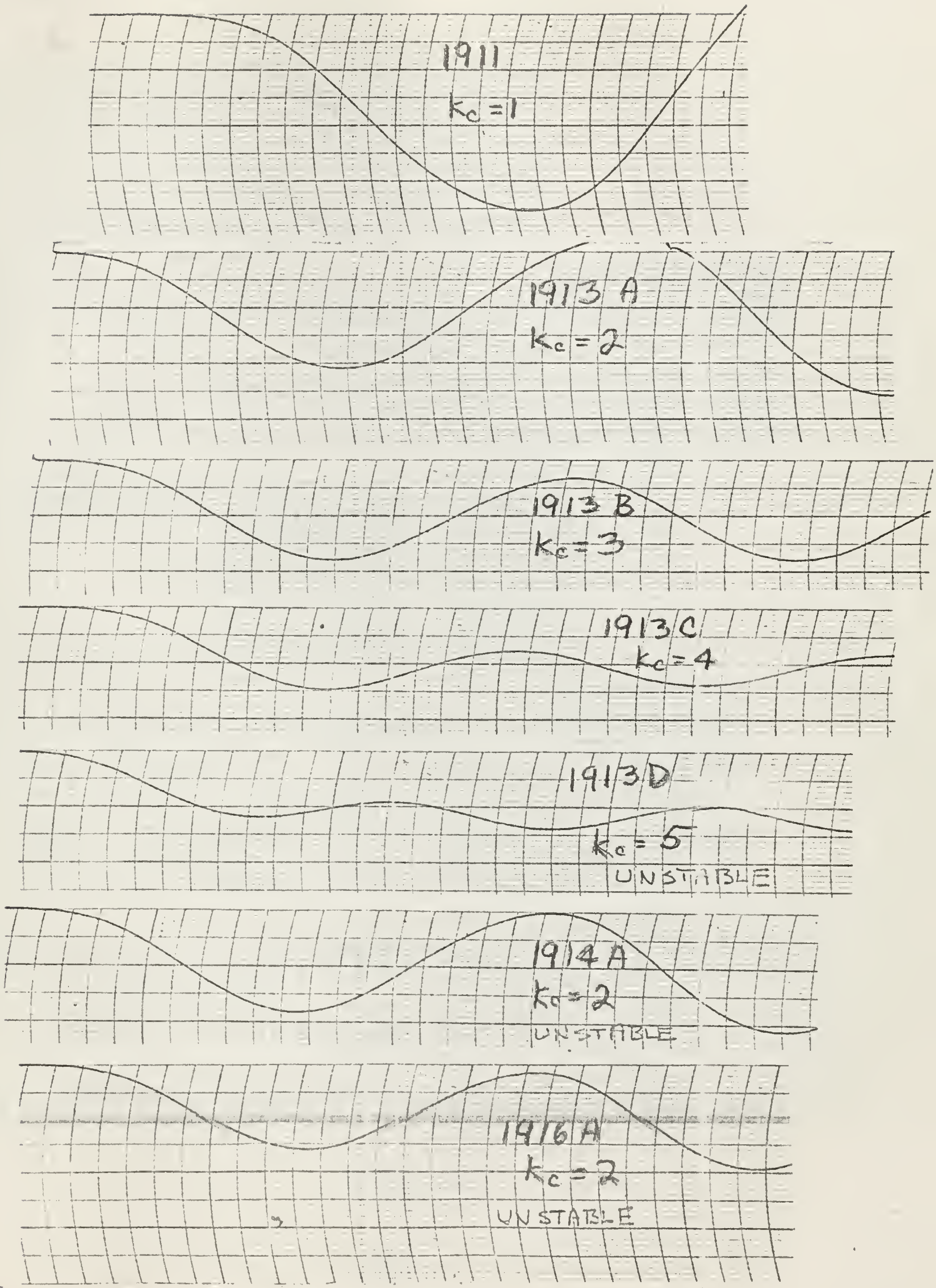

1911
$k_c = 1$
1913 A
$k_c = 2$
1913 B
$k_c = 3$
1913 C
$k_c = 4$
1913 D
$k_c = 5$
UNSTABLE
1914 A
$k_c = 2$
UNSTABLE
1916 A
$k_c = 2$
UNSTABLE

figure D-20

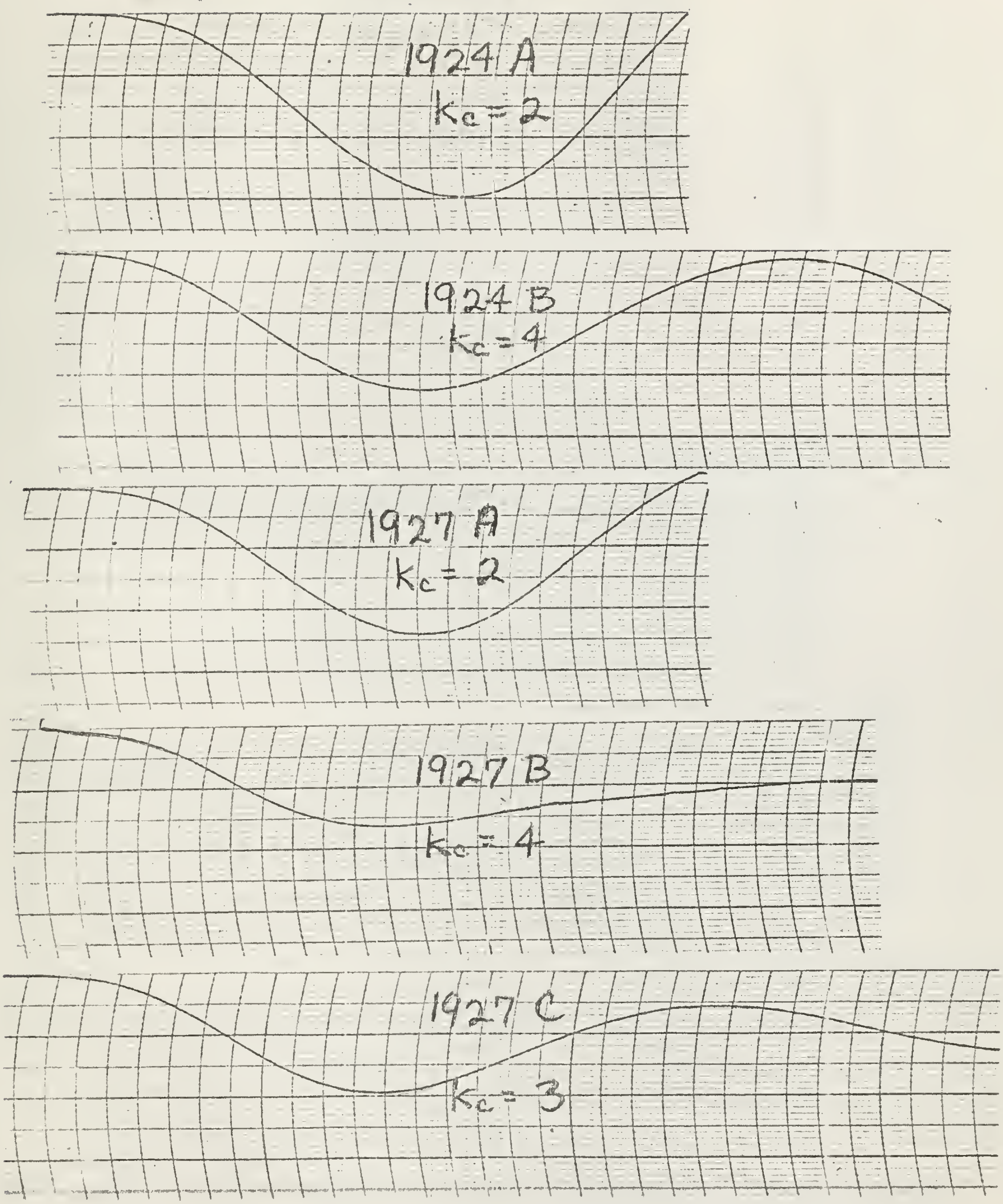


figure D-21

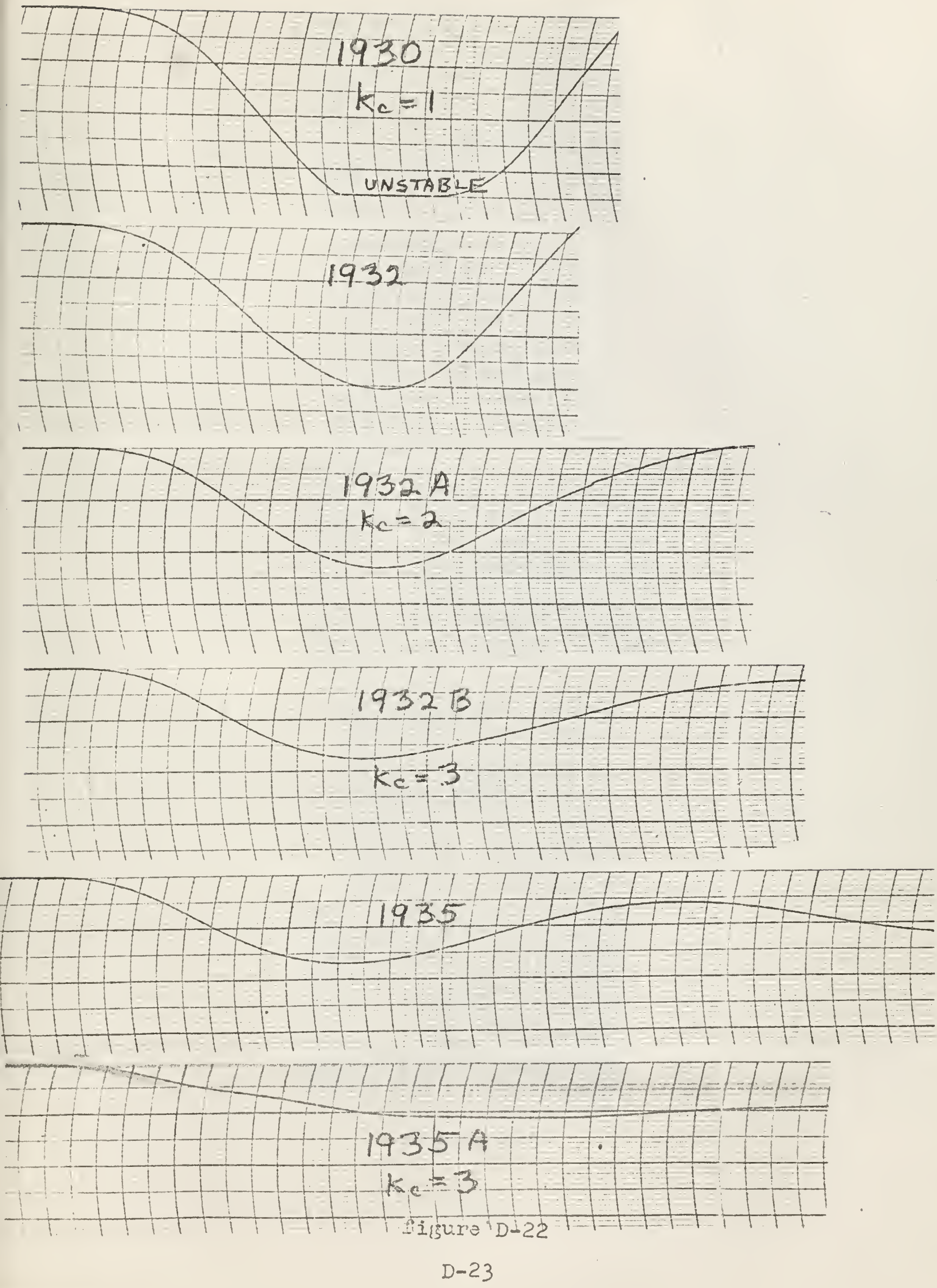


figure D-22

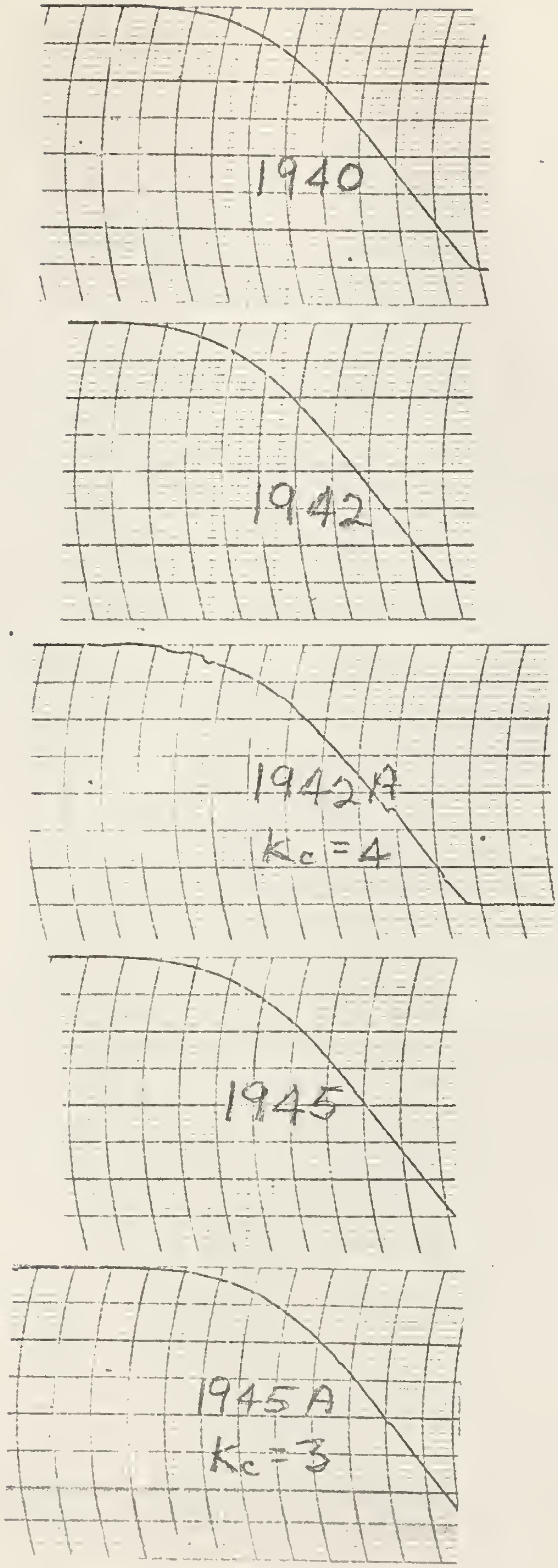


figure D-23

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The application of motor input voltage f
3 2768 002 02947 2
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